## Closed orbit correction in the SSC

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#### Abstract

A global correction scheme proposed for use in the SSC is described. Various features of the SSC lattice that impact the ability to correct the orbit are discussed. Typical results for the residual RMS closed orbit in the arc is calculated to be 0.65mm with peak values of 3mm.

#### I. INTRODUCTION

Most of the techniques associated with closed orbit correction are widely known. The present paper gives a brief description of one such method and discusses the results obtained when it is applied to the SSC collider lattice. The emphasis is on features of the lattice which effect closed orbit correction and it is likely that any of the 8 methods cataloged in ref. [3] would yield similar results. The global scheme described here is very robust and easy to apply. The results of three separate studies are briefly described.

# II. ANALYTIC FORMULATION OF THE OBRIT SMOOTHING ALGORITHM

The closed orbit correction algorithm are more completely described in references [1] and [2] but will be summarized here for the sake of completeness.

The term *reference orbit* is defined to mean the theoretical center line of the accelerator. The term *closed orbit* is defined to mean that orbit which closes on itself in the presence of magnet misalignment and field errors. The closed orbit is described with respect to the reference orbit as are magnet misalignments.

Let  $X_d$  represent the closed orbit at a position d corresponding to a detector. Let  $\Delta x'_a$  represent the change in slope (dx/ds) produced by a magnetic element located at position  $S_a$  in an otherwise ideal lattice characterized by the ideal lattice functions. The relationship between  $\Delta x'_a$  and  $X_{co}(s_d)$  can be thought of as a Green's function and is given in equation 1.

$$X_{d} = \frac{\cos\left(\frac{\mu}{2} - \phi_{da}\right)}{2\sin\left(\frac{\mu}{2}\right)} \left(\sqrt{\beta_{a}\beta_{d}}\Delta X_{a}^{'}\right)$$
(1)

where  $\mu$  is the betatron tune,  $\phi_{da}$  is the phase advance from position d to position a,  $\beta_a$  is ideal beta function at position a,

 $\beta_d$  is ideal beta function at position d.

This can be expressed in more compact form by defining the matrix  $T_a(d)$  as shown in equation 2.

$$X_{d} = T_{a}(d)\Delta X_{a}$$
 (2)

Since the ideal beta functions are linear, the effect of many kicks may be superimposed as shown in equation 3.

$$X_{d} = \sum_{a=1}^{N_{a}} \frac{\cos{(\frac{\mu}{2} - \phi_{da})}}{2\sin{(\frac{\mu}{2})}} \sqrt{\beta_{a}\beta_{d}} \Delta X_{a}^{'}$$
(3)

The orbit correction process proceeds as follows. Let  $X_a^0$  correspond to the closed orbit error at position d before any correction is done. The total closed orbit at position  $X_d$  is then composed of two components  $X_a^0$  and the closed orbit displacements produced by the adjuster kicks as specified in equation 3. This is written symbolically in equation 4.

$$X_{d} = X_{d}^{o} + \sum_{a=1}^{N_{a}} \frac{\cos\left(\frac{\mu}{2} - \phi_{da}\right)}{2\sin\frac{\mu}{2}} \sqrt{\beta_{a}\beta_{d}} \Delta X_{a}$$
(4)

If the number of adjustors and detectors were forced to be equal, we could write a set of N equations in N unknowns which would "cancel" out the error terms  $X_d^0$  to the extent that the actual lattice functions are represented by the ideal lattice functions  $\beta_a$ ,  $\beta_d$ . There would of course be a residual closed orbit error at positions other than  $X_d$ .

In general, the number of detectors will exceed the number of adjusters so it is necessary to define a minimization procedure which can yield the set of corrector strengths  $\Delta X_a^*$ . To this end, we define a *badness function B* as shown in equation 5.

$$B = \sum_{d=1}^{N_d} (X_d - X_d^{bpm})^2$$
 (5)

where  $X_d^{bpm}$  represents the displacement of the beam position monitor at location d.

Equation 5 defines a badness function which is "operational" in the sense that it is a directly measurable quantity. It expresses the fact that the orbit can not be corrected beyond the level at which it can be measured. The exact manner in which  $X_{d}^{bpm}$  is specified will be used in latter section to examine issues associated with BPM alignment.

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The global badness is a function of the set of corrector strengths  $\Delta x'_1$ ,  $\Delta x'_2$ , and  $\Delta x'_n$ . B is explicitly given in equation 6.

$$B = \sum_{d=1}^{N_d} \left( X_d^o + \sum_{a=1}^{N_a} T_a(d) \Delta X' a - X_d^{bpm} \right)^2$$
(6)

The global minimum of B is defined by the set of condition:

$$\frac{\partial B}{\partial \Delta X_a} = 0$$
  $a = 1, 2, \dots Na$  (7)

The set of equations defined by equation 6 and 7 can be expressed in matrix form by defining the vectors Q and v and the square matrix M.

$$M = \begin{bmatrix} M_{11} & M_{1N_a} \\ M_{N_{a1}} & M_{N_aN_a} \end{bmatrix}$$

where

$$\begin{split} \mathcal{M}_{ab} &= \sum_{d=1}^{N_d} T_a(d) T_b(d) \\ \mathcal{Q} &= \begin{bmatrix} \Delta X_1' \\ \vdots \\ \Delta X_{N_d} \end{bmatrix} \\ \upsilon &= \begin{bmatrix} -\sum_{d=1}^{N_d} (X_d - X_d^{bpm}) T_1(d) \\ \vdots \\ -\sum_{d=1}^{N_d} (X_d - X_d^{bpm}) T_{N_a}(d) \end{bmatrix} \end{split}$$

The unknown corrector strengths are now easily calculated from the matrix expression given in equation 8.

$$Q = M^{-1}v \tag{8}$$

It is desirable to have separate families of correctors and detectors so that the beam can be corrected locally in a special region such as the IR region yet have it integrated into the global system in such a way that local correction does not adversely affect the global closed orbit.

This is accomplished by placing a set of correctors and detectors in the local region. The readings of the global set of detectors is weighted and included in badness function.

Let  $NT_d$  be total number of detectors,  $NL_d$  be the number of local detectors,  $NG_d$  be the number of global detectors.

$$B = W_1 \sum_{d=1}^{NL_d} (X_d - X_d^{bpm})^2 + W_2 \sum_{d=1}^{NG_d} (X_d - X_d^{bpm})^2$$

and the system of equations shown in equation 8 is of order  $NL_a$  (the number of local correctors).  $W_1$  and  $W_2$  are arbitrary weighting factors. In general  $NG_d$  will be much larger than  $NL_d$ .

#### **III. IMPLEMENTATION**

The global correction described in the previous section has been implemented in the code TEAPOT and applied to simulating the collider ring of the SSC complex. The current design configuration of the SSC collider calls for one BPM and one steering corrector in each regular cell in each plane. There are 396 regular cells in two arcs in addition, there are 12 BPM's and 12 correctors in each of the 4 interactions regions and 54 additional detector- corrector pairs distributed through the utility sections.

The BPM's themselves are constructed with 4 plates and can give beam position readings in 2 planes simultaneously. The current plan is for only one plane of each BPM to be connected in the arc, although the leads for the other plane will be available and can be connected if necessary

The correctors and BPM's are located in the spool pieces adjacent to the focusing quadrupole for that plane. The BPM is physically mounted on the same shaft with the sextupole and can be closely aligned with the sextupole.

The lattice is composed of two semicircular arcs separated by utility straight sections containing interaction regions and injection sections. There are two low beta interaction regions and two medium beta interaction regions in addition to two utility straights in each ring in the collider. Each of these are treated with a separate family of correctors which are integrated with the global correctors as discussed in the previous section.

The principle magnetic multipole errors in superconducting dipole magnets are given in Table 1. They are dominated by the large systematic sextupole term created by persistent current.

term	systematic low energy	systematic high energy	random
al	0.04	0.04	1.25
b1	0.04	0.04	0.50
a2	0.032	0.032	0.35
b2	2.000	0.800	1.15
a3	0.026	0.026	0.32
b3	0.026	0.026	0.16

Table 1 Principle Magnetic Multipole Errors.

Then equation 5 becomes

The term *residual error* is defined to be the deviation of the closed orbit from the reference orbit after orbit correction has been performed. It has a major impact on accelerator dynamic aperture. The method described here produces an accurate closed orbit whose magnitude and distribution are known. It is produced by three different mechanisms. The relative importance of the three mechanisms is problem dependent and general statements cannot be made. The first mechanism is inaccuracy of the beam measurement process caused by misalignment and imperfect calibration of the BPM's. The second mechanism is simply due to the fact that the beam can be steered off reference in the region between two adjacent corrector locations. In the collider lattice, there are ten dipoles, a defocusing quad and a sextupole which can all contribute to residual closed orbit error. The third mechanism is that the lattice function used in correction algorithm are based on ideal lattice functions rather than "real" lattice functions.

#### **IV. RESULTS**

A number of simulation studies on various aspects of orbit correction have been carried out which will be briefly summarized here.

#### **Baseline Calculations**

Baseline calculations of the residual closed orbit error have been done on the collider lattice for three cases. A lattice without interaction region (FODO lattice), a lattice with uncorrected interaction regions and a lattice with corrected interaction regions. All the lattices had the full set of errors part of which are shown in table 1 and the full set of detectors and correctors in the arcs (396). The runs with corrected IR regions had 12 local BPM's and 12 local correctors in each plane. There were no alignment or multipole errors for the elements in the IR's. Typical results for the residual closed orbit are given in table 2.

	FODO	UNCORRECTED	CORRECTED
	LATTICE	IR's	IR's
All elements	0.7mm	1.2mm	0.7mm
Bends only	0.65mm	0.65mm	0.65mm
Max error	2.7mm	12mm	3.0mm

Table 2 Typical SSC Collider Residual Close Orbit Errors

The simulation concerning corrected IR's is based on a preliminary corrector placement scheme and will be improved. A typical closed orbit plot after correction is shown in Figure 1



Figure 1 Typical Closed Orbit Plot

### **BPM alignment studies**

A study was conducted to determine the effect of aligning BPM's with the chromaticity sextupoles rather than the quadrupoles. Since the sexturpole errors feeddown spread and become random quadrupole errors which produce tune spread, there was possibility that BPM alignment could effect tune shift and hence the dynamic aperture. The result of the study was that the persistent current sextupole fields in the dipoles contributed as much tune shift as the sextupoles. This is more fully described in reference 2.

### One versus two BPM's per cell

A study was done to determine the impact of having one BPM per cell as opposed to 2 BPM's per cell. The BPM at the focusing quad was retained. The basic result was that removing the BPM's a the defocusing quads increased the residual closed orbit by approximately 10%.

#### References

[1]I.Schachinger and R. Talman, Particle Accelerators, 1987, Vol. 22, pp. 57-59

[2]G.Bourianoff and J.Peterson, "BPM Alignment Issues: Quadrupole vs. Sextupole Centering", SSC Collider Note.

[3] J.L.Warren, "Determination of Magnet Misalignments from Measurement of Closed orbit Distortion", Los Alamos PSR Tech note III.