COMPARISON OF SW AND TW NON-SYNCHRONOUS ACCELERATING CAVITIES AS USED IN ELECTRON BEAM STORAGE RINGS**

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We relate the parameters of detuned standing wave (SW) and non-synchronous beam travelling wave (TW) accelerating cavities of equivalent equilibrium performance when used to compensate for radiation and parasitic energy losses by electrons circulating in a high energy electron storage ring.

The relationship is expressed in terms of the coupling parameter β and cavity tuning angle ψ of the TW accelerator's equivalent SW system. A given TW cavity corresponds to a standing wave system possessing specific settings of β and ψ. This is shown for the constant impedance TW waveguide, for which β and ψ can be expressed as explicit functions of TW cavity length l, attenuation factor I, RF electric field phase velocity V_P, and shunt impedance r. Coupling parameter β depends additionally on SW cavity shunt impedance R.

The basis we have used for formulating the equivalence of the two systems follows Travelling Wave Cavity Non-Synchronous Beam Loading theory developed by G.A. Loew[1] and Standing Wave Circuit Analysis theory as described by P.B. Wilson[2].

1. Cavity Voltage Vectors.

The cavities are considered to be designed so as to achieve storage ring circulation of an electron beam of velocity $\simeq$ c and average current i_o. The beam consists of a sequence of beam bunches separated in time by $T_B=2\pi/\omega$. The beam current has first harmonic component i (frequency ω), which for bunches of narrow phase width approaches the infinitely-narrow bunch width value $i = 2i_o$.

The total acceleration received by the beam electrons when the cavity fields have reached equilibrium is identical in the equivalent cavities. The effects due to the accelerating field generated by the RF power source and the retarding field induced by the harmonic current i are illustrated respectively by the voltage vector V_{RF} and V_B of Figure 1a.

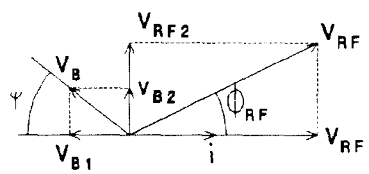

Figure 1a

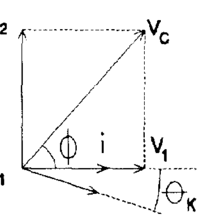

Figure 1b

In Figures 1a and 1b, the phase of the harmonic current vector i also represents the phase of each beam bunch centroid. Figure 1b shows in addition the phase of a bunch electron which is displaced by θ_K relative to the centroid phase.

Since the accelerated electrons are assumed to be relativistic, their relative phases within each bunch do not change during cavity passage. Their phases relative to the cavity electric fields are on the other hand dependent on the cavity tunes. In the "detuned" TW case we consider, the RF field phase velocity is made less than c, and the electrons gain in phase relative to the accelerating wave during passage. In the detuned SW cavity the electrons shift phase relative to the accelerating wave during passage both because of the difference in cavity and beam frequencies, and because of the transit time effect. In either case the integrated energy change of the electron at θ_K due to the RF electric field is

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represented by the projection $V_{RF}Cos(\Phi_{RF}+\theta_K)$, where Φ_{RF} depends on the phase Φ_o relative to the accelerating wave with which the bunch centroid is initially introduced into the cavity.

The effect of the beam-induced cavity field on beam energy is given similarly by the projection $-V_B\cos(\psi-\theta_K)$. The tuning angle ψ is different from zero in the detuned cavities, and has magnitude which depends only on the cavity parameters.

The total energy change V_K of the electron at phase θ_K is given by the projection $V_c\cos(\Phi+\theta_K)$ of the total cavity voltage V_c. In terms of the components V_1 and V_2 of V_c, this is

$$V_K = V_1\cos\theta_K - V_2\sin\theta_K \qquad (1)$$

$$= (V_{RF1}+V_{B1})\cos\theta_K-(V_{RF2}+V_{B2})\sin\theta_K.$$

The detailed dependence of the voltages and phase angles on the TW and SW system parameters provides the connection needed to show the β, ψ equivalence of the two accelerator types. This has been developed using the theories and analyses of references 1 and 2 and is stated without proof in what follows.

2. TW Cavity System

Formulas for V_1 for the constant impedance TW cavity are given in reference 1, which also provides the basis from which V_2 can easily be developed. It is also helpful, in comparing them with their SW cavity counterparts, to write $V_1=V_{RF}+V_{B1}$ and $V_2=V_{RF}+V_{B2}$, thus separating the contributions V_{RF} and V_B due respectively to the RF power source and the electron beam.

Expressed in that way, the components of V_{RF} and V_B are

$$V_{B1} = iV_{B10},\ V_{B2} = iV_{B20}, \qquad (2), (3)$$

$$V_{RF1} = (2IrP_{TW})^{1/2}(C_1\cos\Phi_o + S_1\sin\Phi_o), \qquad (4)$$

$$V_{RF2} = (2IrP_{TW})^{1/2}(C_2\cos\Phi_o + S_2\sin\Phi_o), \qquad (5)$$

Here i is the first harmonic component of the beam current. For sufficiently short bunches the approximation $i=2i_o$ may be used. Shunt impedance r is one-half the "accelerator definition" r_a of shunt impedance usually employed in traveling wave accelerator formulae.

The coefficients in eq's 2 through 5 are;

$$V_{B10} = \alpha r\{\alpha(1-(\delta/I)^2-e^{-Il}[(1-(\delta/I)^2)\cos\delta l \qquad (6)$$

$$- 2(\delta/I)\sin\delta l])-Il\}/I$$

$$V_{B20} = \alpha r\{\alpha(2(\delta/I)+e^{-Il}[((\delta/I)^2-1)\sin\delta l \qquad (7)$$

$$- 2(\delta/I)\cos\delta l])-\delta l\}/I$$

$$C1 = +S2 = \alpha(1+e^{-Il}[(\delta/I)\sin\delta l-\cos\delta l])/I \qquad (8)$$

$$C2 = -S1 = \alpha((\delta/I)-e^{-Il}[(\delta/I)\cos\delta l+\sin\delta l])/I \qquad (9)$$

Parameter δ is the slip rate $\omega(V_P - c)/(V_Pc)$ of the accelerating wave relative to the accelerated electrons, where ω is the beam RF angular frequency and V_P is the phase velocity of the TW accelerating wave; l is the accelerating length of the cavity; $r = r_a/2$ is the cavity shunt impedance per unit length; I is the cavity attenuation factor per unit length; parameter $\alpha = I^2/(I^2 + \delta^2)$.

P_{TW} and phase angle Φ_o appearing in eq's 4 and 5 are respectively the RF power fed to the TW cavity and the injection phase of the beam harmonic current i and the bunch centroids. When the cavity is used to accelerate a stored beam, P_{TW} and Φ_o are together selected to satisfy conditions which permit its optimum capture and stable circulation. Injection of i at Φ_o corresponds to having the beam bunch centroids at the synchronous phase Φ shown in Figure 1.

Stable circulation requires the centroid energy gain $V_{RF1} + V_{B1}$ to be matched to that of the beam circulation loss energy V_{LSS} of the beam. V_{LSS} is specified once the beam energy and ring magnetic bending radii are given. The required value of Φ_o is thus defined in terms of V_{LSS}, power P_{TW} and V_{B1} via eq's. 2 and 4.

3. Equivalent SW System.

The RF-excited and beam-excited TW cavity voltages V_{RF} and V_B have their exact counterparts in the SW system. In the latter case these are expressed in terms of the quality factor Q_o and tuning angle ψ of the cavity, and the system RF coupling parameter β.

V_{RF}, V_B and i are phase-related as shown in figure 1a. As in the TW cavity, ψ and Φ_{RF} designate the phases of the beam-induced and RF accelerating wave voltages relative to the beam current harmonic.

Development of the SW system theory is described in reference 2, where it is shown that

$$V_{RF} = (8\beta P_{SW}R)^{1/2}\cos\psi/(1+\beta), \tag{10}$$

$$V_B = iR\cos\psi/(1+\beta) \tag{11}$$

$$\simeq 2i_oR\cos\psi/(1+\beta) \quad \text{for narrow bunches,}$$

Where

$R = R_a/2$ = SW cavity effective shunt impedance,

Q_o = cavity quality factor,

$\tan\psi = -2Q_o(f-f_o)/(f_o(1+\beta))$,

f_o = frequency to which the cavity is tuned,

f = accelerated beam bunch and RF power source frequency,

i = accelerated beam first harmonic component,

i_o = beam average current.

4. Equivalent ψ, β Formulae.

Equivalent TW and SW systems are understood to be systems which accelerate identical beams so as to cause the beam electrons to experience the same energy and phase change as they circulate in the ring. For this to occur the RF voltages V_{RF} and V_B and their phases relative to that of the beam current in the two cases should be identical.

It is evident from figure 1 that angles ψ, Φ_{RF} and Φ are related to the components of the TW cavity V_{RF} and V_B through

$$\tan\psi = V_{B20}/V_{B10}, \tag{12}$$

$$\tan\Phi_{RF} = V_{RF2}/V_{RF1}, \tag{13}$$

$$\tan\Phi = (V_{RF2}+V_{B2})/(V_{RF1}+V_{B1}). \tag{14}$$

Also $V_{B1} = -V_B\cos\psi$. Substituting for V_{B1} from eq. 2 and for V_B from eq. 11, β is found to be given by

$$\beta+1 = -2R(\cos^2\psi)/V_{B10} \tag{15}$$

$$= -2R(\cos^2(\arctan(V_{B20}/V_{B10}))/V_{B10}$$

Since V_{B20} and V_{B10} depend only on parameters of the TW cavity (see eq's 6, 7), the equivalent CW system coupling parameter β and cavity tuning angle ψ are thus fixed in terms of those parameters and the shunt impedance of the CW cavity serving as the equivalent.

5. RF Power for equivalent performance.

The SW system power level P_{SW} required to produce equivalent beam handling performance in terms of the corresponding TW cavity power P_{TW} is obtained by using eq's 4 through 9, together with 12 and 13, to express V_{RF} as a function of the TW cavity parameters and power. Substituting for V_{RF} in these terms in eq. 10 yields

$$P_{SW}/P_{TW} = \tag{16}$$

$$IrR(C_1^2+C_2^2)(\cos^2(\arctan(V_{B20}/V_{B10}))/\beta(V_{B10})^2$$

References

[1] G.A. Loew, W.W. Hansen Microwave Lab Report #740, Stanford University, 1960.

[2] P.B. Wilson, High Energy Linacs: Applications to Storage Ring Systems and Linear Colliders, SLAC-Pub-2884, February 1982.