MULTIBUNCH BEAM BREAKUP IN HIGH ENERGY LINEAR COLLIDERS

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ABSTRACT

The SLAC design for a next-generation linear collider with center-of-mass energy of 0.5 to 1.0 TeV requires that multiple bunches (~10) be accelerated on each RF fill. At the beam intensity (~ 10^10 particles per bunch) and RF frequency (11 to 17 GHz) required, the beam would be highly unstable transversely. Using computer simulation and analytic models, we have studied several possible methods of controlling the transverse instability: (1) using damped cavities to damp the transverse dipole modes; (2) adjusting the frequency of the dominant transverse mode relative to the RF frequency, so that bunches are placed near zero crossings of the wake; (3) introducing a cell-to-cell spread in the transverse dipole mode frequencies; and (4) introducing a bunch-to-bunch variation in the transverse focusing. The best cure(s) to use depend on the bunch spacing, intensity, and other features of the final design.

1. INTRODUCTION

In this paper, we address the problem of transverse instability of a train of bunches in a high energy linac. The main motivation for accelerating multiple bunches per RF fill is to obtain higher luminosity for a given expenditure of RF energy. The optimal bunch spacing, bunch charge, and number of bunches depend upon many factors other than just the need to be able to control the beam breakup. There is a serious constraint on charge per bunch, due to pair creation at the interaction point: The smooth focusing function of bunch n is taken to be:

\[ y = y_0 + Gs, \]

where \( y \) is the particle energy divided by the rest energy \( mc^2 \) and \( G \) is a constant. We use the smooth-focusing approximation \( k(s) = 1/p(s) \) for the focusing function, where \( p(s) \) is the "average" betatron function at longitudinal position \( s \). The smooth focusing function of bunch \( n \) is taken to be:

\[ k_n(s) = \left[ \frac{\gamma(s)}{\gamma(s)} \right]^{\frac{1}{2}} k_n(0). \]

For the main linacs of the collider, we will assume \( p = 1/2 \), but for other linac subsystems \( p = 1 \) may be desirable to maintain more uniform \( k \), namely \( p \approx 0 \). The transverse dipole wake function is a sum of modes of the following form:

\[ W_L(z) = \sum_{m} W_m \sin(K_m z) \exp \left( -\frac{K_m^2 z}{2Q_m} \right), \]

where \( z \) is the distance behind the exciting bunch; \( K_m \) is the wavenumber and \( Q_m \) is the quality factor of mode \( m \), and the \( W_m \)'s are constant coefficients. Units of \( W_L(z) \) are \( V/m \). The equation of motion (in one transverse plane) for the offset \( x_n \) of bunch \( n \) is:

\[ \gamma(s) x_n'' + \gamma(s) x_n' + \gamma(s) k_n(s) x_n = \frac{Nc^2}{mc^2} \sum_{j=1}^{n-1} W_L [n - j] \ell j x_j(s). \]

Here primes denote derivatives with respect to \( s \). If we assume
the WKB solution

\[ x_1(s) = x_1(0) \left[ \frac{\gamma_0}{\gamma(s)} \right]^{1/2} \exp \left[ \psi_1(s, 0) \right] , \]  

as the motion for the first bunch, and drop a term with rapidly-oscillating integrand, one can show (Ref. 7) that the solution for the transverse motion of bunch \( n \) may be written:

\[ x_n(s) = \left\{ x_n(0) + \frac{N \varepsilon^2}{2 \pi \gamma_0 mc^2 k_0(0)} \int_0^s \left[ \frac{\gamma_0}{\gamma(s')} \right]^{1/2} \exp \left[ i \psi_n(s', 0) \right] \right\} \times \sum_{j=1}^{s-1} W_\perp [(n - j)] x_j(s') ds' \]  

\[ \exp \left[ + i \psi_n(s, 0) \right] , \]  

where

\[ \psi_n(s, s') \equiv \int_{s'}^s k_n(s'') ds'' \]  

is the phase advance of bunch \( n \) between \( s' \) and \( s \). A computer program LINACB1 was written to numerically integrate the equations for \( x_n(s) \).

4. VERY STRONGLY DAMPED WAKE

If the wake is so strongly damped that a bunch only sees a significant wake from the immediately preceding bunch, we can use a simple "daisy chain" model to estimate the transverse blowup of each bunch in the train. Let us assume that the focusing function is the same for all bunches and scales according to Eq. (2) with \( p = 1/2 \). Then, one may show that the equations of motion can be written as if there were no acceleration (see Ref. 7):

\[ x_1' + k_0^2 x_1 = 0 \]  

\[ x_n' + k_0^2 x_n = \frac{N \varepsilon^2 W_\perp(l)}{E_0} x_{n-1} \quad (n > 1) , \]  

where \( k_0 \) and \( E_0 \) are the focusing function and energy at the beginning of the linac, and the longitudinal coordinate \( s \) is to be interpreted as an "effective length"

\[ s_{\text{eff}} \equiv \frac{1}{k_0} \int_0^s k(s) ds \approx 2 \left( \frac{\gamma_0}{\gamma} \right)^{1/2} \quad \text{for} \ \gamma \gg \gamma_0 . \]  

Assuming \( x_1(s) = a_1 e^{i k_0 s} \) where \( a_1 \) is a constant, one finds solutions \( x_n(s) = a_n(s) e^{i k_0 s} \), where

\[ a_n(s) \equiv \sum_{j=0}^{n-1} \left( \frac{-i \alpha s}{j!} \right) a_{n-j}(0) , \]  

and we have defined

\[ \sigma \equiv \frac{N \varepsilon^2 W_\perp(l)}{2 k_0 E_0} . \]  

If the initial conditions are \( a_n(0) = 1 \), this is just the first \( n \) terms of the Taylor series for \( \exp(-i \alpha s) \). Thus, if \( \sigma L \) is of order 1, where \( L \) is the effective length of the linac, there is no significant blowup of bunches beyond the first few in the train. We show an example of this behavior in the next section.

5. EXAMPLES

5.1. Linac Parameters

For illustration, we consider a main linac with accelerating frequency 17.1 GHz, length 3 km, initial energy 18 GeV, and final energy 500 GeV. The beta function is taken to be 3.2 m at the beginning of the linac and scales as \( \gamma^{1/2} \). Keeping the bunch-to-bunch energy variation as small as possible imposes a relation between the number of particles per bunch, \( N \), and the bunch spacing \( \ell \) (Ref. 7):

\[ \ell = c T_F \frac{\gamma_0 \eta}{2} e^{-\sigma} , \]  

where \( T_F \) is the filling time and \( \tau \) is the ratio of the filling time to the attenuation time of the RF structure. The single-bunch loading is

\[ \eta_0 \equiv 4 N \varepsilon \kappa_0 \frac{E_0}{E_\perp} , \]  

where \( \varepsilon \) is the loss parameter of the accelerating mode and \( E_\perp \) is the acceleration gradient. Taking \( T_F = 60 \text{ nsec} \), \( \tau = 0.6 \), \( \kappa_0 = 436 \text{ V} \mu \text{C/m} \), and \( E_0 = 186 \text{ MeV/m} \) gives

\[ \ell \approx (0.216 \text{m}) \frac{N}{10^{10}} . \]  

We shall take \( \ell \) to be 24 RF cycles (about 42 cm) and \( N = 1.67 \times 10^{10} \) in our examples.

5.2. Highly Damped Wake

The number of e-foldings of the wake between bunches is about:

\[ K_0 \ell \approx 100 \quad Q \]  

for \( \ell = 0.12 \text{ m} \) and wavenumber \( K_0 \) of the fundamental transverse mode about 470 to 480 m\(^{-1}\). Let us take as an example \( Q = 35 \) and \( K_0 = 475 \text{ m}^{-1} \), so that there are about 3 e-foldings between bunches.

In Fig. 1, the result of the daisy chain model is compared with the result of the full simulation program LINACB1.

5.3. Damped Wake Combined With Tuning of Wake Zero-Crossings

The combination of lowering the \( Q \)'s and tuning the frequency of the fundamental transverse dipole mode has been discussed in detail in Refs. 7 and 8.

Figure 2 shows an example of "tuning curves", where the maximum transverse blow-up factor of any of the bunches is plotted versus the frequency of the fundamental transverse mode, for various values of \( Q \) (assumed for simplicity to be the same for all transverse modes). The numbers plotted along the curves show the bunch that had the maximum blow-up. Thus, for this example, the curves are independent of the number of bunches in the train, provided there are at least four bunches; current TLC designs have at least 10 bunches per train. Note that even for the higher \( Q \)'s, the tolerance on tuning the fundamental mode frequency is at least ±0.1%, which should not be too difficult to achieve.

5.4. Use of a Spread in Transverse Mode Frequencies

We give an example similar to the previous one, except that we also introduce a frequency spread in each of the transverse modes.

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Fig. 1. Comparison of the results of a daisy chain model (plotted as □) with the results of the program LINACHRU (plotted as •). In each case, the value of the envelope function $|a_n(s)|$ at the end of the linac, for each bunch number $n$, is plotted. The transverse offset $z_n(s) = a_n(s)\exp(iks)$. The focusing function, $k$, is assumed the same for all bunches.

Fig. 2. Maximum transverse amplitude $x_{\text{max}}$ (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of $Q = 20$ to 50. The central value of the fundamental transverse mode wavenumber, where $\lambda_f/\lambda_{W1} = 4/3$, is $477.85$ m$^{-1}$. The range shown about $K_0$ is $\pm 1\%$.

Figure 3 shows tuning curves for $Q = 40$ to 70, with a total spread of $2\%$ in the frequency of each transverse mode distributed uniformly over 200 values; other parameters are as in the preceding example. For $Q = 49$, the blow-up is a factor 2 or less, even with the fundamental transverse mode frequency not tuned to place bunches near wake zero crossings. For the higher values of $Q$ shown, some tuning would be required. Note that $Q$'s p 30 or so are obtainable without slotting the irises, and that according to Figs. 2 and 3, an acceptable solution could be obtained by either tuning the fundamental transverse mode frequency or introducing a $2\%$ spread in the transverse mode frequencies, with $Q \sim 40$.

Fig. 3. Maximum transverse amplitude $x_{\text{max}}$ (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of $Q = 40$ to 70, with a spread in each transverse mode frequency of $2\%$. The central value of the fundamental transverse mode wavenumber, where $\lambda_f/\lambda_{W1} = 4/3$, is $477.85$ m$^{-1}$. The range shown about $K_0$ is $\pm 1\%$.

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