IEEE Transactions on Nuclear Science, Vol. NS-32, No. 5, October 1985

QUADRUPOLE BETATRON ACCELERATOR FOR HIGH CURRENT ION BEAMS

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SUMMARY

Properties of a strong non-neutral ion ring in a quadrupole betatron field are investigated. Superimposed on the axial betatron field, it is shown that the quadrupole field is necessary for the stability of the orbits where the self-fields of the ion ring are not negligible. A closed algebraic expression for the ion limiting current is obtained in terms of the quadarupole field intensity, the channel radius, the transverse temperature of ion beam, and the strength of betatron field. According to the theoretical calculation, high energy ion beam with its current order of one kiloampere can easily be attainable.

In recent years, there is considerable interest in the equilibrium and stability properties of intense relativistic electron rings with applications that include high current betation accelerators.[1-3] Theoretical and experimental results of these research activities indicates that the present high current betatron is one of the best accelerators for high current electron rings. In the same context, the betatron accelerator may also work for a high current ion beam. In betatron accelerators, the radius of the ion ring must be kept constant by shaping the external betatron field in such a way that the betatron flux condition is satisfied. The betatron flux condition has been widely known as a two-to-one rule. In this article, we investigate equilibrium and stability properties of an intense ion ring in a quadrupole betatron accelerator. As shown in Fig. 1, the field configuration of the quadrupole betatron accelerator consists of axial betatron field and strong quadrupole field that provides stability of ion orbits when the self-field of the ion ring is not negligible. The quadrupole field channel is aligned along the azimuthal direction, so that an intense ion ring is rotating through this channel. The externally imposed betatron field acts to confine the ring both axially and radially. Additional focusing force is provided in either radial and axial direction by the quadrupole field. The equilibrium radius of the ion ring is denoted by R_0 and the minor dimension of the ring is denoted by 2a, assuming a circular cross section.

It is shown that the magnetic quadrupole is more effective for the present purpose than the electric quadrupole. In a practical point of view we propose to use the permanent magnets for the magnetic quadrupole, which is made of rare earth cobalt. Assuming the minor cross section of the ion ring in the betatron is an ellipse with its major and minor radii a_{max} and a_{min}, the ion ring current I is limited to

$$I < 1.6 \times 10^{-6}\, \gamma\beta\, \frac{Z}{A}\, \frac{a_{min} a_{max}}{a_c^2}\, (B_0 \ell)^2\, g\!\left(\frac{L}{\ell}\right) + 7.5 \times 10^{6}\, \gamma^3\beta^3\, \frac{A}{Z}\, \frac{a_{min} a_{max}}{a_c^2} - 3.3 \times 10^{-2}\, \frac{\gamma^2\beta}{Z}\, \hat{T}_\perp , \quad (1)$$

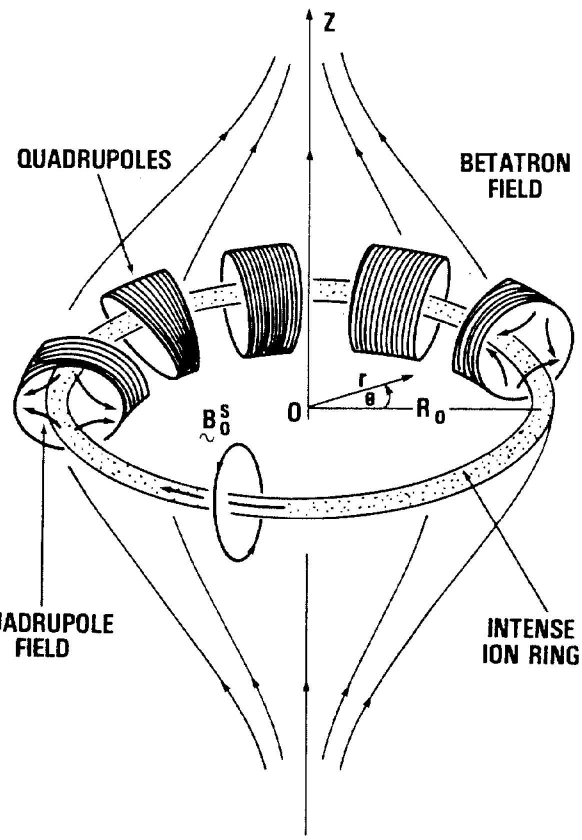

Fig. 1 Schematic presentation of the quadrupole betatron

where B_0 is the quadrupole poletip field in Gauss, ℓ is length of the quadrupole lens in cm, the ion beam current I is in Ampere unit, Z and A are charge state and mass number of ions, $\hat{T}_\perp$ is the transverse temperature in eV and the geometric factor $g(L/\ell)$ is defined by

$$g\!\left(\frac{L}{\ell}\right) = \frac{1 + 4\frac{L}{\ell} + 3\frac{L^2}{\ell^2}}{12\left(1 + \frac{L}{\ell}\right)^2}. \quad (2)$$

In Eq. (2), L is length of the quadrupole free space in cm unit.

The focusing strength θ of the quadrupole magnetic field is defined by[4]

$$\theta^2 = 3.2 \times 10^{-7}\, \frac{Z}{A}\, \frac{B_0 \ell^2}{\gamma\beta a_c}. \quad (3)$$

The ratio a_{min}/a_{max} is defined from[4]

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$$\left(\frac{a_{min}}{a_{max}}\right)^2 = \frac{1 - \left(1 + \frac{2L}{\ell}\right)\frac{\theta^2}{4}}{1 + \left(1 + \frac{2L}{\ell}\right)\frac{\theta^2}{4}}. \quad (4)$$

As a comparison, it is worthy to find the limiting ion current in a conventional betatron accelerator, which is

$$I < 7.98 \times 10^{-7} \gamma\beta \frac{Z}{A} (aB_z)^2. \quad (5)$$

Here a is the beam minor radius in cm and B_z is the betatron field in Gauss. Comparing Eq. (1) with Eq. (5), we can find the ratio I_q/I_β of the limiting currents between the quadrupole and conventional betatrons. That is

$$\frac{I_q}{I_\beta} = 2 \left(\frac{B_0\ell}{B_z a_c}\right)^2 g\left(\frac{L}{\ell}\right). \quad (6)$$

In obtaining Eq. (6), we have neglected the ion temperature. For typical physical parameters, we find that the limiting current of the quadrupole betatron is about two order of magnitude greater than that of the conventional betatron.

An intense ion ring is likely subject to various macro- and micro-instabilities. The most deleterious instabilities appear to be associated with the negative-mass instability. In order to make the stability calculation analytically tractable, we assume that the focussing strength θ of the quadrupole field is much less than unity, which ensures the circular minor cross section of the ion ring. Thus, we consider ion motion in the average external magnetic field provided by periodic quadrupole magnets. Thus, in addition to the betatron field, the focusing force associated with the applied quadrupole field can be determined from the azimuthal component of the effective vector potential

$$A_{q\theta}(r) = - \frac{m}{2q\gamma\beta} \omega_{cq}^2 \rho^2, \quad (7)$$

where $\rho^2 = (r - R_0)^2 + z^2$, the focusing frequency ω_{cq} of the quadrupole field is given by

$$\omega_{cq}^2 = \left(\frac{qB_0\ell}{\gamma m c a_c}\right)^2 g\left(\frac{L}{\ell}\right), \quad (8)$$

and q and m are charge and rest-mass of ions.

The most important physical parameter in the negative-mass instability is the coupling coefficient[5]

$$\mu = \frac{\omega_{cz}^2}{\omega_r^2} - \frac{1}{\gamma^2}, \quad (9)$$

where $\omega_{cz} = qB_z(R_0,t)/\gamma mc$ is the cyclotron frequency of the betatron field, ω_r is the radial betatron frequency of ions defined by

$$\omega_r^2 = \omega_{cq}^2 + (1 - n)\,\omega_{cz}^2 - \omega_{pb}^2/2\gamma^2, \quad (10)$$

n is the field index of the betatron magnetic field and $\omega_{pb}^2 = 4\pi q^2 n_0/\gamma m$ is the ion plasma frequency-squared. For typical experimental parameters, we find $\omega_{cq}^2 \gg \omega_{cz}^2$. Thus, the coupling coefficient μ can easily be a negative value, i.e.,

$$\mu < 0, \quad (11)$$

which ensures stability of the negative-mass instability.[5]

A family of rare earth cobalt provides a strong magnetic field. Thus, the rare earth cobalts including the samarium cobalt are excellent candidates for permanent quadrupole magnets. General properties of permanent quadrupole magnets made by the rare earth cobalt have been presented in the literature.[6] We therefore summarize results of this previous study.[6] For a quadrupole magnet consisting of a reasonably many number of trapezoidal pieces, the poletip field B_0 is expressed as

$$B_0 = 2B_c\,(1 - a_c/a_0), \quad (12)$$

where B_c is the remanent field, and a_c and a_0 are the inner and outer radii, respectively, of the pole magnet. Presently available materials have a remanent field B_c in 10^4 Gauss range and materials with even larger B_c will probably be available in near future.

In order to illustrate the usefulness of Eqs. (1), (2), (3), (4) and (12), we obtain the design parameters of the ion demonstration accelerators, which will produce a 500 MeV proton beam with its current 740 A and pulse length 0.41 microsecond. Total energy of the one beam pulse is 150 kilojoules. This beam is accelerated through a quadrupole betatron. Before the acceleration, the initial parameters of proton beam injected into the quadrupole betatron are; energy 5 MeV, current 100 A and pulse length 3 μs. Average minor radius a of the proton beam during injection is 3 cm. The major radius R_0 of the quadrupole betatron is determined from the initial pulse length τ_i and initial velocity β_{ic} of proton beam. Taking τ_i = 3 μs and $\beta_i = 0.1$ as an initial condition, we have the major radius R_0 = 15 m. The initial betatron field is given by $B_z(R_0,0) \simeq 200$ Gauss for $\beta_i = 0.1$ and R_0 = 15 m. The final betatron field is $B_z(R_0,t) = 2200$ Gauss for 500 MeV final energy.

Design parameters of the quadrupole magnets are obtained from Eq. (12). For a remanent field $B_c = 10^4$ Gauss and the outer radius a_0 = 15 cm, the poletip field B_0 and the channel radius a_c are given by $B_0 = 10^4$ Gauss and a_c = 7.5 cm. Lengths of the quadrupole lens and field free space are arbitrarily chosen by $L = \ell$ = 15 cm. For case of $L = \ell$, the geometric factor $g(L/\ell) = 1/6$. For the injection energy 5 MeV, the maximum proton current I_m = 250 A which is considerably larger than the injection current $I = 100$ A. The focusing strength θ of the quadrupole magnetic field is evaluated from Eq. (3). For $B_0 = 10^4$ Gauss, $\ell = 15$ cm, $a_c = 7.5$ cm and $\beta = 0.1$ corresponding to the injection energy 5 MeV, the focusing strength is given by $\theta = 0.97$. However, the focusing strength θ decreases rapidly with increasing value of proton energy. Since the focusing strength θ is order of unity at the injection, a_{min}/a_{max} in Eq. (4) is considerably small. Solving Eqs. (1) and (4) simultaneously for $\theta = 0.97$ and neglecting the second and third terms in the right-hand side of Eq. (1), we obtain $a_{min} = 1.8$ cm and $a_{max} = 4.5$ cm, which is safely less than the channel radius $a_c = 7.5$ cm. Both a_{min} and a_{max} approach to $a = 2.5$ cm as the proton beam is accelerated from 5 MeV to 500 MeV.

There are several unanswered issues in operation of the quadrupole betatron accelerator. These are: (a) beam injection and extraction, (b) the resistive wall instability, (c) the resonance instability, and (d) the ion resonance instability in case when free electrons present inside the accelerator. All of these issues must be carefully investigated before successful operation of the quadrupole betatron accelerator. However, as shown in the design study of the ion demonstration accelerator, it is concluded that the high current (order of one kiloampere) proton accelerator is easily affordable in the presently known technologies.

ACKNOWLEDGEMENTS

This research was supported by the Independent Research Fund at the Naval Surface Weapons Center.

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