SUCCESSIVE LINEARIZATION METHOD - COMPARISON WITH AN EXACT RESULT

H. Lustfeld
Gesellschaft für Schwerionenforschung mbH, D-6100 Darmstadt, Germany
and
Kernforschungsanlage Jülich, D-5170 Jülich, Germany

Abstract

The successive linearization method has been developed to determine the dynamic aperture of storage rings. In the present paper the method is applied to a simple system that can also be solved exactly. It turns out that the dynamic aperture obtained from the successive linearization method disagrees with the exact result. This indicates that the method should be used with some caution.

1. Introduction

One of the most important quantities in beam dynamics of a storage ring is the dynamic aperture. It is defined as the boundary to that area of phase space in which particles remain stable for a given time and do not get lost.

To determine the dynamic aperture one can apply particle tracking methods. However, to calculate the actual trajectories of protons and heavier particles over sufficiently many turns is often beyond the scope of present computers. Therefore the stability of particle trajectories can only be predicted for short times.

A quite different approach is taken by the successive linearization method. Here it is assumed that, even for long times, the actual solutions of the particle trajectories can be sufficiently well approximated by solutions obtained from a successive linearization of the underlying differential equation. With this and some further assumptions criteria are developed which allow one to directly discern between stable and unstable trajectories and to calculate the dynamic aperture.

Of course the question arises how realistic these criteria are in view of the assumptions involved. It is therefore important to compare the successive linearization method with an exact solution as will be done in this paper. The comparison is carried through for the differential equation

$$\frac{d^2}{ds^2} x + k \cdot x - m \cdot \frac{x^2}{2} = 0$$

with

$$\frac{d}{ds} k = \frac{d}{ds} m = m_0$$

Details are given in section 2. It will turn out that the successive linearization method leads to an incorrect result for the dynamic aperture even if it is driven to arbitrarily high order. Consequently some care should be taken when determining the dynamic aperture by the successive linearization method.

2. Calculations and Results

The successive linearization method has been applied to equations of the type

$$\frac{d^2}{ds^2} x + k \cdot x - m \cdot \frac{x^2}{2} = 0 \quad (1)$$

Here \(x\) is the displacement, \(k \cdot x\) describes the quadrupole focusing whereas \(m \cdot \frac{x^2}{2}\) is due to sextupole contributions and is assumed to be small. Eq. (1) can be obtained from the hamiltonian

$$H = p^2/2 + k/2 \cdot x^2 - m \cdot \frac{x^3}{6} \quad (2)$$

An exact solution is possible if \(k\) and \(m\) do not depend on \(s\) and only this case will be investigated in the following. Then

$$x = p = \pm (2E - k \cdot x^2 + m \cdot x^3/3)^{1/2}$$

or

$$x = \pm (2E - k \cdot x^2 + m \cdot x^3/3)^{1/2} = s \quad (3)$$

3. Conclusion

Applying the successive linearization method to eq. (1) with constant \(k\) and \(m\) leads to the following prediction for the dynamic aperture.

It is infinite if one stops after the first linearization step. This is in disagreement with the exact result where the dynamic aperture is finite and determined by eq. (5). This shows that one has to be cautious when applying the successive linearization method to estimate the dynamic aperture in storage rings.
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References


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