The essential specifications of the TRIUMF KAON Factory beams is 100 mA at 50 GeV. The TRIUMF proposal consists of 5 rings: an Accumulator feeding a 50 Hz Booster synchrotron which raises the energy to 3 GeV. The Booster injects into a 5 times larger accumulator called the Collector which, when it has collected 5 Booster pulses, feeds them into the 10 Hz, 30 GeV synchrotron (the Driver). The fifth ring (the Extender) is a stretcher to meet the requirement for a slow extracted beam. Rings A and B have a radius of 34 = 34 while rings C, D, and E have a radius of 170 m. The distinguishing features of the two synchrotrons (B and D) are that they operate below transition: (g = 9.4 = 9.4), they have high circulating currents (2.8 A x B), and relatively high harmonic numbers (h = 45 & 225). Thus, there is the possibility of many unstable coupled bunch modes but this is mitigated by the fact that the synchrotrons are rapid cycling (10 Hz and 10 Hz). There is, therefore, less time for any instabilities to grow.

Longitudinal Instabilities

The Microwave Instability

The microwave instability occurs if the longitudinal emittance of a beam of given intensity is too small. The accumulated longitudinal emittance in the A ring was chosen to be very large compared with the TRIUMF emittance but not so large that extra cavities would be needed in the Booster (where space is at a premium). In this way, a longitudinal emittance of 0.004 eV-s was arrived at. To test for microwave stability, we cast the Keil-Schnell-Bousard criterion into a form that depends upon longitudinal emittance, e_L (in eV-s), and rf voltage V:

\[ \frac{Z_1}{n} < \frac{1 \ h \ E \ c}{2 \ \pi \ I \ R} \left( \frac{\gamma}{\nu} \right)^{3/2} \left( \frac{\gamma}{\nu} \right)^{3/4} \left( \frac{V \cos \theta_2}{\nu} \right)^{1/4} \]  

I = circulating current, E = total energy of beam, and \( \eta = \gamma^2 - \gamma^2 \). We have taken the form factor F to be 2/3 as appropriate for a parabolic distribution. The longitudinal impedance divided by mode number \( Z_1/n \) has a negative imaginary space charge term and a positive imaginary inductive wall term whose magnitude we take to be 5 \( \Omega \) in all 5 rings.

In rings A and B, the space charge term dominates. Condition (1) is worst at extraction in B where it gives an upper limit on the circulating current of 7.2 A. We are safe even if the inductive wall term is 100 \( \Omega \).

In the Driver, the inductive wall effect dominates and condition (1) is again worst at extraction. If we assume the same emittance 0.004 eV-s and \( \gamma = 30 \), (1) is violated by a factor 5, i.e., we require \( |Z_1/n| \) to be less than a difficult-to-attain 1 \( \Omega \). Two approaches are used together to alleviate the problem; \( \eta \) is made larger by making \( \gamma \) imaginary \( (\gamma = 30) \), and the longitudinal emittance is artificially increased in ring C by a factor of 3. In this way the upper bound on the broad-band impedance becomes 10 \( \Omega \).

Coupled-Bunch Instabilities

The computer code BBI has been used to study these instabilities. Four sources of impedance have been considered for longitudinal coupled-bunch instabilities. The space charge impedance, the broad-band resonator, the kicker modules, and the rf cavities.

Inductive wall effect and space charge: The broad-band resonator (which gives rise to the inductive wall effect at low frequencies) and the direct space charge effect contribute only a real part to the frequency shifts. Landau damping is lost when the magnitude of the frequency shift becomes larger than the half spread in incoherent frequencies. In an effort to retain as much stability as possible, synchrotron frequency spreads have been kept large by lowering the rf voltage so that the bucket area is usually only 20% larger than the bunch. (This is also essential for reducing the Landau tune shift at the beginning of the cycle by increasing the bunching factor.) Towards the end of the acceleration cycles in B and D, however, the bucket area tends to grow rapidly because \( \gamma_c \) is being approached. Owing to beam loading, V cannot be made small enough to maintain a constant bucket area. Landau damping of longitudinal modes is therefore only present in ring A and in ring C after longitudinal emittance blow-up and in the first half of the acceleration cycles in rings B and D.

Kickers: The contribution that the kicker impedances make to the growth rates of the unstable modes is negligible. The reason is that their resonant frequencies (-3 MHz in A, B and C and -1 MHz in D and E) are too small compared with the inverse time length of the bunches (50-300 MHz). BBI actually finds growth rates on the order of 100/s for even numbered within-bunch modes but these are spurious and arise from the fact that BBI uses sinusoidal perturbation modes. The perturbed charge density for even numbered sinusoidal modes does not integrate to zero as it should. Hence, the spectral density functions of these modes do not vanish for zero frequency and this leads to the prediction of their instability in the presence of resonators whose frequencies are very small compared with the bunch frequency. Using Hermitian modes, for example, the kicker induced growth rates are found to be no more than 5/s.

RF cavities: With a final beam power of 3 MW, it is easy to see that the rf system will be one of the most challenging aspects of the KAON Factory to design. The peak beam power requirements of the Booster and Driver are, respectively, 0.53 MW and 5.7 MW. To reduce beam loading effects and the overall cost of the rf system, one tries to minimize the number of cavities, increasing the power per cavity to the maximum feasible. In all rings, fast feedback around the final amplifier stage is required to reduce the shunt impedances (and the cavity Q values) apparent to the beam. In this way, the cavity characteristics of Table 4.4.2.1 have been derived. For stability of the rf system it is necessary that the rf cavities be detuned with respect to the beam. In Table 4.4.2.1 we have listed this detuning as the difference between the cavity resonance frequency and the rf frequency (af) in units of the revolution frequency (fo). Both in ring B and in ring D there is a point near extraction where \( af = f_0 \), i.e., the cavities are tuned to the n = 1 coupled bunch mode.
Table 4.4.2.1. RF cavity characteristics used in BBI

<table>
<thead>
<tr>
<th>Ring</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rg (kHz)</td>
<td>75</td>
<td>29</td>
<td>7.5</td>
<td>54</td>
<td>15</td>
</tr>
<tr>
<td>Q</td>
<td>250</td>
<td>20</td>
<td>13</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>$\Delta\gamma/\gamma_0$</td>
<td>0.05</td>
<td>0.2-1.25</td>
<td>0.22</td>
<td>0.22</td>
<td>1.5</td>
</tr>
</tbody>
</table>

BBI results for this fundamental cavity mode are as follows. In ring A, we are safe for two reasons: we have Landau damping and $1/\tau = 50/s$ for the worst mode ($n = 44$), i.e., only one e-folding time. In the Booster, the maximum growth rate in the region where there is no Landau damping is found to be 4400/s. Actually $1/\tau$ will be about an order of magnitude smaller than this figure because we have not included in the calculation the feedback system which is necessary to deal with the transient beam loading. Even so, the instability will have to be dealt with because it represents more than 4 e-folding times. In ring C, with sufficient feedback to cope with the transient loading of a partially filled ring, the maximum growth rate is only 22/s. In the Driver, $1/\tau = 440/s$ at injection but with a longitudinal emittance blow-up in C, we have Landau damping. Near extraction where there is no Landau damping, $1/\tau = 47/s$ during a time interval of 15 ms. In ring E, there is no Landau damping but the growth rate is only 7/s.

Potentially more serious sources of longitudinal instability are the parasitic modes of the rf cavities. Lacking an experimental prototype of the rf cavities, it is not possible to make accurate predictions. Roughly, cavities in our frequency range can have parasitic resonances with shunt impedances of $\sim 10^5 \Omega$ and quality factors of $\sim 5,000$. In rings A, C and E, these resonances can be made harmless by tuning them so that they lie between revolution harmonics. In rings B and D, the revolution frequency changes with $\beta$ so the best one can do is to ensure that the resonances are swept through quickly, i.e., to make sure that the parasitic resonances fall between revolution harmonics at times when $\beta = 0$ (injection and extraction). An impedance of $10^3 \Omega$ can cause growth rates in rings B and D of 10,000 and 1,000/s respectively. Passive mode damping or the parasitics can reduce these figures by a factor of 10 or so but this is of little help because then the resonances are broader and take longer to cross. In the Booster, the longest crossing time of a resonance with $\beta = 1.0$ is only 0.3 ms. This corresponds to 3 e-folding times, which is tolerable if the particular coupled-bunch mode is later damped (by, say, a low power wide-band feedback system). In the Driver, which has higher Q cavities, the crossing time of a $Q = 5,000$ parasitic is 1 ms. A coupled-bunch mode is therefore only driven for $\sim 1$ e-folding time.

Transverse Instabilities

Mode Coupling

Also known as the PETRA instability or head-tail turbulence, this instability is analogous to the longitudinal microwave instability in that the criterion for its avoidance is equivalent to the transverse beam criterion. When $\sigma$, the rms bunch length, is large compared with $b$, the beam pipe half height, the criterion can be cast into the form

$$|\Delta\omega|/\omega_0 < \nu_0 \sigma/b.$$  

(2)

$\nu_0$ is the synchrotron tune, $\omega_0 = B_0/c/R$ and $\Delta\omega$ is the betatron frequency shift. For our case, the dominant impedance is space charge so the LHS of (2) is simply the difference between the coherent and incoherent Laslett tune shifts ($\Delta\omega_{coh} - \Delta\omega_{inco}$). This quantity is

in all 5 rings of the same order as $\nu_0$. Since $\sigma \gg b$, this instability poses no danger.

Coupled-Bunch instabilities

Since all rings run below transition, negative chromaticities are preferred. It is well known that if the chromaticity is made sufficiently large compared with the inverse length of a bunch then all transverse modes are stable. Quantitatively, we can write this stability criterion as

$$|\xi| > (m+1) \left| n h/(2vb) \right|,$$

(3)

where $v = \text{transverse tune}$, $B = \text{bunching factor}$ and $m = \text{mode number}$. For any given chromaticity, sufficiently high-order modes will be unstable anyway but with steadily diminishing growth rates. Even so, the stability criterion (3) requires unrealistically large chromaticities in all rings except E. Also, particle tracking studies show that in all rings but especially A and B, changing $\xi$ by as little as one unit from its natural value significantly increases magnet aperture requirements. In what follows, therefore, we assume $\xi = -1.3$, the natural value for all 5 rings. Then for modes $m < 4$, (3) implies that ring E is transversely stable, rings A, B and C are not and ring D is unstable below 13 GeV.

We consider three sources of transverse impedance: the broad-band resonator, kickers, and the resistive wall. The computer code BBI was used to calculate risetimes. The broad-band resonator impedance was taken as $2 B^2/5$ times $B$. Kickers were represented as resonators fit to the impedances given by Nassibian. A resistivity of 130 $\mu\Omega$ cm, corresponding to stainless steel, was used for the resistive wall effect.

The BBI results have been summarized in Table 4.4.2.III for the (worst) vertical transverse case. The broad-band resonator has a weak beneficial damping effect. The kicker growth rates are probably optimistic by a factor of 2 or 3 because we have used impedances only for matched kickers. The resistive wall impedance is proportional to $(n - \nu)^{-5}$ where $n$ is the integer just above the tune $\nu$. In order to minimize the resistive wall effect, the transverse tunes in all 5 rings have been chosen to lie just above an integer. Nevertheless, the resistive wall instability is dangerously large in rings C and D. The situation in rings A and C can perhaps be improved with a different choice of wall material. In ring D, however, we have the conflicting requirement of high resistivity for keeping eddy current-induced beam pipe heating and septum components within control. An attractive design in this respect is that of LAMPF II. Our calculations indicate that their proposed beam pipe has up to 30 times smaller resistive wall impedance (0.1 $\text{MD/m}$). This would reduce the resistive wall instability in the Driver to less than 100/s.

Table 4.4.2.III. Transverse BBI results

<table>
<thead>
<tr>
<th>Ring</th>
<th>BB Growth rates (s⁻¹) for Kicker Res. Wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-16</td>
</tr>
<tr>
<td>B at injection</td>
<td>11</td>
</tr>
<tr>
<td>C</td>
<td>-190</td>
</tr>
<tr>
<td>D at injection</td>
<td>-57</td>
</tr>
</tbody>
</table>

Other high Q sources of impedance will exist. For example, parasitic deflecting modes in the rf cavities, if undamped, can have impedances of the same order of magnitude.
As with the longitudinal case, in rings A, C and E, these cavity modes can be tuned to lie between neighboring coupled-bunch modes. For a high Q transverse impedance of 30 MHz/m, growth rates are worst at injection where they are indeed very large: 160,000 m/s and 12,000 m/s for B and D respectively. However, they are saved by the fact that B is also largest early in the cycle so crossing times are only 0.03 ms and 0.3 ms for Q values of 1000 and 5000 in B and D respectively. The resulting number of e-folding times (5 and 3.5) are borderline: narrow-band dedicated feedback for each dangerous cavity mode may be required.

Stabilizing Systems

Longitudinal Microwave Instability

No theoretical investigation has been made for longitudinal emittance blow-up by phase-modulated high harmonic cavities, but the following empirical rule has been found at the CERN PS in a recent machine experiment,[9] on a 3.5 GeV/c flat top, where the principal rf (h = 20) is 9.22 MHz:

\[ \epsilon(t)/\epsilon(0) = \text{const.} \sqrt{\tau(0)} \]

with \( \epsilon \) as the high harmonic excitation voltage (h = 433), \( \tau(0) \) as the initial bunch length in time, and the constant is ~53,000 in units of volts and seconds.

At present it is not clear whether and how these rules scale with rf and drive harmonics, but applying it as it stands to the Collector requires a drive voltage of about 90 kV to achieve the blow-up factor of 3 in 10 ms. The ratio of excitation frequency to rf cannot be maintained, however, as it would mean a 1.3 GHz cavity.

Longitudinal Coupled-Bunch Instabilities

Landau damping by a bunch-to-bunch population spread is excluded because the growth rates due to the rf cavity modes are comparable with the real part of the frequency shifts and an rms population spread of something like 20% would be required. Active damping will therefore be essential. Coupled-bunch modes around the rf frequency (e.g. \( n = h, h \pm 1, h \pm 2 \)) will be damped by feedback loops intimately connected with the rf system. Parasitic cavity modes, if they are all known, can be damped by dedicated feedback loops.

Transverse Coupled-Bunch Instabilities

There are three possible methods of curing coupled-bunch transverse instabilities: Landau damping using either octupoles or a variation in bunch population, or direct damping of unstable modes by feedback.

Tune Spread: In all rings except E and D near extraction (in both of these cases the beam is usable anyway), the frequency shift is dominated by the space charge effect. The required tune spread are thus simply the difference between the coherent and incoherent Laslett tune shifts. For rings A and B the incoherent tune shifts are already large as we allow them (5v = 1/6) and so additional tune spread due to octupoles is excluded. In rings C and D where the shift is only ~0.1, there is room for the additional spread of 0.03 required to Landau damp all the transverse modes. In both cases, the required tune spread can be achieved with a pair of dc octupoles per superperiod (the superperiodicity is 12). With lengths of 0.2 m, the required strengths would be 750 T/m\(^2\) in front of a D-quad for vertical tune spread and 300 T/m\(^2\) in front of an F-quad for horizontal tune spread.

Decoupling: The criterion for decoupling the individual bunches can be written:

\[ dN/dN_{\text{rms}} \cdot \omega_0/\omega_{\text{cloc}} > \text{growth rate} \]

where \( N \) is bunch population and \( \omega_{\text{cloc}} \) is the local coherent Laslett tune shift. Vertical instabilities due to the resistive wall effect and kickers can be damped in rings A and B with as little as 1% population spread. In rings C and D, the vertical resistive wall growth rate is larger while \( \omega_0 \) is 5 times smaller; a 10% population spread is required. For horizontal instabilities, bunch decoupling is a less promising method even though the resistive wall effect is 4 times smaller than in the vertical case. This is because the horizontal \( \omega_{\text{cloc}} \) is 30 times smaller than the vertical one. Only in ring A (where there are no horizontal kickers) is there some hope of using this method; a 4% population spread is required in this case.

One can show that for each MHz/m of transverse (horizontal) impedance, population spreads of 300% and 100% are needed in rings B and D respectively. Hence, we cannot realistically expect to use this method to damp instabilities caused by, for example, high-Q transverse parasitic rf cavity modes.

Feedback: Feedback is introduced as an artificial impedance which depends upon pick-up and deflector geometry and upon the gain.

Assuming a 4 m deflector in the Driver and a standard pickup geometry, the gain required to cancel the vertical resistive wall impedance is 64 db. With this deflector geometry, we require a maximum deflecting voltage of 380 V per mm of beam position error. Allowing a maximum error of 4 mm and assuming push-pull operation into a \( \times 30 \) \( \times \) 30 ohm stripline, one ends up with 2 \( \times \) 4.1 kW wideband (350 MHz) amplifiers. Lenetkenh's deflector brings this figure down, but it remains an expensive item. A wideband feedback system with a gain essentially determined by the low frequency impedance is a luxury and the calculations suggest another solution: a narrow-band system (~3 MHz) with the final tube in the tunnel working directly into a higher impedance electrostatic deflector. For 4 defectors and push-pull, the tubes have to deliver a peak voltage of 1.5 kV.

For the Booster, the required gain is 45 db for the resistive wall effect and the required maximum deflector voltage is 46 V per mm offset. To allow for a 4 mm position error, one has to provide 2 \( \times \) 85 W push pull into 50 ohm. With respect to this modest power one can envisage a bandwidth of up to 20 MHz, say, to cope with unpredictable kicker impedances.

In both B and D, parasitic rf cavity resonances will require dedicated narrow band feedback loops. These can be specified once we have built the cavities and measured these modes.

References

[8] H.A. Thiessen, these proceedings.