# STABILIZING CHARACTERISTICS OF POST-COHPLERS 

S. Machida, T. Kato and S. Fukumoto

National Laboratory for High Energy Physics Oho-machi, Tsukuba-gun, Ibaraki-ken, 305, Japan

Abstract: A model of Alvarez linac equipped with post-couplers was constructed. The field distribution and the resonant frequency of the accelerating mode were measured to study its properties against perturbations. The experimental results were compared with an analysis using equivalent circuits. Post-couplers have an effect to distort the field distribution as well as to stabilize it depending upon the length of the posts.

## Introduction

At KEK, an extension of the proton linac was planned and a model tank with the post-coupled structure was constructed. As the result of the experiment, we obtained the interesting fact. If the two passbands, which are an accelerating and a post one, get too close together, the field is distorted. When the field is distorted, there is a remarkable change in the accelerating frequency.

In the other stabilized structures than the postcoupled, nothing is known about the distortion. It is clear that information on the distortion is important for understanding the mechanism of the post-coupled structure and also for the other stabilized structures.

## Experimental Results

## Perturbations

Main parameters of the model tank are shown in Table 1 . We introduced static perturbations. We moved both end plates to the same direction by the same amount. Three kinds of perturbations were prepared. The amounts of the movement were,
$\begin{array}{r}1-s t \text { cell } \\ 15-t h \text { cell }\end{array} \quad \begin{array}{r}-1 \mathrm{~mm} \\ 1 \mathrm{~mm}\end{array}, \begin{array}{r}3 \mathrm{~mm} \\ -3 \mathrm{~mm}\end{array}, \begin{array}{r}-5 \mathrm{~mm} \\ 5 \mathrm{~mm}\end{array}$,
where we take positive value when a cell is lengthened. Corresponding frequency shifts were,
$\begin{array}{rrr}1-\text { st cell } \\ 15-\text { th cell }\end{array} \quad \begin{array}{r}-1.5 \mathrm{MHz} \\ 1.0 \mathrm{MHz}\end{array}, \begin{array}{r}3.5 \mathrm{MHz}, \\ -3.0 \mathrm{MHz}\end{array} \quad \begin{array}{r}-6.0 \mathrm{MHz} \\ 4.0 \mathrm{MHz}\end{array}$.
We will obbreviate these cases as $\Delta 1 \mathrm{~mm}, \Delta 3 \mathrm{~mm}$ and $\Delta 5$ mm respectively.

## Ficld Distribution

We studied the behavior of the field distribution versus the post length inserted into the tank. First, the field without perturbations was measured as shown in Fig. 1 (a). Due to the perturbation, the field was deformed (Fig. 1(b)). When the post length was about 12.5 cm , the field was distorted (Fig. 1(c)). After the post passed this point, the field distribution showed an inverse slope as before, and when the post length was about 13.5 cm , the field became flat (Fig. 1(d)). The field was flatter than that without perturbations. When posts were inserted further, the field distribution had a slope again in the same direction as that for without posts.

To represent the slope of the field numerically, we define the distortion parameter as follows 11 .

$$
D_{x}=\sum_{i}\left|F_{i}-\bar{F}\right|
$$

where $F_{i}$ is the field level of each of the fifteen cells, iand $\bar{F}$ is the average of them. Using $D$, the dependence of the field on the post length is shown in

Fig. 2. In this figure, $D_{x}$ of perturbations, $\Delta 1 \mathrm{~mm}$ and $\Delta 3 \mathrm{~mm}$, and without the x e perturbations are also represented. In all cases, the field is distorted at about 12.5 cm and stabilized at about 13.5 em .

## Accelerating Frequency

For all perturbations, the dependence of the accelerating frequency on the post length is shown in Fig. 3. When the posts were around 12.5 cm , where the field was distorted, the accelerating frequency went down and then jumped up above the previous value. Then, the accelerating Erequency fell down and went up slowly. When the post length was about 13.5 cm , where the field stabilized, no sign denoting the stabilization was observed in the frequency.

## Table 1

Main parameters of the model tank

| Number of Cells | 15 |
| :--- | :---: |
| Frequency | 401.05 MHz |
| Tank Length | 2.51 m |
| Inside diameter | 0.45 m |
| Dríft-tube $\quad$ Outer diameter | 8.0 cm |
|  | Bore diameter |
| Energy |  |
| Beta | 1.5 cm |
|  |  |
|  |  |
|  | $0.20 .60-28.78 \mathrm{Mev}$ |

## Analysis of Equivalent Circuit

## Equivalent Circuit

For the post-coupled structure, an equivalent circuit has been proposed by T. Nishikawa \{2! and shown in Fig. 4. The $L_{0}, L_{1}$, and $L_{2}$ represent the inductances of a drift-tube, a stem, and a post. The $C_{0}, C_{1}$, and $C_{2}$ represent the capacitances between adjacent drift-Eubes, a drift-tube and the outer wall, and a post and a drift-tube. The $M_{0}, M_{1}$, and $M_{2}$ represent the mutual inductances between one half of a drift-tube and the other, adjacent stems, and adjacent poses.

Suppose that there is the phase shift $\theta$ between adjacent stems and adjacent posts. We define $k_{c}$ as

$$
\begin{align*}
& 1 / k_{c}=\frac{2 L_{2}}{L_{1}\left(1-k_{1}^{2}\right)}\left[\frac{L^{2}}{\omega_{1}^{2}}\left(1-k_{1}^{2}\right)-1+k_{1} \cot \right\}  \tag{1}\\
& +\frac{2 L}{L_{2}} \frac{\omega_{2}^{2} / \omega^{2}-1+k_{2} \cos \theta^{2}}{\left(\omega_{2}^{2} / \omega^{2}-1\right)^{2}-F_{2}^{2}}, k_{1}=M_{1} / L_{1}, k_{2}=M_{2} / L_{2} \\
& \omega_{1}=1 / \overline{L_{1} C_{1}}, w_{2}=1 / L_{2} C_{2}
\end{align*}
$$ then

$$
\begin{align*}
& V_{n}=-\left(1-\frac{w_{1}^{2}}{w^{2}}-k_{c}\right) A_{n}-\left(R_{0}+R_{n}\right) A_{n+1}, \\
& V_{n+1}=\left(k_{n}+k_{n}\right) A_{n}+\left(1-\frac{w_{0}}{w^{2}}-k_{n}\right) A_{n+1},  \tag{2}\\
& k_{n}=M_{n} / I_{n}, w_{n}=1 / \sqrt{L_{-} G_{i}}, A_{n}=j w I_{n} .
\end{align*}
$$

We join N cells and define the boundary condition as $\mathrm{V}_{1}=\mathrm{V}_{\mathrm{N}}=0$. These equations lead to

$$
\begin{aligned}
& \left.\frac{B_{L}+2_{L}}{2} \cdot A_{n-1}+u-R_{L}\right) A_{n}+\frac{E_{L}+E_{c}}{2} A_{A_{T}}=\frac{\omega_{1}}{\omega+A_{n}} A_{n} \\
& \left(1 \cdot k_{6}\right) A_{1}+\left(R_{2}+R_{6}\right) A_{2}=\frac{\omega_{i}}{\omega} A_{1} \text {, } \\
& \left(k_{C}+k_{C}\right) A_{N-1}+\left(1-k_{C}\right) A_{N}=\frac{w^{2}}{t_{C}} A_{N} .
\end{aligned}
$$

This relation suggests the expression in the matrix form

$$
\left\{\begin{array}{cc}
1-k_{L} & k_{c}+k_{L}  \tag{4}\\
\left(k_{c}+k_{c}\right) / 2 & 1-k_{L}\left(k_{L}+k_{c-}\right) / 2 \\
-
\end{array}\right] A=\frac{u^{2}}{w^{2}} A
$$

The $n$-th element of a eigen-vector gives the amplitude of the $n$-th cell.
$\frac{\text { Properties of } k}{\text { The } k+k}$ tube and the outer wall. Without perturbations the voltage $v$ between the drift-tube and the outer wall is zero. Chen perturbations are introduced into $n-t h$ cell and the voltage $v$ is produced, the shunt current $i_{c}$ between them flows $\mathrm{a}_{\mathrm{s}} \mathrm{i}_{c}=\mathrm{v}_{\mathrm{c}} /\left(k_{c}+\mathrm{k}_{0}\right)$. If $\mathrm{k}_{\mathrm{c}}+\mathrm{k}_{0}$ is infinite, $i$ always becomes zero and the current at $n-t h$ cell is not changed by perturbation. If $k_{c}+k_{o}$ is zero, very low voltage makes much shunt current Elow and the current to the next cell vanishes at n-th cell.

The denominator of Eq . (1) is separated into two terms, a term of stems and that of posts.

$$
\begin{align*}
& \left(1 / k_{c}\right)_{\text {stem }}=\frac{2 L_{c}}{L_{i}\left(1-k_{1}^{2}\right)}\left[\frac{\omega^{2}}{\omega_{1}{ }^{2}}\left(1-k_{1}^{2}\right)-1+k_{1}\right],  \tag{5}\\
& \left(1 / k_{c}\right)_{\text {pot }}=\frac{2 L_{0}}{L_{2}} \frac{1}{\omega^{2} / \omega^{2}-1-k_{2}} \tag{6}
\end{align*}
$$

Here, we assumed that the phase shifts between adjacent stems and between adjacent posts are zero.

If the. lengths of posts are changed, the value of $\omega_{2}$ and so the denominator of Eq. (6) changes. When $\omega_{2}={ }^{2} \omega \sqrt{1+k_{2}}$, Eq. (6) becomes infinite, thus $k_{c}$ approaches to zero. Because the absolute value of $\mathrm{k}_{0}$ must be smaller than unity, $k_{c}+k_{0}$ gets near zero necessarily.

In an Alvarez structure, $\omega_{1}$ is located below a half of the accelerating frequency $\omega$. Thus Ey. (5) becomes positive, if $k$ is positive. As a result of combination with two terms, when $\omega_{2}$ becomes below $\omega$ and these terms cancel each other on condition that $k$, is negative, the absolute value of $k_{c}$ becomes infi ${ }^{2}$ nite.

Three-Cell Tank Model
We will examine the relation between the value of $k$ and the field distribution using three-cell tank model. First of all, we will introduce perturbations into the equivalent circuits. The accelerating frequency $\omega_{0}$ of $n-t h$ cell is rewritten to $\omega_{0}^{\prime}$ and Eq. (3)

$$
\begin{equation*}
\frac{E_{c}+k_{c}}{2} A_{n-1}+\left(1-k_{c}\right) A_{n}+\frac{k_{1}+k_{c}}{2} A_{n+1}=\frac{w_{1}^{\prime 2}}{w^{2}} A_{n} \tag{7}
\end{equation*}
$$

To make eigen-value problem as Eq. (4), we move the perturbed term to the left side. Let $\omega_{0}{ }^{\prime}=\omega_{0} / \sqrt{1+\delta}$, then

$$
(1+5)\left[\frac{E_{u}+k_{c}}{2} A_{r-t}+\left(1-R_{c}\right) A_{n}+\frac{k_{c}+k_{c}}{2} A_{n+1}\right)=\frac{w_{0}}{w-1} A_{n}{ }^{(8)}
$$

Now, we investigate the behavior of the field distribution and the accelerating Erequency against perturbations. Suppose that the frequencies of hoth cell become

$$
\begin{equation*}
\omega_{i}^{\prime}(1-c t)=\omega_{i} / \sqrt{1+\delta}, \omega_{i}^{\prime}(3 r d)-\omega_{1} / \sqrt{1-\varepsilon} \tag{9}
\end{equation*}
$$

The eigen-value problem representing this tank becomes

$$
\left\{\begin{array}{ccc}
(1+\delta)\left(1-R_{c}\right) & (1+\delta)\left(k_{c}+R_{c}\right) \\
\left(k_{c}+k_{-}\right) / 2 & 1-k_{c} & \left(R_{L}+k_{c}\right) / 2
\end{array}\right\} A=\frac{\omega_{5}^{2}(10)}{\omega^{2}} A
$$

We neglect $\delta^{2}$, then
$\frac{\omega_{c}^{2}}{\omega^{2}}=1+R_{c}, 1 \cdots R_{L}, 1-R_{c}-2 R_{c}$.
For the 0 -mode, the eigen-vector becomes

$$
A=\left\{\begin{array}{c}
\left.(1+\delta)\left(k_{c}+k_{c}\right) /(1+\delta) k_{c}+k_{c}-\delta\right)  \tag{12}\\
1 \\
\left.(1-\delta)\left(k_{c}+k_{c}\right) /(1-\delta) k_{c}+k_{c}+\delta\right)
\end{array}\right\} A_{2} .
$$

Here, we define the tilt parameter $T$ as

$$
\begin{align*}
T & =\left(A_{3} / A_{2}-A_{1} / A_{2}\right) / 2 \\
& =\frac{-\delta\left(k_{c}+E_{c}\right)\left(k_{c}+1\right)}{\left((1+\varepsilon) k_{c}+R_{0}-\delta\right]\left((1-\delta) k_{c}+R_{c}+\delta\right)}  \tag{13}\\
& \simeq-\delta\left(k_{c}+1\right) /\left(k_{c}+k_{c}\right) .
\end{align*}
$$

The $T$ has two variables, $\delta$ and $k$. When $\delta$ increases, T becomes large in the negative direction in proportion to $\delta$. When the absolute value of $k$ increases, the denominator of $T$ becomes large. The field becomes flat and that is the stabilization. On the contrary, when $k_{c}+k_{0}$ gets near zero, the denominator of $T$ becomes Cmall and the field slopes steeply. The field is distorted.

These consideration proves whether the field is distorted or stabilized depends upon the value of $k_{c}$, and that the distortion occurs peculiarly when the re are errors of the frequency in some cells.

## Comparison with Experiment

## $\frac{\text { Field Distribution }}{\text { Using the repren }}$

Using the representation with the equivalent circuits, we will compare the result of the analysis with those of the experiment.

The 1 -st and 15 -th cells are perturbed in the experiment, three kinds of $\delta$ are taken,

$$
\delta=0.005, \quad 0.015, \quad 0.025
$$

these $\delta$ correspond to the frequency shift of $\Delta 1 \mathrm{~mm}, \Delta 3$ mm and $\Delta 5 \mathrm{~mm}$ respectively.

We decrease $\omega_{2}$ by the insertion of posts. Put $\delta$ $=0.025$. As $\omega_{2}$ decreases, $k_{c}$ decreases slowly and the field distribution is little affected (Fig. 6(a)). When $\omega_{2}$ gets near $\omega \sqrt{1+k_{2}}, k_{c}$ becomes $-k_{0}$ and the field is distorted. The distribution completely changes around this frequency (Fig. 6(b) (c)). As $\omega_{2}$ decreases furthermore, the field recovers from the distortion and has an inverse tilt (Fig. 6(d)). Then $k$ becomes infinite in the negative direction and next in the positive one. The field becomes completely flat (Fig. 6(e)). After the stabilization point, $k$ changes slowly with the decrease of $\omega_{2}$ and the slope of the field is produced again in the same direction as before (Fig. 6(f)). The corresponding data of the experiment are shown in Fig. 5.

The distortion parameter calculated from the equivalent circuit is shown in Fig. 7. The field distribution obtained by equivalent circuits agrees fairly with the experimental one.

## Accelerating Frequency

For each $\hat{\delta}$, the dependence of $\omega$ on $\omega_{2}$ is shown in Fig, 8. At around $\omega_{2}=355 \mathrm{MHz}\left(k_{c} \cong-k_{0}^{2}\right)$, the frequency transition occurs proportionally to the amount of $\delta$.

## Conclusions

[^0]equivalent circuits. In this analysis, the parameter $k$ was defined, which corresponds to an element of the impedance between a drift-tube and the outer wall. It was shown that the field is stabilized when $k_{c}+k_{0}$ becomes infinite and the field is distorted when $k_{c}^{c}+k_{0}^{0}$ is zero.

## References

[1] J. Ungrin, S.O. Schriber and R.A. Vokes, "PostCoupler and Stem Current Measurements for Iligh Current CN Drift-Tube Linacs", IEEE Trans. Nucl. Sci. NS-30, No. 4, 3013, 1983.
[2) T. Nishikawa, unpublished communication, 1967.


Fig. 1 Electric field on axis of the model tank, before and after stabilization and distortion by posts.


Fig. 2 Distortion parameter vs. the length of posts inserted into the model tank, taking endplates perturbation for parameter.


FJ.g: 3 Resonant frequency ohift by insertion of posts, traking endplates perturbation for parameter.


Fig. 4 Equivalent circuit for an Alvarez structure with stems and posts.
 $\omega_{2}$ in the experiment.


Fig. 6 Electric field dependence on the post frequency $\omega_{2}$ in analysis of equivalent circuit.


Fig. 7 oistortion parameter ox vs. The post frequency $\mathrm{c}_{2}$, taking endplates perturbation for parameter.


Fig. 8 Resonant irequency shift vs, the post frequency $\omega_{2}$, taking endolates perturbation for parameter.


[^0]:    The stabilization by post-couplers was confirmed. However, in some cases, it was shown also that postcouplers have a harmful effect on the field. To understand the mechanism of post-couplers, we analyzed

