# RELATIVISTIC KLYSTRON WAKEFIELDS* 

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#### Abstract

Monopole, dipole and quadrupole wake potentials are calculated for two cavitics in a standing-wave relativistic klystron, using two independent programs, TBCI and AMOS. Reflections from model terminations which may distort long-range wakes can be mitigated either by using a very long pipe or by


 using absorptive materials at the pipe boundaries.
## Introduction

In order for the relativistic klystron (RK) to fully achieve its potential as a high-gradient power source at high frequencies for the next generation of linear colliders and compact accelerators, it is important to control the wakefields produced by the electron bunches in the driver beam. The minimization of energy spread and transverse beam breakup is particularly relevant for two beam accelerator schemes [1] in which reacceleration of the electron beam is an integral part. Accurate calculations of the wakefields produced by electrons passing through various cavities are essential for investigation into beam stability problems. In this paper, we have calculated the monopole, dipole and quadrupole wake potentials for typical relativistic klystron cavities. Calculations of the wakefields for the reacceleration cavities as well as beam dynamics are currently in progress and will be reported in subsequent papers.

We have chosen for our study here two standing-wave cavities used in the recent SL4 experiment [2] performed at the Accelerator Rescarch Center (ARC) at Livermore by the SLAC/LLNL/LBL collaboration. The beam pipe radius of the SL4 relativistic klystron is 7.0 mm for the first four cavities, and narrows downs to 4.6 mm for the penultimate and output cavities in order to extract power at 11.4 GHz from a subharmonic drive. The lowest dipole modes of the first four SL4 cavities have frequencies above the cutoff frequency of the $T E_{11}$ mode in the beam pipe; while those for the last two cavities have frequencies below cutoff. Wakefields with dominant frequencies above cutoff are damped as they exit the cavity region and propagate into the beam pipe. Long-range wakefields with dominant frequencies below cutoff, on the other hand, ring for a long time as they are trapped in the region near the cavity.

In order to verify the numerical accuracy of our models, we compare the wake potentials from two independent computer programs, TBCI [3] and AMOS [4]. AMOS is a newly developed finite difference code which solves Maxwell's equations in the time domain for cylindrically symmetric structures with linear material properties. The results of both TBCI and AMOS agree very well with each other for a variety of test cases. However, for wakes with frequencies above cutoff, both programs calculated spurious reflections from open boundaries of the beam pipe. These can be eliminated using long pipes, but at the expense of long computer run time. We have employed a method [5] in the AMOS code of using lossy dielectrics with finite conductivities at the open ends of the beam pipe to absorb the outgoing waves. We are able to show that a significant part of the spurious reflections at the open ends can be mitigated with this method.

## Wakefields of the SL4 Relativistic Klystron Cavities

The wake potential witnessed by a unit charge at a distance s behind a bunch of total charge $Q^{\prime}$ can be expressed in a multipole expansion as:

$$
\begin{aligned}
\vec{W}(s) & =\left.\frac{1}{Q^{\prime}} \int_{-\infty}^{\infty} d z(\vec{E}+\vec{v} \times \vec{B})\right|_{t=\frac{z+s}{c}} \\
& =\frac{-\partial w_{0}(s)}{\partial s} \hat{z}+w_{1}(s) r^{\prime} \hat{x}+\cdots
\end{aligned}
$$

where $z$ is the beam axis, $r^{\prime}$ is the radial displacement of the center of the bunch from the axis, and $v$ is the velocity of a rigid bunch. [6] The first term in the expansion corresponds to the longitudinal monopole wake. The second term corresponds to the transverse dipole wake; and so on. Taking the Fourier transforms of the wakes, the longitudinal and transverse inpedances are defined as:

$$
\begin{gathered}
Z_{10}(\omega)=\frac{1}{c} \int_{-\infty}^{\infty} d s \frac{\partial w_{0}}{\partial s} e^{i \omega s / c} \\
Z_{\perp 1}(\omega)=\frac{-i}{c} \int_{-\infty}^{\infty} d s w_{1}(s) e^{i \omega s / c}
\end{gathered}
$$

Figure 1 shows a TBCI model of cavity 3 in the SL4 relativistic klystron. The 7 mm beam pipe radius gives a cutoff frequency of 16.4 GHz for the $T M_{01}$ mode, and 12.6 GHz for the $T E_{11}$ mode. Figures 2 to 4 are the monopole, dipole and quadrupole wakes for this cavity calculated with TBCI. These figures show the bunch shape, the longitudinal wake, the transverse wake and the azimuthal wake on a relative scale. The model for cavity 5 looks similar to Fig. 1. In this case, the 4.6


Fig. 1 TBCI model of SL4 relativistic klystron.
mm beam pipe radius gives a cutoff frequency of 25.0 GHz for the $T M_{U 1}$ mode, and 19.1 GHz for the $T E_{11}$ mode. Figures 5 to 7 are the wake potentials for this cavity calculated with TBCI. The beating seen in Fig. 7 is apparently caused by two quadrupole modes with frequencies very close to each other, as a result of the dimensions chosen for the nose and the gap of this cavity.

From these figures, it is seen that the dominant frequencies of the RK wake potentials are below cutoff, except for the dipole wakes (Fig. 3) for cavity 3 which are above cutoff and as a result are strongly damped as the fields propagate down the beam pipe. In this special case, as noted earlier [7], the


Fig. 2 SL4 cavity 3 monopulse wake for 0.3 meters.


Fig. 3 SL4 cavity 3 dipole wakes for 0.7 meters using a 70 cm beam pipe.


Fig. 4 SL4 cavity 3 quadrupole wakes for 0.3 meters.
long range part of the wake is numerically dependent on the length of the model beam pipe. Spurious reflections from the open ends of the beam pipe can be eliminated in principle by making the beam pipe long enough so that for a given simulation time, reflected waves from the open ends would not have sufficient time to reach back to interfere with the wakes generated directly by the bunch. In Fig. 3 we have shown the dipole wakes calculated with TBCI for cavity 3 with the beam pipe extending 35 cm on either side of the cavity. (The cavity itself has a gap of only 3.5 mm .) Reflections are effectively suppressed for this long pipe calculation.

In practice, the necessity of modeling cavities with very long pipes in order to obtain accuracy for the long range wakes makes the problems cumbersomely large, and computation times unrealistically long. We are therefore motivated to find alternative ways to obtain accurate wakefields for cavity design and beam dynamics calculations. We examine next a new wakefield code AMOS [4] which allows one to design an absorptive load and shorten the beam pipe.


Fig. 5 SL4 cavity 5 monopole wake for 0.3 meters.


Fig. 6 SL4 cavity 5 dipole wakes for 0.3 meters.


Fig. 7 SL4 cavity 5 quadrupole wakes for 0.3 meters.

## Comparison of AMOS and TBCI

The RK cavity dimensions are modified somewhat for the purpose of this comparison in order to accommodate the current limitations of the AMOS program. In particular, we have idealized the cavity model to eliminate curved surfaces and to move boundary lines of the structure to coincide with the edges of the uniform rectangular meshes. Figure 8 shows schematically the regions (cross hatched) of the klystron and possible resistive loads at the ends of the beam pipe. Using this model, we have calculated the monopole, dipole and quadrupole wake potentials for open or closed boundary conditions, and for varying lengths of the beam pipe. Results from AMOS and TBCI for the idealized models without the loads are in excellent agreement. Figure 9 shows, for instance, the dipole transverse wakes for a cavity with 7 mm radius calculated with the two programs. The two wakes overlay perfectly on this scale. Compared with the long pipe calculation of Fig. 3, these short
pipe calculations also illustrate that when Maxwell's equations are solved with the first order radiative boundary conditions used in TBCI or AMOS, the wake potentials are sensitive to reflections from the open boundaries, as is evidenced by the broad secondary peak in Fig. 9.

## Absorption Model for Boundary Termination

We now use the AMOS code to investigate here a theoretical method [5] to mitigate reflections from terminations. The method uses the fact that lossy dielectrics can be used to absorb rf power. Instead of open pipes, we model the terminations with absorptive materials having $\epsilon_{r}=1$ but with a gradual increase of conductivity from zero (vacuum) to a maximum value, before capping off with perfect metal. A parabolic distribution of conductivities is used to simulate the effects of absorption over a distance of about one wavelength $(\lambda=2.1 \mathrm{~cm})$. Although only three regions are shown at each end of the pipe in Fig. 8, 120 regions with 60 absorptive materials are used in the actual calculations. The model for the 7 mm radius cavity consists of a total of 444 zones along the beam axis, 40 zones for the pipe radius, 20 zones across the gap, and 44 zones along the roof of the cavity. Each side of the square zone has a length of 0.175 mm . In this model, the beam pipe extends only 3.69 cm on each side of the cavity.

In Fig. 10, we plot the value of the secondary peak, which is a measure of the partial reflections, of the wake potential as a function of the maximum value of conductivity used in the absorption model. If the dielectrics have very small values of conductivities, they behave essentially like vacuum. Thus large reflections are expected from the metal boundaries. On the other hand, if the dielectrics have high conductivities, they behave like metals; and large reflections are also expected. For a fixed absorption length, there is an optimum value of the maximum conductivity which provides maximum absorption, and hence minimum secondary peak of the wake potential. For the 7 mm radius cavity, the "best" conductivity model has a maximum conductivity of $0.55 \mathrm{mho} / \mathrm{m}$ at the ends of the beam pipe. Our sensitivity study of the parabolic absorption model has so far only varied the value of the maximum conductivity for a fixed absorption length. Effects of other functional forms of the absorption models and of the lengths of the absorption regions are open to further investigations.
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Fig. 8 Idealized AMOS model of SL4 cavity 3 with resistive loads at the beam pipe ends.


Fig. 9 Transverse dipole wakes from TBCI and AMOS for 0.3 meters for pipe length $=7.46 \mathrm{~cm}$.


Fig. 10 Max. value of secondary peak vs max conductivity of dielectric load.

