Examination of the Stability of the Advanced Imaginary γ_t Lattice

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Abstract

An advanced imaginary-γ_t lattice for the Fermilab Main Injector is examined for its response to quadrupole field errors, quadrupole misalignment errors, as well as its dynamical aperture. We find that the lattice is tunable except near an integer tune. The misalignment sensitivity factors are acceptable and can be lowered if the low-beta triplet quadrupoles are specially aligned. The dynamical aperture is very large provided that a family of harmonic sextupoles is installed.

I. Introduction

A transitionless lattice with imaginary-γ_t for the Fermilab Main Injector was proposed in a previous paper[1]. That lattice is very compact and fulfil all the necessary restrictions and requirements. The natural chromaticities were corrected with two family of sextupoles of reasonable strengths. The lattice was then subject to variation in particle momentum. The changes in beta-functions and dispersion turned out to be relatively small.

In this paper, we are going to study some other properties of the lattice. In Sect. II, the response to quadrupole field-gradient errors is examined. We find that the lattice is tunable and the changes in beta-functions, dispersion, and betatron tunes are small provided that the integer tune is avoided. In Sect. III, the misalignment sensitivity factors are computed analytically and are found to be acceptable. These factors can be reduced tremendously if the low-beta triplet quadrupoles are specially aligned. In Sect. IV, the dynamical aperture is studied at injection energy 8.9 GeV, where aperture demand is tight, and at 19 GeV during acceleration, where the eddy current is at a maximum. The tracking code[6] TEAPOT was used. We find that the aperture can be made very large by installing a family of harmonic sextupoles to counteract the nonlinearities brought in by the chromaticity sextupoles.

II. Dependence on Gradient Errors

The betatron-function and tune dependence on quad-gradient errors were examined using SYNCH. A gradient error of $\pm 3\%$ was introduced into the focussing and defocusing FODO cell quadrupoles including those in the zero-dispersion straight sections. The starting tunes were $\nu_x = 19.714$ and $\nu_y = 19.188$. We expect the tune change to follow the linear relations:

$$\begin{aligned} \Delta\nu_x &= c_{xF}(\Delta G_F/G_F) + c_{xD}(\Delta G_D/G_D)\ , \\ \Delta\nu_y &= c_{yF}(\Delta G_F/G_F) + c_{yD}(\Delta G_D/G_D)\ , \end{aligned}$$

where the coefficients are given by $c_{xF} = \hat\beta_x N_F G_F/4\pi$, $c_{xD} = -\check\beta_x N_D G_D/4\pi$, $c_{yF} = -\check\beta_y N_F G_F/4\pi$, $c_{yD} = \hat\beta_y N_D G_D/4\pi$. Here, $\hat\beta_x = 77.42$ m and $\check\beta_x = 25.86$ m ($\check\beta_y = 25.59$ m and $\hat\beta_y = 76.78$ m) are the horizontal (vertical) betatron functions at the FODO F-quad and D-quad, respectively; $N_F = 50$ and $N_D = 26$ are, respectively, the number of FODO F- and D-quads, whose strengths are $G_{F/D} = \pm 0.0454$ m^{-1}. We obtain $c_{xF} = +13.98$, $c_{xD} = -2.43$, and $c_{yF} = -4.62$, $c_{yD} = +7.21$, which are verified by the simulation results in Figs. 1 and 2. For a $\pm 3\%$ variation in G_F (G_D), the horizontal tune changes by ± 0.42 (∓ 0.07) while the vertical tune changes by ∓ 0.14 (± 0.21).

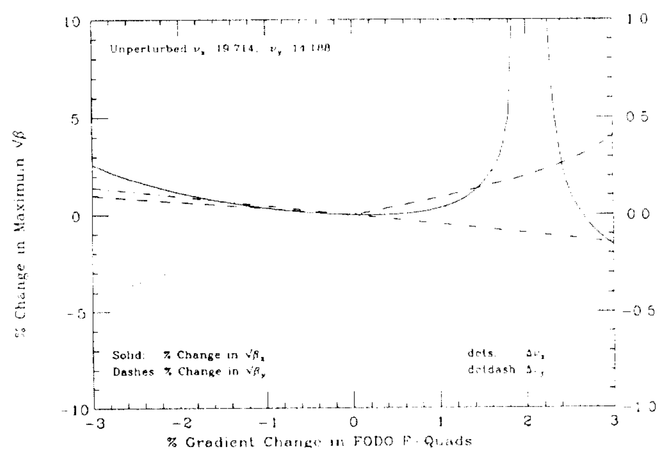

Fig. 1. *The betatron functions and tunes dependence on a gradient error in the focussing quadrupoles in the FODO cells.*

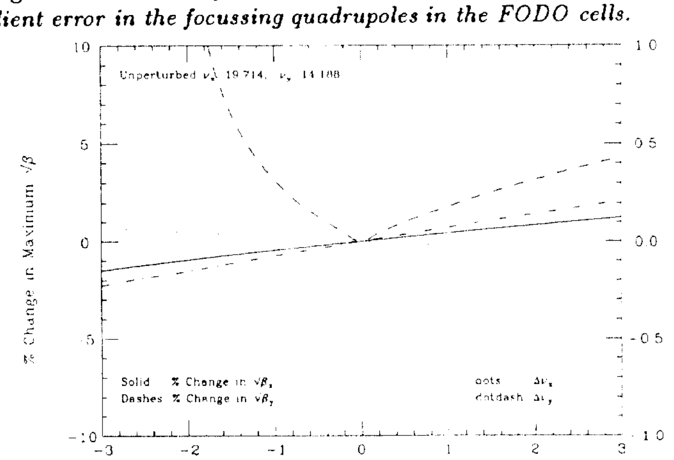

Fig. 2. *The betatron functions and tune dependence on a gradient error in the defocusing quadrupoles in the FODO cells.*

Figures 1 and 2 also show that the betatron functions are

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very stable (the changes in $\sqrt{\beta_x}$ and $\sqrt{\beta_y}$ are less than 2.5%) until one of the corresponding tunes, ν_x or ν_y, reaches an integer value. The designed lattice with the imaginary γ_t, as well as the present Main Injector design based on the FODO-cell structure, have only two-fold symmetry due to the some special injection and extraction constraints. The integer-tune stop band of the betatron functions in the ring with the two-fold symmetry is to be expected[2]. (The forbidden values of the tune in the accelerator with N superperiods are $\nu = N/2,\ N,\ 3N/2,\ \ldots$).

The horizontal dispersion showed a very similar dependence on the horizontal tune as the horizontal betatron functions. It becomes unstable when the horizontal tune reaches an integer values[3]. This is expected, because the dispersion at location s is given by

$$D_z(s) = \frac{1}{2\sin\pi\nu_z}\int_0^C ds \frac{\sqrt{\beta_z(s)\beta_z(s')}}{\rho(s')}\cos(\pi\nu_z - |\phi_z(s) - \phi_z(s')|)\ ,$$

where $z = x$ or y and $\rho(s)$ is the local radius of curvature. It should be mentioned that it is possible to change the tunes to $\pm$ one unit without noticeable change in all betatron functions if the integer tunes are avoided.

III. Misalignment Errors

It is impossible to align all beam elements perfectly. Transverse misalignment errors can lead to offsets of the closed orbit, which must be corrected during operation. If all the misalignments are random, uncorrelated, and have a variance $\langle z^2\rangle$ where $z = x$ or y, it is easy to show that the closed-orbit offset $z_{co}(s)$ at location s has a variance given by

$$\langle z_{co}^2(s)\rangle = \frac{\langle z^2\rangle\beta_z(s)}{2\sin\pi\nu_z}\sum_i \beta_{zi}\left[\left(\frac{B'\ell}{B\rho}\right)_i \cos(\pi\nu_z - |\phi_{zi} - \phi_z(s)|)\right]^2 .$$

In the above, B' is the field gradient of the quadrupole and ℓ its length, the summation runs over all the quadrupoles in the lattice, and the thin-lense approximation has been assumed. The computation was performed by reading a SYNCH output file[4], the $\langle z_{co}^2(s)\rangle$'s were computed at each quadrupole, and the maxima were recorded. We obtain the misalignment sensitivity factors:

$$S_z = \left[\frac{\langle z_{co}^2(s)\rangle}{\langle z^2\rangle}\right]_{\max}^{\frac{1}{2}} = \begin{cases} 46.1 & \text{horizontal} \\ 70.0 & \text{vertical} \end{cases}$$

Since S_z follows a Rayleigh distribution, there is a 98% probability that the closed-orbit offset will fall within $2S_z$.

These misalignment sensitivity factors may appear to be large. This is because first the lattice size is large and secondly there are too many low-beta insertions. Although the beta-functions in the low-beta triplets are only $\sim$ 50 m, the middle triplet quadrupoles are over 4 m in length while the FODO quadrupoles are only $\sim$ 1 m long. These 4 m quadrupoles alone contribute about 90% in the above summation. Usually triplet quadrupoles are specially aligned. This may reduce the sensitivity factors significantly.

The maximum closed-orbit offsets due to the quadrupole misalignment errors were also computed with SYNCH. In SYNCH, when we enter DX or $DY = 0.1$ mm, the code should generate uniform random misalignments in the range (−0.05, 0.05 mm). Thus $\langle z^2\rangle = (1/12)\times 10^{-2}$ mm^2, and the expected $\langle z_{co}^2\rangle^{1/2}$ should be 1.33 and 2.02 mm, respectively, for the horizontal and vertical. However, the SYNCH results give 2.23 and 3.88 mm, about two times too large. We do not know exactly the reason of the discrepancy. However, we do discover that the SYNCH results were not very random.

IV. Dynamical Aperture Study

Correction sextupoles S_F and S_D can only be placed near the F-quadrupoles and D-quadrupoles in the FODO regions of the lattice, because the dispersion elsewhere is rather small. We have only 44 S_F and 22 S_D to correct for the natural chromaticities and their strengths $k_{sf} = -0.056$ m^{-2} and $k_{sd} = +0.126$ m^{-2} are therefore relatively large. This implies relatively large nonlinearities and small dynamical aperture. The second-order tuneshifts are given by

$$\begin{aligned}\nu_x &= 19.714 + 4.82\times 10^2\epsilon_x/\pi - 1.56\times 10^4\epsilon_y/\pi\ ,\\ \nu_y &= 14.188 - 1.56\times 10^4\epsilon_x/\pi - 7.49\times 10^3\epsilon_y/\pi\ ,\end{aligned}$$

where ϵ_x and ϵ_y are, respectively, the *unnormalized* horizontal and vertical emittances measured in m-r. It is clear that an emittance of 40π mm-mr normalized at the injection energy of 8.9 GeV (the required acceptance for the Main Injector) will produce a tunespread as large as 0.067, thus hitting a resonance. Such a small dynamic aperture can become a real difficulty for the imaginary-γ_t lattice. In fact, unacceptable small apertures have been reported in the tracking of some SSC injector lattices involving negative dispersion[5].

In order to lower the nonlinearities, we install a family of 44 harmonic sextupoles S_{DA}, two in each of the regular and long-straight blocks near the triplet quadrupoles where the dispersion is about 1 m, and two in each of the zero-dsipersion blocks in the zero-dispersion region. We also include systematic multipole errors in the dipoles. Since the dipoles are not superconducting, random errors are small and are neglected. At the injection energy of 8.9 GeV, the errors provided from the magnetic measurements of the prototype dipoles are $b_2 = 0.0327\times 10^{-4}$ and $b_4 = 1.782\times 10^{-4}$ at one inch. These multipoles in the dipole field $B(x)$ at offset x are defined as

$$B(x) = B_0\left(1 + \frac{1}{2!}b_2x^2 + \frac{1}{4!}b_4x^4 + \ \ldots\ \right)\ ,$$

and B_0 is the vertical field at the center. The harmonic sextupole strength was set at $k_{sda} = +0.08$ m^{-2} (not necessary the best choice), the correction sextupoles were fitted to $k_{sf} = -0.0598$ m^{-2} and $k_{sd} = +0.1446$ m^{-2} for natural chromaticity cancellation. The new tuneshifts become

$$\begin{aligned}\nu_x &= 19.714 + 4.43\times 10^2\epsilon_x/\pi - 2.81\times 10^2\epsilon_y/\pi\ ,\\ \nu_y &= 14.188 - 2.81\times 10^2\epsilon_x/\pi + 1.33\times 10^3\epsilon_y/\pi\ .\end{aligned}$$

Comparing with the above, the nonlinearities have been reduced in some cases by more than one order of magnitude. Note that the coefficients have opposite signs which further lower the tuneshifts to less than 0.002 when the normalized emittances are 40π mm-mr at 8.9 GeV.

The lattice was next tracked with TEAPOT. We launch the particles with 0.2% momentum offset at the middle of a FODO D-quadrupole at a certain horizontal offset. The maximum vertical offset that gave 35000 turn survival was recorded. In this way we scanned the whole x-y space and obtain the aperture plot shown in Fig. 3.

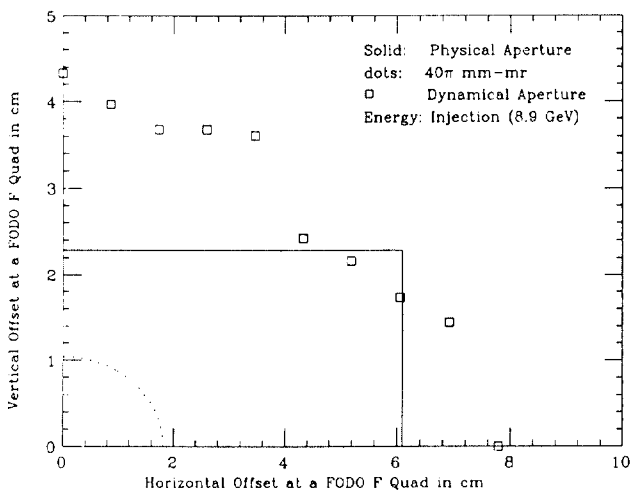

Fig. 3. *Dynamical aperture at a FODO F quad obtained from tracking at the injection energy 8.89 GeV. The required aperture is 40 π mm mrad.*

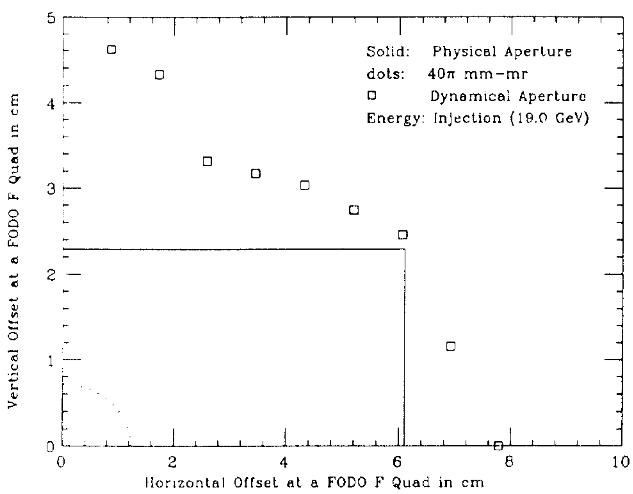

Fig. 4. *Dynamical aperture at a FODO F-quad obtained from the tracking at 19 GeV when the eddy currents are largest.*

The lattice was next tracked with 0.2% off momentum at 19 GeV when the eddy currents have a maximum; the corresponding b_2 was $+0.262 \times 10^{-4}$ at 1 inch. The strength of the harmonic sextupoles was chosen as $k_{sda} = +0.045\ \mathrm{m}^{-2}$ (again not necessary the best choice), the correction sextupoles were fitted to $k_{sf} = -0.0887\ \mathrm{m}^{-2}$ and $k_{sd} = +0.0875\ \mathrm{m}^{-2}$ for the natural chromaticities cancellation. The second-order tuneshifts

$$\begin{aligned} \nu_x &= 19.714 + 5.07 \times 10^2 \epsilon_x/\pi - 4.13 \times 10^2 \epsilon_y/\pi \ , \\ \nu_y &= 14.188 - 4.13 \times 10^2 \epsilon_x/\pi + 9.01 \times 10^2 \epsilon_y/\pi \ , \end{aligned}$$

were still small. The second-order sextupole tuneshifts were still small. The trackings were performed with particles having 0.2% momentum offset. The 35000-turn survival aperture is plotted in Fig. 4, which is not much different from Fig. 3. We should point out that the apertures shown in both plots are not real. This is because the magnetic field was not presented correctly at distances over 25.4 mm. However, the plots do illustrate that large aperture can be achieved in imaginary-γ_t lattice. Also the tunespread expressions presented above were for on-momentum particles. However, since the chromaticities had been fitted to zero, the tunespread coefficients do not vary significantly when the momentum has a 0.2% offset.

V. Conclusion and Discussions

The above analysis shows that our imaginary-γ_t lattice is very stable. It is tunable; all the betatron functions, dispersion, as well as the tunes show only small changes with respect to the gradient errors in the FODO quads, provided that the integral tunes are not encountered. The sensitivity of the closed orbit on quadrupole misaligments is acceptable. Since most of the closed-orbit offset comes from the 4-m quadrupoles in the low-beta triplets, special alignment of these quadrupoles will lower the sensitivity factor significantly. For example, if the fronts and ends of these quadrupoles are aligned independently, the sensitivity factors will drop by a factor of $\sqrt{2}$.

Although there have been reports[5] that lattices with negative dispersion have small dynamical apertures even if harmonic sextupoles are introduced, our lattice does behave otherwise. With harmonic sextupoles properly installed, the 35000-turn survival dynamical aperture for 0.2% off-momentum particles has been domonstrated to be very large both at injection when the beam size is big and during acceleration when the eddy-current contribution is large. The reason for the success is probably due to our proper control of the dispersion function, which is perfectly matched block to block and only varies between -2.77 and 2.82 m. We want to point out that in many lattices with negative dispersion, the dispersion is allowed to flow freely without control so that it may reach ± 5 m and even ± 15 m. With such big dispersions, the off-momentum particles can easily be thrown sideway into the bad-field region of the dipoles and get lost. This may explain why those lattices exhibit small aperture.

References

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