STORAGE RING PARAMETERS FOR HIGH GAIN FEL

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1. Introduction

We use one-dimensional free electron laser (FEL) theory to find criteria for choosing electron beam and undulator parameters for operation of a high gain FEL at 400 Å, described in Refs. [1] and [2]. The criteria are (i) moderate electron beam energy (<1 GeV), (ii) high peak current (several hundred amperes), (iii) small emittance (~10^{-8} m-rad), (iv) small relative momentum spread (~0.001), and (v) narrow undulator gap (~3 mm). Results of two-dimensional simulations on FEL performance are also presented.

2. Predictions of 1-D FEL Theory

To summarize the results of one-dimensional FEL theory, it is convenient to introduce the following dimensionless parameter [3]:

$$\rho = \left(\frac{K^2[JJ]\, r_e n \lambda_u^2}{32\ \pi\gamma^3}\right)^{1/3} \quad , \tag{1}$$

where $K = eB\lambda_u/2\pi mc$, e = electron charge, B = peak value of the undulator magnetic field, λ_u = period of undulator magnet, m = electron mass, c = velocity of light, r_e = classical electron radius, n = electron density, γ = electron energy/mc^2, and

$$[JJ] = [J_0(\xi) - J_1(\xi)]^2 \quad , \quad \xi = \frac{K^2}{4(1 + K^2/2)} \tag{2}$$

For parameters of interest in this paper, ρ is the order 10^{-3}.

For the moment, we assume that all electrons have the same energy. The characteristics of intensity growth develop as follows: Near the entrance of the undulator, where small-signal theory applies, the gain G is given by

$$G = 536(\rho z/\lambda_u)^3 \quad , \tag{3}$$

where z is the distance from the undulator entrance. Farther along, the laser power P grows exponentially [4] from the initial power P_{in} with an exponential rate proportional to ρ:

$$P = \frac{1}{9} P_{in} e^{gz} \quad , \quad g = 4\pi\sqrt{3}\ \rho/\lambda_u \quad . \tag{4}$$

Eventually, the growth stops because electrons are captured in an ponderomotive potential well (bucket), and the laser saturates near the point $z = z_{sat}$ with a peak power P_{sat}. These quantities are approximately given by

$$z_{sat} \approx \lambda_u/\rho \quad , \tag{5}$$

$$P_{sat} \approx \rho P_{beam} \quad . \tag{6}$$

Here $P_{beam} = \hat{I}E/e$ is the power in the electron beam, $\hat{I}$ and E being the peak electron current and electron energy, respectively.

Taking $\hat{I}$ = 100 A, E = 1 GeV, and $\rho = 1\times10^{-3}$, which are typical values considered here, Eq. (6) gives a peak laser power of 100 megawatts. Assuming a beam pulse length of 100 ps and a repetition time of 100 ms, we then obtain an average laser power of 0.1 watt. Equation (5) implies that the number of periods in the undulator N is about $\rho^{-1} \approx 1000$. Random errors in such a long undulator should be carefully controlled in order not to degrade the FEL performance [5].

3. Effects Due to Energy Spread, Emittance and Diffraction

For beams with finite energy spread, g in Eq. (4) is replaced by $g' = 8\pi\mu_i\rho/\lambda_u$, where μ_i is the largest positive imaginary part of μ that satisfies the following dispersion relation [6]:

$$\mu - (1 - \rho\mu) \int dx\, \frac{f(x)}{(\mu - x)^2} = 0 \quad , \tag{7}$$

where f(x) is the distribution function in the variable

$$x = \frac{\gamma - \gamma_r}{\gamma_r \rho} \quad . \tag{8}$$

The resonant energy γ_r is defined in terms of the laser wavelength λ by the relation

$$\lambda = \lambda_u \frac{1 + K^2/2}{2\gamma_r^2} \quad . \tag{9}$$

By analyzing Eq. (7), we find that the growth rate g' is reduced significantly from the ideal value g unless

$$\sigma_\gamma = \sqrt{\left\langle\left(\frac{\gamma - \gamma_r}{\gamma_r}\right)^2\right\rangle} \lesssim \rho \quad . \tag{10}$$

The saturation level of the laser will also be reduced from the value given by Eq. (6) if the condition (10) is violated.

In addition to the natural energy spread, the emittance contributes an effective energy spread given by

$$(\sigma_\gamma)_{eff} = \frac{K\pi}{2\sqrt{2}\gamma\lambda} \sqrt{\epsilon_x^2 + 5\epsilon_y^2} \quad , \tag{11}$$

where $\epsilon_x(\epsilon_y)$ is the horizontal (vertical) emittance. For the cases of interest here, the effective energy spread, although not negligible, is usually smaller than the energy spread in the beam, σ_γ.

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When the beam emittance, and hence the beam cross section is sufficiently small, the diffractive effect can become significant, leading to a corresponding reduction in gain. However, the results of our numerical simulation have indicated that in a high-gain FEL the radiation tends to stay close to the electron beam so that one-dimensional theory summarized here is qualitatively a good guide. Recently, this "optical guiding" phenomenon has been studied theoretically by several authors [7,8].

4. Parameter Optimization

Equation (1) can be written alternatively as follows:

$$\rho^3 = \frac{1}{16\pi}\,\frac{r_e}{ec}\,\frac{K^3[JJ]}{2(1+K^2/2)}\,\frac{\lambda}{\gamma^2}\,\frac{\hat{I}}{\sqrt{\varepsilon_x\varepsilon_y}} \quad . \qquad (12)$$

In obtaining this expression, we have assumed a uniform focussing force in the undulator, expressed by the equivalent horizontal and vertical β-functions

$$\beta_x = \beta_y = \frac{\lambda_u\gamma}{K\pi} \quad . \qquad (13)$$

It is well-known that the alternating field in an undulator provides a focusing force in the vertical direction. Focusing in the horizontal direction can be provided either by tilting or by shaping the pole surfaces of the undulator [9].

Equation (12) leads to the following criteria to maximize ρ and hence optimize the FEL performance for a given optical wavelength λ: large peak current, small emittance, low beam energy, and large K, which implies a small undulator magnet gap. These requirements are sometimes in conflict with each other, and careful trade-offs are necessary for an optimum design [2,10]. For example, the energy cannot be too small because limitations due to both the coherent instabilities and the intrabeam scattering become severe at lower energies.

We have chosen λ = 400 Å as our nominal wavelength. Through a detailed study [2] of several specific examples of storage rings, and taking into account various multiparticle phenomena [10] and lattice structure effects [11], we have found that the optimum value of beam energy is about 750 MeV. Other storage ring parameters are, ε_x about 10^{-8} m-rad, $\hat{I}$ from 200 to 400 A, and σ_γ about 0.002.

For the undulator parameters, we assume a steel-permanent magnet hybrid structure, for which the following relation is valid [12]

$$B = 3.33\, e^{-x(5.47 - 1.8x)} \quad \text{(Tesla)} \quad , \qquad (14)$$

where x is the ratio of the magnet full gap to the undulator period λ_u. Althrough a small gap is perferred (for large K and thus large ρ), it should not be too small otherwise the effective energy spread given by Eq. (11) could become large and degrade the performance. If we choose a gap of 3 mm, the rest of the undulator parameters are found to be λ_u = 2.34 cm, K = 3.65 and β_x = 3.05 m. From these values and Eq. (12), we find that ρ is about 1 to 1.5 x 10^{-3}.

5. FEL Performance

The one-dimensional theory summarized in Section 2 provides a basic guideline for designing an FEL storage ring system. For a more quantitative evaluation of FEL performance, we used the two-dimensional particle-simulation code FRED [13] developed at Lawrence Livermore National Laboratory. The code follows the evolution of the optical field along the undulator axis. An important aspect of FRED is that it takes into account diffraction effects, which could be a priori important when the beam cross section is small. As we mentioned in Section 3, the results indicate that the diffractive tendency can be countered by focusing effects in high-gain FELs.

FRED was originally designed to study amplifier FELs, and it is necessary to specify an input power P_{in} to run the code. Therefore, we need to find P_{in} appropriate to the initial noise level in the electron beam as it enters the undulator. This is a subject that has not yet been settled. However, Ref. [3] estimates the maximum amplification in intensity to be of the order N_e, where N_e is the number of electrons contained in the length of one radiation wavelength. From this it follows that $P_{in} \approx N_e^{-1} P_{sat}$. Using the values $P_{sat} \approx$ 100 MW and $N_e \approx 10^5$, which are typical for the present case, one obtains $P_{in} \approx$ 1 kW. We have used this value in our simulation.

We have evaluated the FEL performance corresponding to various beam conditions studied in Ref. [2]. The results of our calculation agree qualitatively with the predictions of one dimensional theory in that the cases with higher ρ yield higher output power. Quantitatively, however, the output power levels were between a few and a few tens of megawatts,

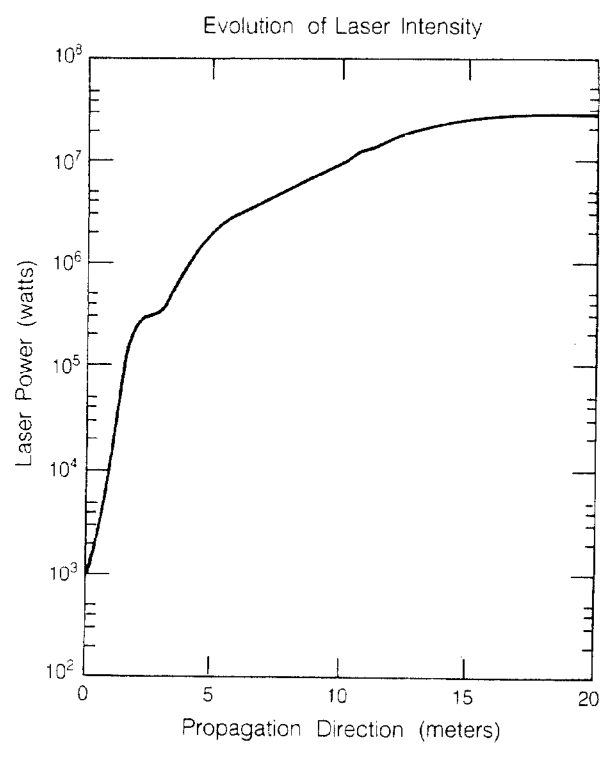

Figure 1. The evolution of laser intensity corresponding to the beam parameters E = 750 MeV, $\varepsilon_x = \varepsilon_y = 4.7 \times 10^{-9}$ m-rad, and σ_γ = 0.002.

much smaller than the several hundred megawatts expected from Eq. (6). The discrepancy is probably due to the large energy spread σ_γ: The ratios σ_γ/ρ for the cases studied here are of order or greater than unity, so that the gain could be reduced significantly as discussed before.

Another feature of the FRED results not understood from simple one-dimensional theory is a very rapid rise in the laser power from the input level of 1 kW to about 100 kW in the first few meters of the undulator, as can be seen in Fig. (1). When the input power level was set at 100 kW in one computation, the initial rapid rise disappeared. Proper interpretation of this result seems to require a better understanding of how coherence develops from initial noise in high-gain FELS.

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