Tracking With Space Charge in the Fermilab Booster

S. STAHL and C. ANKENBRANDT
Fermilab*, P.O.Box 500, Batavia, IL 60510

1 Introduction

The tracking program TEAPOT[7] has been adapted to simulate beam behavior in the Fermilab Booster in order to elucidate the causes of observed limitations on beam intensity and brightness. In addition to space charge, the effects of rf, field ramp, finite aperture and gradient errors are included. The evolution of beam intensity and emittance in the initial milliseconds following injection has been examined and compared with the behavior of the real beam. The model has also been used to extrapolate the performance of the Booster following the upgrade of the Linac.

2 The Fermilab Booster

The Fermilab Booster accepts H^- ions from the Fermilab Linac at a kinetic energy of 203 MeV and accelerates them to a kinetic energy of 8 GeV for injection into the Main Ring. The combined-function magnets which constitute the basic linear lattice are part of a resonant circuit whose frequency is 15Hz. Thus, the acceleration process proceeds from injection to extraction in 1/30th of a second with no magnetic field ramp "porch" during capture of 200 MHz Linac bunches into 30 Mhz (h=84) Booster rf buckets. The sources of rf voltage in the Booster are nine pairs of rf cavities placed in some of the six meter straight sections, where the horizontal dispersion is at its minimum value, 1.8 m.

The charge-exchange process used for injection allows beam from the Linac to be injected directly onto the phase space occupied by previously injected beam circulating in the Booster. Typically from one to four turns of beam from the Linac are injected, each turn corresponding to about 5×10^{11} protons. The outcome of this process is a brighter beam than originally emitted from the Linac. However, the injected protons do not continue to occupy the same volume of phase space indefinitely as more protons are injected. At a threshold of about 1.5×10^{12} protons injected into the Booster the emittance of the beam extracted from the machine is observed to increase linearly with increasing charge.[1]

3 The Model

TEAPOT treats all elements as "thin." That is, a delta-function "kick" is delivered to a particle at each element. The particles are propagated from element to element through drift regions. Tracking through the thin element lattice–an approximation to the real lattice–is exact. The program allows the tune and chromaticity to be fit by varying the strength of selected elements. The nominal tunes in the Booster are $\nu_x = 6.81, \nu_y = 6.84$, while the bare chromaticities are approximately $\chi_x = -18, \chi_y = 13$.

Motion in the longitudinal plane is also included in the code. For purposes of the simulation, each pair of rf cavities is lumped into one rf accelerating gap at the center of the drift, making 9 gaps in all. (In the case of the Booster, where the synchrotron tune is as high as 0.1 at injection, representing all rf cavities as one lumped element introduces an artificially high amount of synchro-betatron coupling and subsequent emittance growth.) The phase and voltage of each rf element as a function of time may be independently determined.

Any simulation of the effects of space charge in the Booster should include the physical aperture of the machine, since significant injection losses occur at higher intensities—for example, 70 percent efficiency is typical at 3×10^{12} protons injected. The Booster aperture, inferred from scans performed at injection, is set to ± 4.4 cm in the horizontal plane and ± 2.2 cm vertically.

3.1 Space Charge

The effect of the mutual repulsion of the beam constituents in the simulation is modeled as a series of delta-function kicks delivered at discrete locations around the ring. Cpu requirements preclude the use of a large number of macroparticles to represent the distribution, from which the field is to be calculated. Therefore, the distribution is assumed to be Gaussian, for which an analytic expression for the field is known.[2] The moments of the distribution and a

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conditions	emittance	$\Delta\nu_{max}$	eff
dc beam	9.5 π	1.2	100 %
rf	11 π	0.8	89 %
accel	10 π	1.1	100 %
quad err	12.5 π	0.6	74 %

Table 1: Normalized 95 % emittance, maximum tune shift and transmission after simulating 4×10^{12} protons for 1024 turns in the Booster.

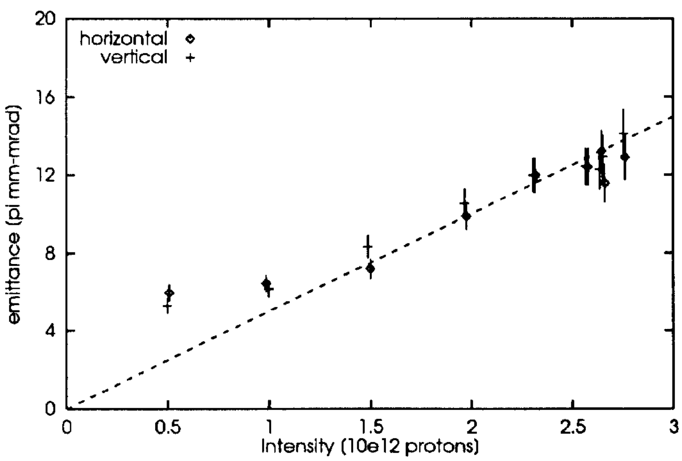

Figure 2: Simulation results for emittance vs. intensity in the Fermilab Booster.

tune shift of 0.5 ± 0.1, which is slightly higher than the experimental result. We conclude that it is primarily the presence of half-integer driving terms in the Booster which limit the achievable phase space density.

5 After the Upgrade

Once an acceptable model is established, it can be used to extrapolate the performance of the machine into a new regime. Following the Linac upgrade now underway, the injection energy of the machine will be raised to 400 MeV. It is argued that the increase in $\beta\gamma^2$ from 0.83 to 1.45 will allow the phase space density to increase by a factor of 1.74, given the same limiting space charge tune shift. Assuming the same fractional error fields though, the observed increase in normalized emittance is somewhat greater. This is due to the fact that at the higher injection energy the beam is no longer as constrained by the aperture. In the emittance vs. intensity plot for the higher energy, shown in Fig. 3, the slope of the fit to the data indicates a limiting space charge tune shift of 0.5. The increase in achievable phase space density is 1.5. This improvement in performance, while not as substantial as that inferred from simply scaling the energy, is still sufficient to meet the requirements of the Main Injector. The Main Injector proposed intensity of 6×10^{10} protons per bunch, which exceeds

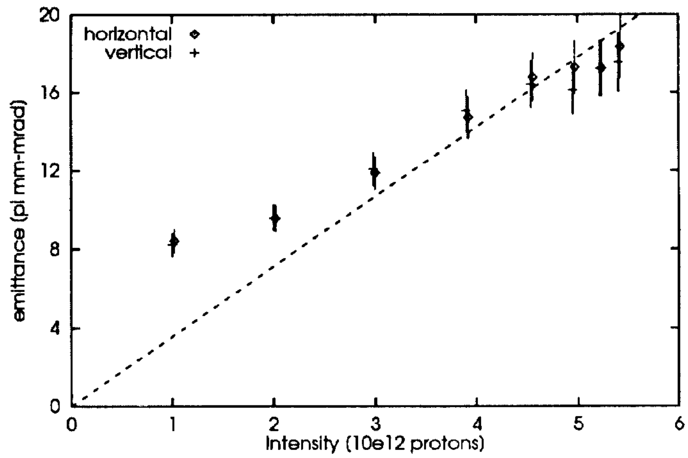

Figure 3: Simulation results for emittance vs. intensity in the Fermilab Booster at an injection energy of 400 MeV.

the current record for intensity in the Booster, appears to be achievable. At that intensity, the normalized transverse emittance is projected to be approximately 17π mm-mrad, which satisfies the Main Injector specification of 20π mm-mrad.

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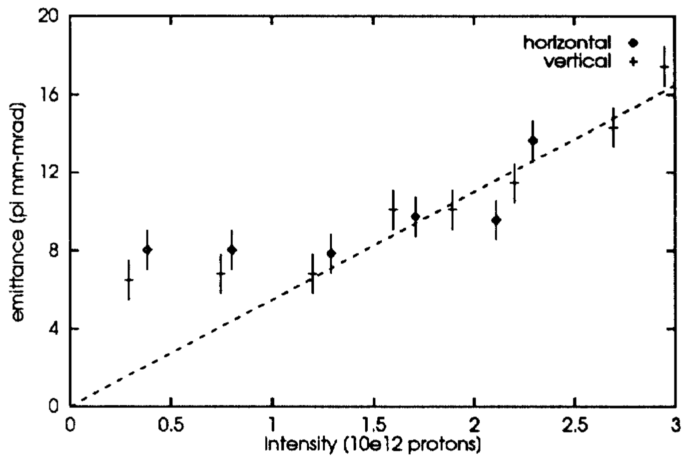

Figure 1: Achievable emittance as a function of intensity in the Booster.

rational function approximation[3] to the exact expression are used to calculate the field.

The number of macro-particles tracked and the number of space-charge kicks per turn are determined empirically by varying these parameters and observing the point at which the same conditions give unacceptably divergent results. The number of macro-particles used in the simulations is 400, and the number of space-charge kicks per turn is 96.[1]

4 Results

The "best" recorded emittances in the Booster as a function of intensity over a period of time appear as plotted in Figure 1. These data were obtained at 8 GeV. The line drawn on the figure suggests that there is a limit to the phase space density of the beam. The best fit to the data, expressed in terms of the maximum space charge tune shift for a round Gaussian beam, indicates a tune shift limit of 0.4 ± 0.1.

Experimental indication that the beam blowup occurs at injection is sparse. The flying wires are not able to resolve beam sizes within a millisecond, so "following" the beam size as it blows up is not possible. However, flying wire data do suggest that the blowup is in the first millisecond or so after injection.

4.1 Simulation

The simulation effort is aimed at replicating and explicating the observed dependence of emittance on intensity. The advantages of simulation are the ability to probe the distribution being tracked in great detail and to turn physical effects on and off.

[1] A typical run of 1024 turns consumes 3 hours of cpu time on an Amdahl 5890. This corresponds to approximately .03 cpu seconds per particle-turn.

Tracking a DC beam in a linear lattice without errors reveals an intrinsic space charge limit at $\nu = 6$. This has previously been observed in simulation by Parzen.[6] It appears to be a space-charge induced structure resonance, and might be called envelope instability.[5] The Booster consists of 24 nominally identical cells. The modulation of the beam by space charge resonates with this structure at $\nu = 6$. The resonance $2\nu_x - 2\nu_y = 0$ strongly couples the two planes, so increases in one emittance eventually are accompanied by increases in the other. However, at a current equivalent to the peak current at 4×10^{12} protons injected this effect raises the 95 % normalized emittance to 9.5 π, which is considerably less than the observed emittance in the Booster at that intensity.

A more realistic model of the Booster is developed by introducing the effects of rf, magnetic field ramp, and magnetic field errors and observing their effects on the final emittance and the maximum obtainable tune shift in the machine. Table 1 summarizes these results for a beam intensity of 4×10^{12}. The "rf" entry lists the results when the rf is turned on and the beam is bunched. The emittance increase is greater than in the dc beam case, and so the maximum space charge tune shift is smaller Presumably this is due to the effect of synchrotron oscillations driving particles across space-charge driven resonances, in particular the $4\nu_x = 24$ resonance.

The introduction of acceleration actually improves the situation somewhat. This is not unexpected, since the effect of an increase in energy is to reduce the space charge tune shift. (During the first 1024 turns, the kinematic factor $\beta\gamma^2$ increases from .83 to 1.08.)

The large tune shifts induced in the beam at high intensity and the nearness of the half-integer make half-integer driving terms in the Booster of particular concern. To investigate the effect of such driving terms, a random Gaussian distribution of quadrupole errors is generated with the standard deviation set to one half of one percent of the nominal quadrupole strength. For the Booster, this introduces a stopband width at $\nu = 6.5$ of approximately .015. (In the absence of correction, the stopband width at the half-integer in the Booster is approximately 0.1.) The result is a significant increase in the emittance. It is to be noted that the maximum tune shift is not simply the distance from the half integer tune to the operating point. Since the nonlinear nature of the space charge force and synchrotron motion of the particles combine to drive different portions of the distribution onto resonance at different times it is perhaps more appropriate to consider the rms tune shift as representative of the beam, as suggested by Machida.[4] However, the situation is complicated by the losses and the width of the stopband—tripling the stopband width depresses the maximum tune shift below 0.4. The net result, including magnet position errors inferred from closed orbit data—which causes a further degradation in efficiency, approximately 10 %—is depicted in Fig. 2. This is to be compared with Fig. 1. The agreement is fairly good. Fits to the data indicate a limiting space charge