

Compensation of RF Transients During Injection into the Collector Ring of the TRIUMF KAON Factory

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Abstract

The Collector ring in the KAON Factory proposed at TRIUMF [1] is designed to accumulate five 3 GeV proton pulses in order to match the time cycles of Booster (50Hz) and Driver (10Hz). The averaged beam current in the ring varies in steps at the injection of each pulse from Booster. RF cavities need to be retuned at every injection to maintain the proper bunching voltage as well as the phase relation between the cavity voltage and the beam current. Since the Collector will be heavily beam-loaded and cavity tuning will be slow compared with jumps in beam current, each injection is expected to cause substantial disturbance to the cavity voltage and to the beam. The disturbance is called an injection transient. This paper studies possible control schemes of the rf system during the injection period in the Collector by using an equivalent circuit model and numerical solutions of the coupled nonlinear differential equations. The simulations are used to suggest suitable values for the fast feedback gain and the peak tuning rate. This paper is an abridged version of a design note [2].

I. EQUATIONS FOR CAVITY VOLTAGE

Equations for cavity voltage response V when the cavity is tuned by varying the inductance L have been given by Wang [3]. We introduce the quantities $\Omega^2 = 1/(LC)$ and $\alpha = 1/(2RC) = \Omega/(2Q)$, where R is shunt resistance and C capacitance. The circuit equation is:

$$2\alpha R [\dot{I} - 2I\dot{\Omega}/\Omega] = V [\Omega^2 - 4\alpha\dot{\Omega}/\Omega] + 2\dot{V} [\alpha - \dot{\Omega}/\Omega] + \ddot{V}.$$

For the Collector, the term $\dot{\Omega}/\Omega$ is negligible for $\dot{\Omega}/2\pi$ less than 36 GHz/sec. Since the largest tuning rate we shall model is 3 GHz/sec, it is clear that the time derivative of Ω can be ignored.

We shall introduce a general feedback I_f which is injected into a summing point along with the generator current I_g . We make a specific assignment to I_f later in the text. Suppose a wave with angular velocity ω will maintain synchronism with the synchronous particle. Then the cavity voltage V , beam image current I_b , generator current I_g and feedback current I_f can be written:

$$\begin{aligned} V &= \mathbf{V} e^{j\omega t} & \mathbf{V} &= V(t) e^{j\psi_v} e^{j\phi_v(t)} \\ I_b &= \mathbf{I}_b e^{j\omega t} & \mathbf{I}_b &= I_b(t) e^{j\psi_b} e^{j\phi_b(t)} \\ I_g &= \mathbf{I}_g e^{j\omega t} & \mathbf{I}_g &= I_g(t) e^{j\psi_g} e^{j\phi_g(t)} \\ I_f &= \mathbf{I}_f e^{j\omega t} & \mathbf{I}_f &= I_f(t) e^{j\psi_f} e^{j\phi_f(t)}. \end{aligned}$$

The total current driving the cavity is $I_t = I_g + I_b + I_f = \mathbf{I}_t e^{j\omega t}$. The evolution equation for the phasors is:

$$\ddot{\mathbf{V}} + 2(\alpha + j\omega)\dot{\mathbf{V}} + (\Omega^2 - \omega^2 + 2j\alpha\omega)\mathbf{V} = 2\alpha R(\dot{\mathbf{I}}_t + j\omega\mathbf{I}_t). \quad (1)$$

A. Steady State Conditions

In the steady state $\phi_v = \phi_b = \phi_g = \phi_f \equiv 0$ and the amplitudes are V^0, I_b^0, I_g^0, I_f^0 . We define a reference point $\psi_v = 0$ and use the minimum power condition $\psi_g = 0$. Below transition energy, $\psi_b = -(\pi/2 + \mu_b)$ where μ_b is the synchronous phase angle. We can now solve (1) for the resonance frequency Ω and the generator current I_g^0 .

$$\tan \Psi_0 \equiv \frac{(\Omega^2 - \omega^2)}{2\alpha\omega} = \frac{I_b^0 \cos \mu_b - I_f^0 \sin \psi_f}{(V^0/R)} \quad (2)$$

$$I_g^0 = (V^0/R) + I_b^0 \sin \mu_b - I_f^0 \cos \psi_f. \quad (3)$$

Ψ_0 is called the tuning angle and is positive so $\Omega > \omega$ when below transition energy. Evidently the tuning conditions will not change when feedback is applied if $\psi_f \equiv 0$, and this is the condition we adopt.

B. Non-Steady State

We assume $V(t)$ is slowly varying so \ddot{V} can be neglected compared with $j\omega\dot{V}$. We suppose the conventional loops which control the phase and amplitude of the generator are slow so that \dot{I}_g/I_g and $\dot{\phi}_g$ are small compared with ω . \dot{I}_b and $\dot{\phi}_b$ will be slowly varying since the synchrotron frequency ($\omega_s/2\pi$) is much less than the radio frequency (i.e. $\omega_s \ll \omega$). Hence these quantities can be neglected. The dynamics of the feedback have not yet been specified so we retain all terms in I_f, ϕ_f . Since $\alpha/\omega \approx 1/(2Q)$ terms in α/ω can be neglected compared with unity when $Q \gg 1$.

Suppose we set $\phi_f = \phi_v$. This has the effect of unmixing the feedback, at least to first order. The voltage amplitude is controlled by I_f and the voltage phase is basically independent of I_f . Further, let us suppose that $I_f = -H \times (V/R)$. This gives a feedback similar to that described by Lee [4]. Finally, the cavity equations with local proportional feedback are:

$$\dot{V}/\alpha = -V(1+H) - VH \times (\dot{\phi}_v/\omega) + R[I_g \cos(\phi_g - \phi_v) + I_b \sin(\phi_b - \phi_v - \mu_b)] \quad (4)$$

$$\dot{\phi}_v/\alpha = \tan[\Psi(t)] + (H/\omega)(\dot{V}/V) + (R/V)[I_g \sin(\phi_g - \phi_v) - I_b \cos(\phi_b - \phi_v - \mu_b)] \quad (5)$$

Let $I^0 = V^0/R$. Along with the assignment $I_f = -HV/R$ goes the steady-state condition $I_g^0 = I^0(1+H) + I_b^0 \sin \mu_b$.

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II. BEAM EQUATIONS

We consider rigid dipole oscillations of the beam. The synchronous energy is E_s , and the beam energy is E_b . Let $\Delta E = E_b - E_s$. The rate of energy change is:

$$d/dt(\Delta E/\omega) = e[V(t) \sin(\mu_b + \phi_v - \phi_b) - V^0 \sin \mu_b]/2\pi h. \quad (6)$$

Here e is the electron charge. Let $\eta_s = (1/\gamma_i^2 - 1/\gamma_s^2)$ be the slip-factor. The phase advance equation is:

$$d\phi_b/dt = |\eta_s| \omega (\Delta E) / (\beta_s^2 E_s). \quad (7)$$

III. PHASE AND RADIAL LOOPS

The equations above (4-7) are supplemented by those for the analogue loops which control the generator current. For instance $d\phi_g/dt = \Delta\omega_g(t)$. The frequency deviate is controlled by the radial loop (dimensionless gain K_r) and the phase loop (gain K_p per second) according to:

$$\tau_p \times \frac{d}{dt} \Delta\omega_g = \Delta\omega_g + 2K_p(\phi_b - \phi_v) - \frac{|\eta_s| \omega}{\beta_s^2 E_s} K_r \Delta E. \quad (8)$$

This equation defines the gains and their units. The mutual loop time constant is τ_p . The phase loop sets the damping rate and the radial loop alters the coherent frequency.

IV. RESULTS

Equations (4-8) were solved numerically for a variety of boundary conditions. The simulation results, $\phi_b(t) - \phi_v(t)$, are represented graphically. The coordinate axes are defined as follows. The abscissa is the number of synchrotron oscillations for zero beam loading and zero radial loop gain. The ordinate for phase is degrees, assuming a bucket of $\pm 180^\circ$ for zero synchronous phase angle. The figures are supplemented by a commentary.

A. Machine Parameters

We now consider the effect of injection transients in the KAON Factory Collector ring. The injection beam phase angle is $\phi_b = 0$, and the synchrotron frequency is 7 kHz. The quality factor is $Q = 5000$ and $R/Q = 100$ ohm at the fundamental resonance. The harmonic number of the machine is $h = 225$. The radio-frequency is $\omega/2\pi = 60$ MHz. The natural cavity damping time is $\tau_c = 1/\alpha = 26$ μ s. The cavity voltage is 150 kV, implying the steady-state drive current is $I^0 = 0.3$ Amp. When all five batches are present, the beam current component at the radio frequency is $I_b^0 = 6$ A. Hence the peak beam loading ratio is $I_b^0/I^0 = 20$, and the incremental loading per batch is $\Delta I_b^0/I^0 = 4$.

B. Injection Transients and Compensation

Figure 1 shows the case of one batch already in the ring, and the arrival of a fresh batch. There is no local feedback, and no phase/radial control. The cavity starts to retune to the new equilibrium value immediately after the injection

of the second batch. The incremental detuning is 30 kHz per batch, and the tuning rate is 150 MHz/sec. The arrival of the new batch is such a large perturbation that the coherent phase motion is unbounded. Further, stability cannot be restored by simple-minded phase/radial or tuning loops.

One possibility, figure 2, is to let the peak tuning rate increase so that the cavity comes back on tune well before one phase-oscillation of the beam. Unfortunately, the minimum value to restore bounded phase-oscillations is outrageous: 3 GHz/sec and so this concept is impractical. The phase/radial loop is disabled.

C. Voltage Proportional Feedback

Figure 3 shows the effect of enabling the local proportional feedback with gain set at $H = 100$. In other respects the parameters are as for figure 1. The feedback reduces the cavity phase (in radian) and relative amplitude errors to a few parts per 1000. The synchrotron motion is bounded ($\phi_b - \phi_v = \pm 2^\circ$) but *undamped*.

Figure 4, shows the effect of enabling the phase loop with critical gain $K_p = 2.333$ per cavity damping time, and $K_r = 0$. The phase-loop time constant is $\tau_p = 50$ μ s. The phase difference ($\phi_b - \phi_v$) is damped to $\pm 0.1^\circ$ in six phase oscillations, and the damage to the beam is negligible. The frequency modulation depth is ± 10 kHz. Reducing the time constant τ_p or increasing the gain K_p will reduce the oscillations still further.

The fast feedback gain $H = 100$ far exceeds that required to produce a large coherent bucket, a value of $H = 25$ would have been quite adequate. However, the phase oscillations would be four times larger ($\pm 8^\circ$) and this is not acceptable.

For the case of figure 4, the transient persists long after the cavity has retuned; and this suggests that a slower tuning rate might still give acceptable results. This is confirmed: in figure 5 the tuning rate is only 16 MHz/sec. The phase angles ϕ_b and ϕ_v deviate from their nominal zero values for as long as the cavity takes to retune, but the phase difference $\phi_b - \phi_v$ (which is what counts) damps to zero just as quickly as when the tuning rate is 150 MHz/sec. The use of 16 MHz/sec is only for example's sake, and is not meant to suggest the optimum. Ideally, the tuning time and phase oscillation damping time should be similar, suggesting an optimum tuning rate of ≈ 30 MHz/sec.

The effectiveness of the local proportional feedback in reducing the *amplitude* error of the cavity voltage diminishes¹ with the amount of beam current already circulating before arrival of a fresh batch. Consequently, when the fifth batch is injected the relative error is 3 times as great as when the second batch is injected. Since $\Delta V/V_0$ is less than 1%, this is not a serious problem and could be rectified by increasing the feedback gain to the delay limited value of $H = 200$.

¹The control over ϕ_v is not so severely compromised.

V. CONCLUSION

The compensation of injection transients in the Collector provides strong motivation to use local proportional feedback with an open loop gain in excess of 100. When this is done, a tuning rate of 16 MHz/sec is adequate for injection transient compensation, as demonstrated in figures 4,5.

It is seen that fast feedback with large gain is far more effective than fast tuning in reducing the injection transients. A large fast feedback gain has already been recommended² to reduce periodic transients (due to revolution harmonics close to the radio frequency) and we see that reduction of injection transients is another reason for pressing toward the value $H = 200$. Note, however, that the feedback bandwidth³ scales with the gain; hence while $H = 100$ needs 1.2 MHz bandwidth, $H = 200$ requires 2.4 MHz. Further, the power tube must be fully capable of sourcing the quadrature current demanded by the fast feedback during the 1.4 ms required for tuning.

Care is required in choosing the gains and time constants of these loops. A first evaluation suggests a phase-loop bandwidth in excess of 20 kHz is required, (nearly 3 times the synchrotron frequency) when the open loop gains are $K_p = 45$ kHz and $K_r \leq 1$. However, more sophisticated models with PID controllers might well lead to different conclusions for the loop parameters.

VI. REFERENCES

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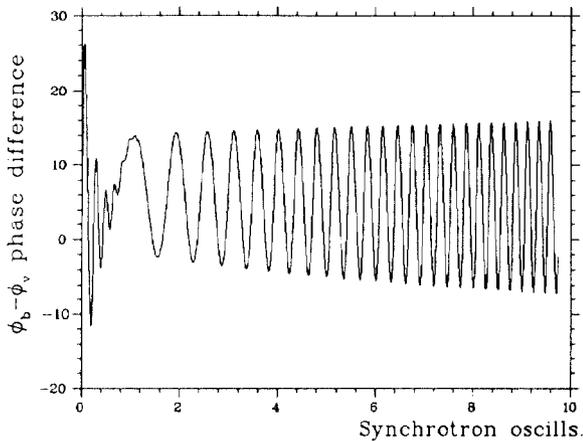


Fig.1: No transient compensation.

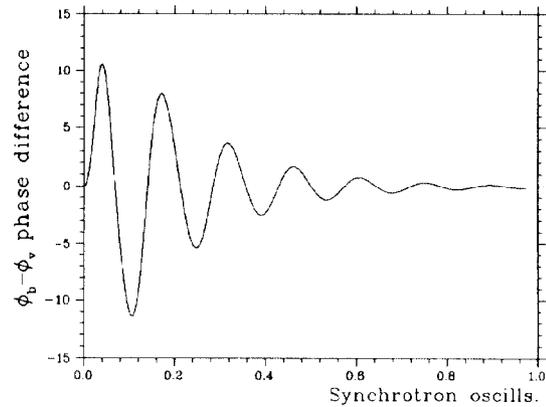


Fig.2: Compensation by fast tuning.

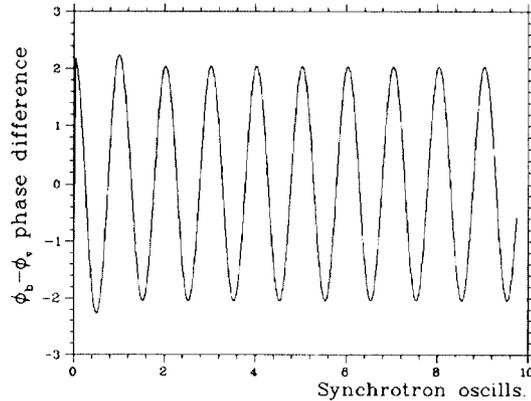


Fig.3: Proportional voltage feedback.

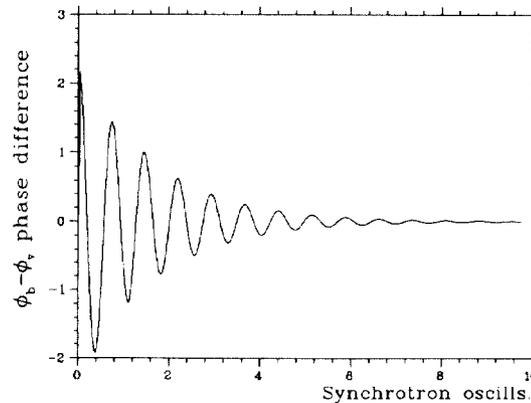


Fig.4: Voltage feedback with beam-phase loop.

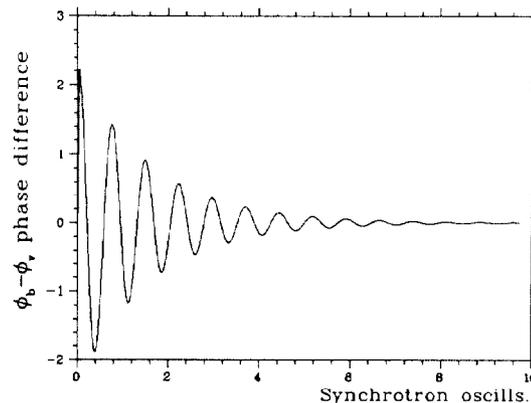


Fig.5: Voltage feedback with slow cavity tuning.

²See section 4.2.5 of reference [1].

³The bandwidth is H times the natural cavity bandwidth.