WAKE POTENTIALS OF A RELATIVISTIC POINT CHARGE CROSSING A BEAM-PIPE GAP: AN ANALYTICAL APPROXIMATION

G. Dôme,
European Organization for Nuclear Research (CERN), CH-1211 Geneva 23, Switzerland

Abstract

Although much work has been devoted recently to the numerical calculation of wake potentials, there is at present no analytical expression for the wake of a point charge crossing a gap in a beam-pipe of finite diameter. For a relativistic charge the wake potential can be computed in a simple manner from an integral over the electric field at the radius of the beam-pipe. In the case of a pill-box cavity, our basic assumptions are that the resonant frequencies and the field at the radius of the beam-pipe are essentially the same as in a cavity without beam hole. The summation over the infinity of modes can still be carried out as far as reflections from the outer cavity wall do not contribute; this is sufficient to compute the total loss for short bunches.

Geometry

Let us consider the gap between a double-step cross section change in a round beam-pipe.

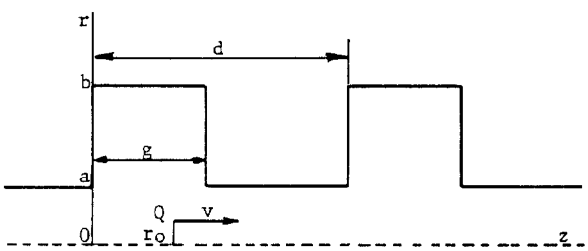

Fig. 1 Periodic structure repeating the gap g with a period d.

In the literature [1,2], the periodic structure of Fig. 1 has been used many times as a simple model for numerical computations. In the following, we shall neglect the coupling between cells, so that the wake potential per cell will be assumed to be the same as in the limiting case $d = \infty$, i.e. for a single cavity formed by a double step in the diameter of the beam-pipe.

Longitudinal wake potential

The longitudinal wake potential $w_{\parallel}(\tau)$ seen by a unit test charge which follows an exciting charge Q with the same velocity v but at a distance τ in time is defined by

$$w_{\parallel}(\tau) = -\frac{1}{Q}\int_{-\infty}^{+\infty} E_{\parallel}(z,\tau+\frac{z}{v})\,dz \qquad (1)$$

with a Fourier transform

$$W_{\parallel}(\omega) = \int_{-\infty}^{+\infty} w_{\parallel}(\tau)\, e^{-j\omega\tau}\,d\tau = -\frac{1}{Q}\int_{-\infty}^{+\infty} E_{\parallel}(z,\omega)\, e^{j\omega\frac{z}{v}}\,dz \qquad (2)$$

The exciting charge traverses the cavity at $r_o, \phi_o=0$ whereas the test charge is at r,ϕ. The electromagnetic fields in the cavity can be expressed as infinite sums [3] over all the solenoidal and irrotational modes E_{mpn}; for both kinds of modes,

$$E_z \sim J_m(k_r r)\cdot\cos m\phi\cdot\cos(\frac{n\pi}{g}z) \qquad (3)$$

where $k_r b = j_{mp}$ is the p^{th} root of the $J_m(x)$ Bessel function. In what follows the subscript ℓ will be used as a short notation for the doublet (p,n); the angular frequency ω_ℓ of a E_{mpn} mode is given by

$$\omega_\ell^2 = c^2 k_\ell^2 \quad \text{with} \quad k_\ell^2 = k_r^2 + (\frac{n\pi}{g})^2 \quad \text{and} \quad k_r = \frac{j_{mp}}{b} \qquad (4)$$

In the limiting case of infinite Q_ℓ and of ultra-relativistic particles (v=c), one obtains for a pill-box cavity without beam hole:

$$W_{\parallel}(\omega) = -\sum_{p=1}^{\infty}\frac{\varepsilon_m}{\varepsilon_o \pi g}\,\frac{J_m(j_{mp}\frac{r_o}{b})\,J_m(j_{mp}\frac{r}{b})}{j_{mp}^2\left[J'_m(j_{mp})\right]^2}\cos m\phi\,\cdot \qquad (5)$$

$$\cdot\sum_{n=-\infty}^{+\infty}\frac{j\omega}{\omega^2-\omega_\ell^2}\left[2-(-1)^n e^{j\omega\frac{g}{c}}-(-1)^n e^{-j\omega\frac{g}{c}}\right]$$

where ε_m is Neumann's symbol: $\varepsilon_m=1$ when $m=0$, $\varepsilon_m=2$ when $m\neq 0$.

Now introduce a beam-pipe of radius "a". For $r\leq a$, the integral (2) selects the Fourier component of $E_{\parallel}$ whose phase velocity along z is equal to v. Since this component varies radially as $I_m(kr/\beta\gamma)$, for ultra-relativistic particles the integral (2) is equal to

$$\frac{I_m(\frac{kr}{\beta\gamma})}{I_m(\frac{ka}{\beta\gamma})}\cdot\left[\text{Integral computed on the gap at } r=a\right] \qquad (6)$$

$$\approx (\frac{r}{a})^m\cdot\left[\text{Integral at } r=a\right] \quad \text{when} \quad \frac{ka}{\beta\gamma} \ll 1$$

The same remark applies to the integral for the exciting charge at r_o.

The basic assumption is that for $r\geq a$, E_z of each mode is still given by (3), which entails that the resonant frequencies are still given by (4). [In fact, they are increased by $O(a^3/b^2g)$.] With this assumption and taking (6) into account, (5) becomes for a cavity with beam hole:

$$W_{\parallel}(\omega) = -N_{\parallel}\cdot\sum_{p=1}^{\infty}\frac{\varepsilon_m}{\varepsilon_o\pi g}\,\frac{\left[J_m(j_{mp}\frac{a}{b})\right]^2}{j_{mp}^{\;2}\left[J'_m(j_{mp})\right]^2}\,\cdot \qquad (7)$$

$$\cdot\sum_{n=-\infty}^{+\infty}\frac{j\omega}{\omega^2-\omega_\ell^{\;2}}\left[2-(-1)^n e^{j\omega\frac{g}{c}}-(-1)^n e^{-j\omega\frac{g}{c}}\right]$$

where

$$N_{\parallel} = (\frac{r_o}{a})^m\,(\frac{r}{a})^m\cos m\phi \qquad (8)$$

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The Fourier transform to time domain reads

$$w_{\parallel}(\tau) = N_{\parallel}\cdot\sum_{p=1}^{\infty}\frac{\varepsilon_m}{\varepsilon_o\pi g}\,\frac{\left[J_m(j_{mp}\frac{a}{b})\right]^2}{j_{mp}^2\left[J_m'(j_{mp})\right]^2}\cdot$$

$$\cdot\sum_{h=-1}^{+1}(1+\delta_{ho})(-1)^h\,H(\tau-h\,\frac{g}{c})\cdot \tag{9}$$

$$\cdot\sum_{n=-\infty}^{+\infty}\cos(nh\pi)\,\cos\omega_\ell(\tau-h\,\frac{g}{c})$$

where $H(\tau)$ is Heaviside's unit function: $H(\tau)=0$ for $\tau<0$, ½ for $\tau=0$, 1 for $\tau>0$.

Poisson summation formula yields, for any real h:

$$\sum_{n=-\infty}^{+\infty}\cos(nh\pi)\cdot\frac{\sin(\omega_\ell\tau)}{\omega_\ell} = \frac{g}{c}\,\mathrm{sgn}(c\tau)\cdot$$

$$\cdot\sum_{n=-\infty}^{+\infty}H\left[(c\tau)^2-(2n-h)^2g^2\right]\cdot J_o\left[k_r\sqrt{(c\tau)^2-(2n-h)^2g^2}\right] \tag{10}$$

This relation converts a sum over modes in the z-direction into a sum of wave fronts bouncing back and forth between the planes z=0 and z=g.

Normal mode description

Eq. (10) can be used to show that in (9), $H(\tau-hg/c)$ may be replaced by $H(\tau)$ without changing the result, in full agreement with relativistic causality.

After some manipulation, (9) can be rewritten as

$$w_{\parallel}(\tau) = N_{\parallel}H(\tau)\cdot\sum_{\ell}K_{m\ell}\,\cos(\omega_\ell\tau) \tag{11}$$

where

$$K_{m\ell} = \frac{\varepsilon_m}{\varepsilon_o\pi g}\cdot\frac{2\left[J_m(j_{mp}\frac{a}{b})\right]^2}{j_{mp}^2\left[J_m'(j_{mp})\right]^2}\,\varepsilon_n\left[1-(-1)^n\cos(\omega_\ell\,\frac{g}{c})\right]$$

$$= \frac{\left|\int_0^g E_{z\ell}\cdot e^{j\omega_\ell\frac{z}{v}}\,dz\right|^2_{\text{at } r=a}}{\varepsilon_o\int\left|\vec{E}_\ell\right|^2 dV} \tag{12}$$

i.e. $K_{m\ell}=(\omega R/Q)_\ell=2k_{m\ell}$ of P. Wilson [4]; here R = "Circuit R" = ½ "Linac R".

This is the form which was first proposed by P. Wilson and K. Bane [5] in 1977, essentially for $c\tau>g$. In 1980, using causality arguments, K. Bane [6,7] proved this form to stay valid for all τ under rather general circumstances, in particular for a periodic structure.

Wave front description

With (10), (9) can be put into the alternate form

$$w_{\parallel}(\tau) = N_{\parallel}\cdot\frac{\partial}{\partial(c\tau)}\left[\frac{\varepsilon_m}{\varepsilon_o\pi}\,U(c\tau)\right] \tag{13}$$

where

$$U(c\tau) = H(c\tau)\cdot\sum_{h=-1}^{0}(-1)^h\,F_m(x_{oh},y) + \sum_{n=1}^{\infty}H(c\tau-2ng)\cdot$$

$$\cdot\sum_{h=-1}^{+1}(1+\delta_{ho})(-1)^h\,F_m(x_{nh},y) \tag{14}$$

and

$$F_m(x,y) = \sum_{p=1}^{\infty}\frac{2\left[J_m(j_{mp}y)\right]^2}{j_{mp}^2\left[J_m'(j_{mp})\right]^2}\,J_o(j_{mp}x)$$

with

$$y = \frac{a}{b},\quad x_{nh} = \frac{1}{b}\sqrt{(c\tau-hg)^2-(2n-h)^2g^2} \tag{15}$$

Transverse wake potential

It is defined by

$$\vec{w}_\perp(\tau) = \frac{1}{Q}\cdot\frac{c}{v}\int_{-\infty}^{+\infty}\vec{F}_\perp(z,\tau+\frac{z}{v})\,dz \tag{16}$$

where $\vec{F}_\perp = \vec{E}_\perp+\left[\vec{v}\times\vec{B}\right]_\perp$

When computed from the sum over all solenoidal and irrotational modes E_{mpn} of a pill-box cavity without beam hole, its Fourier transform compared to (5) is found to be

$$\vec{W}_\perp(\omega) = c\cdot\mathrm{grad}_\perp\,\frac{W_{\parallel}(\omega)}{j\omega} \tag{17}$$

which is merely an expression of the Panofsky-Wenzel theorem [7,8]

$$\frac{\partial}{\partial(c\tau)}\,\vec{w}_\perp(\tau) = \mathrm{grad}_\perp\,w_{\parallel}(\tau)\qquad\text{for a given m} \tag{18}$$

Let

$$N_\perp\Big|_{\substack{r\\ \phi}} = (\frac{r_o}{a})^m\cdot m(\frac{r}{a})^{m-1}\begin{bmatrix}\cos m\phi\\ -\sin m\phi\end{bmatrix} \tag{19}$$

For a cavity with beam hole, $\vec{w}_\perp(\tau)$ is given by an expression analogous to (9) where $N_{\parallel}$ is replaced by $c/a.\vec{N}_\perp$, and $\cos\,\omega_\ell(\tau-hg/c)$ is replaced by $\omega_\ell^{-1}\sin\omega_\ell(\tau-hg/c)$.

Normal mode description

With $K_{m\ell}$ given by (12) we have

$$\vec{w}_\perp(\tau) = \frac{c}{a}\,\vec{N}_\perp\cdot H(\tau)\sum_{\ell}K_{m\ell}\,\frac{\sin(\omega_\ell\tau)}{\omega_\ell} \tag{20}$$

Wave front description

With $U(c\tau)$ given by (14),

$$\vec{w}_\perp(\tau) = \frac{1}{a}\vec{N}_\perp \frac{\varepsilon_m}{\varepsilon_o \pi} U(c\tau) \tag{21}$$

The function $F_m(x,y)$

In the following we take $x \geq 0$, $0 \leq y \leq 1$.

From the asymptotic expansion of Bessel functions it can be seen that

$$-\frac{\partial}{\partial x} F_m(x,y) = \frac{1}{\pi x y} \sum_{p=0}^{\infty} \sum_{q=-1}^{+1} (1+\delta_{qo})\, H(2p+2qy) \cdot$$

$$\cdot \left\{ \cos\left(\frac{p-q}{2}\pi\right) \cdot \left[x^2-(2p+2qy)^2\right]^{\frac{1}{2}} \cdot H\left[x^2-(2p+2qy)^2\right] \right. \tag{22}$$

$$\left. +(-1)^m \sin\left(\frac{p-q}{2}\pi\right) \cdot \left[(2p+2qy)^2-x^2\right]^{\frac{1}{2}} \cdot H\left[(2p+2qy)^2-x^2\right] \right\}$$

+ higher order terms + regular terms.

This relation converts a sum $F_m(x,y)$ over modes in the r-direction into a sum of wave fronts propagating radially, being diffracted at r=a and reflected at r=b. The singularities occur at $x=(2p+2qy)$ or, with (15), at

$$b x_{nh} = \sqrt{(c\tau-hg)^2-(2n-h)^2 g^2} = 2pb + 2qa$$

i.e.

$$c\tau_{nh} = \sqrt{(2pb+2qa)^2+(2n-h)^2 g^2} + hg \tag{23}$$

where $p,n = 0,1,2,\ldots$ $q,h = -1,0,+1$, $2n-h \geq 0$.

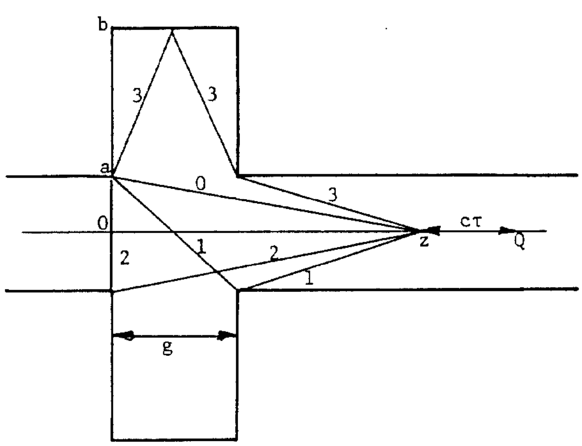

Fig. 2 A few diffracted rays

The primary wakefields are produced by diffraction at the square corners r=a of the cavity; secondary wakefields are then produced by reflection of the primary diffracted fields at the outer wall r=b, and/or subsequent diffraction at the square corners. The diffracted fields eventually catch up the test particle which is a distance $c\tau$ behind the exciting particle, at a distance z in the beam-pipe. We now show that the values (23) of $c\tau$ for which $w_{\parallel}(\tau)$ and $\vec{w}_\perp(\tau)$ have singularities are the limiting values of $c\tau$ for which the catching-up occurs far (i.e. for $z\to\infty$) in the beam-pipe.

For the diffracted ray 0, catching up occurs at a distance z such that

$$c\tau + z = \sqrt{a^2+z^2}$$

$\lim_{z\to\infty} c\tau = c\tau_o = 0;$ this case corresponds to p=0, q=0, n=0, h=0.

For the diffracted ray 1,

$$c\tau + z = \sqrt{(2a)^2+g^2} + \sqrt{a^2+(z-g)^2}$$

$\lim_{z\to\infty} c\tau = c\tau_1 = \sqrt{(2a)^2+g^2} - g;$ this case corresponds to p=0, q=1, n=0, h=-1.

For the diffracted ray 2,

$$c\tau+z = 2a + \sqrt{a^2+z^2}$$

$\lim_{z\to\infty} c\tau = c\tau_2 = 2a;$ this case corresponds to p=0, q=1, n=0, h=0.

For the diffracted ray 3,

$$c\tau+z = \sqrt{(2b-2a)^2+g^2} + \sqrt{a^2+(z-g)^2}$$

$\lim_{z\to\infty} c\tau=c\tau_3 = \sqrt{(2b-2a)^2+g^2}-g;$ this case corresponds to p=1, q=-1, n=0, h=-1.

If $x+2y<2$ (i.e. $\tau<\tau_3$), the series for $F_m(x,y)$ can be summed analytically. The result is

$$F_m(x,y) = \frac{1-y^{2m}}{2m} - f_m\left(\frac{x}{2y}\right) \tag{24}$$

the first term being (-log y) when m=0;

$$f_m(u) = \frac{2}{\pi}\int_0^{\theta=\mathrm{arc\,sin}\,u} \frac{\sin(2m\theta)}{2m\sin\theta} \cdot d\sin\theta + \delta_{mo} \cdot H(u-1) \cdot \log u \tag{25}$$

where arc sin u = $\pi/2$ if $u \geq 1$; m=0,1,2,...

Case $c\tau < \min(c\tau_3,\ 2g)$

Then (14) can be expressed analytically and reduces to

$$U(c\tau) = H(c\tau) \cdot \left[f_m\left(\frac{\sqrt{(c\tau+g)^2-g^2}}{2a}\right) - f_m\left(\frac{c\tau}{2a}\right)\right] \tag{26}$$

The condition $\tau<\tau_3$ means that τ is so short that waves reflected on r=b have not enough time to catch up the test particle; this condition is met for all particles within bunches shorter than $c\tau_3$. For $\tau<\tau_3$, $U(c\tau)$ cannot depend on b; this is indeed the case for the expression (26).

From (25),

$$U(c\tau) = H(c\tau) \cdot \frac{\sqrt{2g\ c\tau}}{\pi a} + \ldots \quad \text{when } c\tau\to 0 \tag{27}$$

Therefore, from (13) and (21):

$$w_{\parallel}(\tau) = N_{\parallel} \cdot H(c\tau) \frac{\varepsilon_m}{\varepsilon_o\ \pi^2\ 2a}\sqrt{\frac{2g}{c\tau}} + \ldots$$

$$\text{when } c\tau \to 0 \tag{28}$$

$$\vec{w}_\perp(\tau) = \vec{N}_\perp \cdot H(c\tau)\ \frac{\varepsilon_m}{\varepsilon_o\ \pi^2\ a^2}\sqrt{2g\ c\tau} + \ldots$$

When (26) is used in (13) or in (21), there results a simple analytical expression for $w_{\parallel}(\tau)$ or $\vec{w}_\perp(\tau)$ when $\tau<\tau_3$. This expression exhibits the same general behaviour as the wake potentials obtained from (11) or (20) by summing many modes and computing the tail of the series with the optical resonator model; the main difference is the behaviour for $c\tau \to 0$, but this difference would appear only in extremely short bunches.

Impedance function $Z(\omega)=Z_R(\omega)+jZ_I(\omega)$

The impedance function is identical to $W_{\parallel}(\omega)$ or $\vec{W}_{\perp}(\omega)$ except for the normalization factor: $N_{\parallel}$ or $\vec{N}_{\perp}/a$ are replaced by a^{-2m}. From (28), the asymptotic behaviour of the averaged $\overline{Z}_{\parallel}(\omega)$ at high frequencies is

$$\overline{Z}_{\parallel}(\omega) = N_{\parallel} \frac{\varepsilon_m}{\varepsilon_o\, c\pi^{3/2}} \left| k\, \frac{(2a)^2}{2g} \right|^{-\frac{1}{2}} e^{-j\frac{\pi}{4}\,\mathrm{sgn}(\omega)} \tag{29}$$

for $k\,\frac{(2a)^2}{2g} >> 1$

In the optical resonator model [9], $\overline{Z}_{\parallel}(\omega)$ behaves ultimately as $\omega^{-3/2}$, which corresponds to a finite $w_{\parallel}(o)$. Nevertheless, the $\omega^{-1/2}$ law fits rather well some results computed with BCI [10]. From (11),

$$W_{\parallel}(\omega) = \int_{-\infty}^{\infty} w_{\parallel}(\tau)\, e^{-j\omega\tau}\, d\tau = N_{\parallel}\cdot \sum_{\ell} K_{m\ell} \left[\frac{\pi}{2}\,\delta(\omega-\omega_\ell) + \frac{\pi}{2}\,\delta(\omega+\omega_\ell) + \frac{j\omega}{\omega_\ell^2-\omega^2} \right] \tag{30}$$

For $\omega << \omega_o$ (the lowest resonant frequency of the cavity),

$$W_{\parallel}(\omega) \approx N_{\parallel}\cdot j\omega L \quad \text{where} \quad L = \sum_{\ell} \frac{K_{m\ell}}{\omega_\ell^{\,2}} \tag{31}$$

Using (12) one obtains

$$L = \frac{\mu_o}{\pi}\,\varepsilon_m b \cdot \sum_{p=1}^{\infty} \frac{2\left[J_m(j_{mp}\,\frac{a}{b})\right]^2}{j_{mp}^{\;3}\left[J'_m(j_{mp})\right]^2}\, \mathrm{th}(j_{mp}\,\frac{g}{2b}) \tag{32}$$

If $\frac{g}{b}<<1$, $L \approx \frac{\mu_o}{2\pi}\,\varepsilon_m g\cdot F_m(0,\frac{a}{b}) = \frac{\mu_o}{2\pi}\,\varepsilon_m g\cdot$

$$\cdot\begin{cases} \log\frac{b}{a} & \text{for } m=0 \\ \frac{1}{2m}\left[1-(\frac{a}{b})^{2m}\right] & \text{for } m=1,2,\dots \end{cases} \tag{33}$$

If $\frac{g}{b} > 1$, $L \approx \frac{\mu_o}{\pi}\,\varepsilon_m\, b\cdot f(\frac{a}{b})$

$$\text{where } f(y) = \sum_{p=1}^{\infty} \frac{2\left[J_m(j_{mp}\,y)\right]^2}{j_{mp}^{\;3}\left[J'_m(j_{mp})\right]^2} \tag{34}$$

$$\begin{cases} \frac{\pi}{2}\, f(y) = \frac{2y}{(2m)^2-1} + \frac{\pi}{2}\, a_m\, \frac{(-\frac{1}{2})_m}{m!}\, y^{2m} + 0(y^{2m+2}) & y<<1 \\ a_o = 0.775238, \quad a_m \gtrsim 1 \text{ for } m=1,2,\dots & \\ \frac{\pi}{2}\, f(y) = (1-y)^2\left[-\log(1-y) + \dots\right] & y\to 1 \end{cases}$$

Loss parameter for a Gaussian bunch with an rms length $c\sigma$

Longitudinal loss parameter

$$2k_{\parallel} = \int_0^{\infty} d\tau\cdot e^{-\frac{\tau^2}{4\sigma^2}}\, w_{\parallel}(\tau) \Bigg/ \int_0^{\infty} d\tau\cdot e^{-\frac{\tau^2}{4\sigma^2}} \tag{35}$$

For short bunches, when $c\sigma < \min\,(c\tau_3,\ 2g)$, one can use (26) in (13) in order to compute (35). There is no simple analytical expression for (35) but this integral is easily computed numerically. In most cases which were compared with results of TBCI [11], (35) gave agreement to within 20-30%. Obviously, (26) only involves the dimensions a and g, and therefore it could not represent the influence of small variations in the geometry of the gap. Nevertheless, the results obtained from the simple expression (26) are much better than "order of magnitude" estimates.

If $c\sigma << c\tau_1$, $\quad 2k_{\parallel} \approx N_{\parallel} \frac{\varepsilon_m}{\varepsilon_o\,\pi^2 a} \left[\sqrt{\frac{g}{c\sigma}+1} - 1\right]$ (36)

If $\omega_o\sigma > 1$, $\quad 2k_{\parallel} \approx N_{\parallel}\,\omega_o^2\, L\, e^{-\omega_o^2\,\sigma^2}$ (37)

where L is given by (32).

Transverse loss parameter

It is given by an expression analogous to (35), and the same remarks apply.

If $c\sigma << c\tau_1$, $\quad 2\vec{k}_{\perp} \approx \vec{N}_{\perp}\, \frac{2\varepsilon_m}{\varepsilon_o\,\pi^{5/2}}\, \frac{c\sigma}{a^2} \left[\sqrt{\frac{g}{c\sigma}+1} - 1\right]$ (38)

If $\omega_o\sigma > 1$, $\quad 2\vec{k}_{\perp} \approx \vec{N}_{\perp}\, \frac{L}{\sqrt{\pi}} \cdot \frac{c}{a\sigma}$ (39)

where L is given by (32).

As a function of σ, $k_{\perp}$ varies as $\sqrt{g.c\sigma}/a^2$ for small σ and as σ^{-1} for large σ.

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