Measurements are reported of beam current dependent bunch lengthening in the SRS. The results show that even for the short (~200 ps) high frequency (500 MHz) bunches the functional dependence of bunch length on beam current is characteristic of that which one might expect to find in the presence of turbulent bunch lengthening. A study of the relationship between single bunch current and measured bunch length is made under varying conditions of r.f. power and energy. The information obtained is used to provide an estimate of the broad band impedance of the electron storage ring.

Introduction

It is well known that in electron storage rings a blow-up in the electron bunch length will occur above a certain threshold current. The phenomenon of bunch lengthening has been measured on many machines [1,2,3], and theories have been advanced to explain the observed effects [4,5]. A knowledge of the bunch length is an important parameter in any machine as it determines the peak current and so is important in determining instability behaviour.

This paper is the first report of bunch length measurements on the Synchrotron Radiation Source (SRS) operating in the single bunch mode. The bunch length is measured for a variety of machine conditions and, as will be seen, the bunch length shows a marked dependence on current and is particularly large at the SRS injection energy of 605 MeV.

Apparatus

Bunch length measurements were carried out using a previously reported image dissector tube [6]. This device, whose normal application is as a fast TV camera tube, uses the visible portion of the synchrotron radiation spectrum to produce a Gaussian waveform which represents the longitudinal electron density distribution of the bunch. The final image is displayed on a conventional CRO screen and recorded photographically. The full-width half heights (FWHH) of the recorded traces are then measured to determine the standard deviation, $\delta$, of the density distribution (FWHH = 2.35 $\delta$). A typical dissector output trace for single bunch operation is shown in figure 1. The temporal calibration of the trace is determined using a multibunch beam where the time between adjacent bunches is fixed by the value of the main r.f. operating frequency.

Results

The results of the measurements of the single bunch length as a function of the parameters, current ($I$), synchrotron oscillation frequency ($f_s$) and energy ($E$) are summarised graphically in figures 2 through to 4.

The growth of the longitudinal blow-up as current is accumulated at injection energy is readily apparent (fig.2). In order to investigate the functional relationship between the bunch length and current the experimental values were plotted on a log-log scale. Within the limits of experimental error the plotted data is consistent with a linear fit between $\ln \delta$ and $\ln I$. A least squares fit to the experimental points supports a relationship of the form,

$$\frac{\delta}{\text{FWHH}} = k \frac{I}{f_s}$$

where $k$ is a constant and $a = 2.8 \pm 0.2$.

Figure 2. Dependence of single bunch length on beam current. $E = 605$ MeV, $f_s = 135$ kHz.

The magnitude of the error bars shown on the experimental points are determined by the accuracy limit in measuring the oscillogram trace heights and half-widths (using dividers) and not by the accuracy of the scope.
calibration which is negligible in comparison with the error in measuring the heights and widths.

The dependence of bunch length on the synchrotron oscillation frequency was determined by stacking and storing a fixed single bunch current (15 mA) at injection and altering the value of \( f_s \) by gradually increasing the r.f. cavity volts. In fact, while increasing \( f_s \) through the range shown in figure 3 some beam loss was experienced, however the measured values of bunch length could be normalised to 15 mA using the empirical relationship discussed above (as will be seen later this relationship has been verified in multibunch at a different \( f_s \) value from that shown in figure 2, so this normalisation was carried out with confidence).

Also shown, for comparison, in figure 3 is the natural bunch length which results from the equilibrium between radiation damping and quantum fluctuations, and is calculated from the analysis of Sands [7].

Finally the variation in the longitudinal blow-up with increasing energy was studied. The results are shown in figure 4 and again the solid line indicates the calculated natural bunch length for the machine parameters pertaining at the time of the measurement.

**Discussion**

The rapid increase in bunch length as current is increased at injection energy is perhaps the most striking feature of the data presented here. This phenomenon has also been confirmed in multibunch operation (see figure 5) where the data, analysed in the same manner as that of the single bunch, yields a relationship of the form,

\[
\left( \frac{\sigma_z}{\sigma_0} \right) = k' \left( \frac{I}{I_0} \right) ^ {3/2}
\]

where \( k' \) is a constant and \( b = 2.95 \pm 0.1 \).

Bunch lengthening in other storage rings has been found to be in good agreement with the turbulent instability model where the bunch lengthens with increasing current to provide sufficient Landau damping for stability (see e.g. ref. 2). The strength of the dependence of \( \sigma_z \) on \( I \) in this model is similar to that found on the SRS and is given by [2],

\[
\left( \frac{\sigma_z}{\sigma_0} \right) = \frac{2 \pi e R f_0 F}{F \sigma_z E} \frac{I_0}{Z/p}
\]

where \( R \) is the radius of the ring, \( c \) is the speed of light, \( e \) is the electronic charge, \( f_0 \) is the orbit frequency, \( F \) is a form factor (equal to 6 for a longitudinal Gaussian distribution), \( I_0 \) is current per bunch and \( Z/p \) is the longitudinal broad band impedance.

It should be noted that turbulent bunch lengthening is most applicable to rings with low r.f. frequencies (~ 50 MHz) and long electron bunches which
probe the vacuum chamber at low frequencies where $z/p$ is constant. Clearly, from figures 3 and 4, as the SRS bunch length falls off with increasing $f_r$ and as the dependence of $\sigma$ on these latter parameters is not as one would expect from turbulent bunch lengthening and presumably other competing effects determine the equilibrium length. However if we assume that at low energy and long bunch lengths the constant $k$ in equation 1 is proportional to $z/p$ then an estimate of the SRS broad band impedance is made possible and is $\approx 20 \, \Omega$. This value is not untypical of that which one might expect for a radiation source with its large number of vacuum chamber cross-section variations.

A puzzling feature emerges from figures 2 and 5. If one compares the 1 mA single bunch length (from figure 2) with the 160 mA multibunch length (from figure 5) it is apparent that the multibunch length exceeds the single bunch length by a factor of $\approx 2.3$, this despite the fact that the current per bunch is the same in both cases. A preliminary study of the effects on the single bunch length with current in other r.f. buckets has been made. Currents of 1 mA have been stacked in two bunches 160 ns apart (diametrically opposite) and 4 ns apart. However the measured bunch length is no different from the 1 mA single bunch case. Continued studies of the effects on bunch length of filling different numbers of bunches at varying spacings and intensities is planned.

Figure 3 shows a monotonic decrease in $\sigma$ with increasing $f_r$ as would be expected from both simple Sands type theory and from the turbulent bunch model. The functional dependence between the parameters is not accurately known from the relatively few data points but it appears to fall off rather slower than the inverse-square dependence given by the turbulent model. Similarly the dependence of bunch length on energy is non-simple, falling rapidly from injection energy to 1.0 GeV and then staying fairly constant up to 1.8 GeV. The most noteworthy feature here is that the bunch length decreases with increasing energy (at least between 0.6 GeV and 1.0 GeV) in contrast to the predictions of the natural bunch length model were $\sigma \propto E^{1/2} (V_0 \cos \phi)^{-1/2}$, where $V_0$ is the accelerating voltage and $\phi$ is the synchronous phase angle.

Conclusions

The single bunch length in the SRS has been measured as a function of the machine parameters, current, energy and synchrotron oscillation frequency. No theory has been advanced to explain the dependence of the measured bunch lengths on these parameters but the approximate one-third power law dependence of length on current has been used to obtain an estimate of the broad band impedance of the electron storage ring.

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