THE BEAM DYNAMICS STUDY IN A COMPACT SYNCHROTRON

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Abstract

A new computer code has been developed for compact synchrotron design. Based on a numerical integration of the exact equations of motion, the systematic beam dynamics study is carried out, with either computed or measured magnet field including magnet edges. Significant discrepancies of machine parameters reveal some unique features of the beam dynamics in small synchrotrons. Interactive procedures between the beam dynamics predictions on the one hand and the magnet design on the other are then explored so that cost-effective design can be achieved with minimum effort.

1. Introduction

The worldwide effort to design and construct a compact electron storage ring as source of synchrotron radiation for X-ray lithography has become considerably more intense during the past few years. The design objective here should be the minimization of machine size and complexity while maintaining the optical source quality of the beam. In such machines, edge fields often comprise 20% to 50% of the total machine length. The bending angle of each magnet ranges from 45° to 180° instead of at most a few degrees which is common in high energy synchrotrons. The fringe multipole field of the magnets will undoubtedly play a nontrivial role in determining basic machine parameters. Therefore, the conventional treatment for simulating particle motion in synchrotron, which uses the isomagnetic approximation plus thin lens kicks (both linear and nonlinear), no longer accurately models the closed orbit of the machine.

A new scheme has been implemented in our study of the beam dynamics for a compact synchrotron design at the Texas Accelerator Center (TAC). Based on the exact integration of the particle orbit, all relevant lattice functions and beam parameters can be obtained with and without the fringe field. One of the goals of this project is to explore the feasibility of interactive procedures between the beam dynamics predictions on the one hand and the magnet design and field measurements on the other so that cost-effective design can be achieved with minimum effort.

2. The Computer Code — COMSYN

An improved 4th order Runge-Kutta method is used to integrate the exact equations of motion in a rectangular coordinate system. With time as an independent variable, the equations of motion have the following form:

\[ \ddot{x} = \frac{q}{m} (V_x + B_x \dot{y} - B_y \dot{z}) \]
\[ \ddot{y} = \frac{q}{m} (V_y + B_y \dot{z} - B_z \dot{x}) \]
\[ \ddot{z} = \frac{q}{m} (V_z + B_z \dot{x} - B_x \dot{y}) \]

Here, we keep the electric field components in the equations so that this formalism can be used for a future study of the accelerating effect on beam dynamics in the compact synchrotron. Since these equations have the simplest form, the CPU time is significantly reduced in the beam tracking. The choice of time as an independent variable also allows us to avoid the finite difficulty in limiting the slope in the cartesian trajectory if one integrates over the entire 180° bend, as noted by Moser.

The accuracy of the integration was checked by tracking a reference particle many turns in the synchrotron. The maximum departure of the particle track from the theoretical ideal orbit was within 2 x 10^-11 m with 500 integration steps in each superperiod. It is good enough for our design study at present.

This program has been applied to the TAC compact synchrotron design study, and the results are given in the following sections.

3. Calculations of The Lattice Functions with Isomagnetic Field

![Fig. 1. Lattice function distribution along reference orbit over a half of superperiod. The curves are computed by COMSYN. The stars indicate the value of the lattice functions at each end of the magnet elements computed by DIMAD.](image)

Five particles with the different initial coordinates are tracked over one superperiod in transverse phase...
space. Based on the simulation all of the lattice functions ($\beta$-function, $\alpha$-function, $\gamma$-function, tune, natural chromaticity) can be obtained. The results indicate that these lattice functions are in a good agreement with the values calculated by the code DIMAD\cite{5}, as shown in Fig. 1. For example, the difference in tune value computed by DIMAD and by COMSYN is less than $10^{-6}$. The CPU time needed for the lattice calculations by COMSYN is 20 seconds with SUN computer, which is comparable with DIMAD (18 seconds).

### 4. Calculation and Harmonic Analysis of Magnetic Field

A computer code for three dimensional field calculation, MAGNUS\cite{6}, is used to compute the field distribution in the fringe area of the TAC dipole. The output is stored as a data set $B(r, \theta, z)$ in the computer system for a later use. The field components at the particle position that are needed for the beam tracking are then obtained from the three-dimensional spline interpolation. For comparison, two different versions of the dipole end winding are studied. Fig. 2 shows their boundary configurations and the corresponding field profiles along the reference orbits.

![Fig. 2. Mesh generated by computer and the edge field profile of two different versions of end windings B1(left) and B2(right).](image)

The field drop beneath zero in B1 (see Fig. 2) is caused by the coil crossing the reference orbit at lower level. It can be expected that this change in the field sign would introduce a strong multipole field in the area. This has been demonstrated by the harmonic analysis of the field profiles, as shown in Fig. 3. We intend to carry out a precise field mapping program for the TAC dipole using a Hall probe. The final design of the dipole will be determined based on the beam dynamic study using the computed field as well as the measured field of this magnet.

![Fig. 3. Sextupole and octupole components of the dipoles B1(left) and B2(right) along the reference orbit.](image)

### 5. Beam Simulation with The Computed Edge Field

The method in section 3 is used to find lattice functions with the computed edge field. It is obvious that the lattice functions will be affected by the fringe field, as indicated by Tanaka's study\cite{9}. Our calculations show that for the dipole B1, the horizontal tune is shifted down by 2.3% only but the vertical tune by 10.9% from 1.160 to 1.03. This shows that the vertical tune with the edge field is very close to an integer resonance. To eliminate this problem, the strength of quads has been readjusted in our calculations and the tune is restored to the original design value. The fast Fourier transform (FFT) is also used in the tune calculation. The results are the same as that obtained by the matrix method mentioned as above for the linear field.

The dynamic aperture of the TAC ring was calculated by beam tracking. The dynamic aperture is defined as a region where particles are stable without hitting a vacuum chamber over one thousand turns. The chamber dimensions used are $\pm 30$ mm in the horizontal plane and $\pm 10$ mm in the vertical plane. The calculation of the dynamic aperture was performed with a CRAY. For 2000 integration steps each turn, the needed CPU time is about 1 minute for each 100 turns. Fig. 4 shows the phase space at the center of the focusing quadrupole. Fig. 5 shows the dynamic aperture for the two different edge fields B1 and B2. The rectangle indicates the 10 standard deviation area which is needed for the acceptable quantum lifetime of the beam. It can be seen that the second version of the end winding, B2, provides relatively weak multipole filed.
so that its dynamic aperture is much wider than that of the first version, B1. This result is consistent with the conclusion from the harmonic analysis of the edge field described in section 4. Fig. 6 gives the results of the tune computation by the FFT. A strong coupling between the horizontal and the vertical motions can be noted near the linear vertical tune in the tune spectrum of B1. The calculations also indicate that the dynamic aperture depends not only on the edge field profile but also on the position of the operating point(\(\nu_x, \nu_y\)). This study is in progress.

References


Fig. 4. Phase space at the center of the focusing quadrupole.

Fig. 5. Dynamic aperture at the center of the focusing quadrupole. The two curves refer to the different edge field B1 and B2.

Fig. 6. The tune spectrum in vertical motion with different edge fields B1 (top) and B2 (bottom). They are calculated by Fast Fourier Transform. The highest peak in the figures corresponds to the linear tune values. A coupling between the horizontal and the vertical directions can be seen in the top figure.