Ion Beam Steering with a High Intensity Electron Beam

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Summary

In conventional theory, steering or bending an ion beam of high energy and high current requires very intense magnetic fields, which are both uneconomical and bulky. This problem is even more severe for singly charged ion beam with very high atomic number, which require large magnetic field energy both to bend and also to focus the beam against its self electric field. In this paper we present a new and simple technique, which will substantially alleviate these problems.

Introduction

We consider here the possibility of mixing the ion beam with an electron beam moving at the same speed, but with a much higher number density. We require

$$\frac{m_i}{m_e} >> \frac{N_e}{N_i} >> 1, \qquad (1)$$

where N_e and N_i are line number densities of electrons and ions, and m_e and m_i are electron and ion rest mass, respectively. The transverse applied magnetic field in this case is required only to bend the electron beam, which in turn traps the ion beam by its own attractive electrostatic potential. Therefore the center of mass radius of curvature and applied magnetic field energy can be reduced substantially. Although these two beam axes are slightly separated due to polarizing effect of the magnetic field on their opposite charge, under certain conditions they will however remain together as a stable configuration. The required magnetic field strength and field energy for bending the comoving beams are reduced by the ratios N_i/N_e and $(N_i/N_e)^2$, respectively, from their values for bending the ion beam alone. This substantial savings in bending field energy coupled with similar reductions in the field energy for beam focussing (see below) make this technique especially attractive.

It is well known that a non-neutral particle beam will rapidly expand and be lost due to its self electrostatic field, if external focussing force is not being applied. Several techniques have been developed in order to accomplish this objective. These techniques include an externally applied magnetic field along the beam direction of propagation[1]; discrete magnetic quadrupole lenses[2]; and continuous magnetic quadrupole field generated by four helically wound conducting wires of alternatively opposing current,[3] superimposed on an axial magnetic field. In principle, all of these techniques can be used to confine an ion beam, but in general they are more efficient in confining electrons due to the light electron mass. In fact, for comoving beams where relation (1) holds, the magnetic field energy needed for confinement is reduced by the ratio m_eN_e/m_iN_i from its value in the absence of electrons.

Thus in this paper we will explore the possibility of bending the ion beam by mixing it with a comoving electron beam.

Analysis

To investigate the properties of comoving electron and ion beams in the presence of an external transverse magnetic field, B_z, perpendicular to the propagation direction of the beams, we treat the electron and ion beams as rigid body, i.e. the beam maintains fixed density and current profiles with respect to its axes. This is a reasonable assumption, since the self fields do not directly affect the lateral displacement of the beams axes. Therefore, the equations of motions for lateral displacements of the ion and electron beam axes (x_i and x_e, respectively) relative to their original propagation direction can be expressed as:

$$\frac{d^2x_i}{dt^2} = \frac{ZeB_z\beta}{\gamma m_i} + F_{ei}(x_e - x_i)/\gamma m_i N_i \qquad (2)$$

$$\frac{d^2x_e}{dt^2} = -\frac{eB_z\beta}{\gamma m_e} + F_{ie}(x_i - x_e)/\gamma m_e N_e$$

where Z is the charged state of ions, γ is the relativistic factor, $\beta = \frac{v}{c}$, and $F_{\alpha\delta}(x)$ is the transverse (electric and magnetic) static force exerted on a column of species δ by a column of species α.

This force can be expressed as:

$$F_{\alpha\delta} = e_\delta \int n_\delta \, [E^\alpha - (\beta B^\alpha)]_x \, dS_\delta$$
$$= \frac{e_\delta}{\gamma^2} \int n_\delta E_x^\alpha \, dS_\delta, \qquad (3)$$

where n is the number density, E^α and B^α are self electric and magnetic field of the species α, the subscript x denotes the direction of the force, and the integral is to be carried out over the cross section of column of species δ.

We assume both ion and electron beams possess Gaussian profile, i.e.

$$n_\alpha(r) = \frac{N_\alpha}{\pi R_\alpha^2} e^{-r^2/R_\alpha^2}, \qquad (4)$$

where R is the beam radius. Then after some simple geometric calculations, we can write Equation (3) as

$$F_{\alpha\delta}(x) = \frac{2e_\alpha e_\delta N_\alpha N_\delta}{\pi R_\delta^2 \gamma^2} e^{-x^2/R_\delta^2} \int_0^\infty dr$$

$$[e^{-r^2/R_\delta^2}(1 - e^{-r^2/R_\alpha^2}) \int_0^{2\pi} e^{\frac{2rx\cos\theta}{R_\delta^2}} \cos\theta d\theta] \tag{5}$$

$$= \frac{2\sqrt{\pi} e_\alpha e_\delta N_\alpha N_\delta}{R_\delta \gamma^2} [e^{-z} I_{1/2}(z) - \eta^{1/2} e^{z\eta - 2z} I_{1/2}(z\eta)]$$

where $z = x^2/2R_\delta^2$, $\eta = R_\alpha^2/(R_\alpha^2 + R_\delta^2)$, and $I_{1/2}$ is modified Bessel function of order one half. By using the identity

$$I_{1/2}(z) = (e^z - e^{-z})\sqrt{\frac{1}{2\pi z}}, \tag{6}$$

we can simplify $F_{\alpha\delta}(x)$ to:

$$F_{\alpha\delta}(x) = \frac{2e_\alpha e_\delta N_\alpha N_\delta}{x\gamma^2} (1 - e^{-x^2/(R_\alpha^2 + R_\delta^2)}). \tag{7}$$

The equation of motion of the relative separation between the two beam axes $(r_i - r_e)$ can be obtained from Equation (2). Using the force form of Equation (7), we can write this equation of motion as

$$\frac{d^2x}{dt^2} = A - D\frac{1}{x}[1 - e^{-x^2}], \tag{8}$$

where we now define $x = |x_i - x_e|/(R_i^2 + R_e^2)^{1/2}$, and the constant A and D are given by

$$A = \frac{eB_z\beta}{\gamma m_e(R_i^2 + R_e^2)^{1/2}} \tag{9}$$

$$D = \frac{2N_i Ze^2}{\gamma^3 m_e(R_i^2 + R_e^2)}.$$

In obtaining Equation (8), the constraints in Equation (1) have been applied.

Integrating Equation (8) over x with the initial condition $\frac{dx}{dt} = 0$ at $x = 0$, we obtain

$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 = x\,[A - \frac{D}{2} G(x)], \tag{10}$$

where

$$G(x) = \frac{1}{x}[\ell n(x^2) + E_1(x^2) + 0.57721]. \tag{11}$$

and E_1 is the exponential integral. Equation (10) denotes the kinetic energy of electron beam in the frame of reference moving with the ion beam.

We are mainly interested in the maximum separation of the two beams, x_m, which occurs when $dx/dt = 0$, or

$$G(x_m) = \frac{2A}{D} = \frac{\gamma^2\beta B_z(R_i^2 + R_e^2)^{1/2}}{ZeN_i}. \tag{12}$$

G(x) decreases away from its peak value of $\simeq 1.0$ at $x \simeq 2.1$, and has the following asymtotic form

$$G(x) = \begin{cases} x, & x \ll 1 \\ \frac{1}{x}[\ln(x^2) + 0.577], & x \gg 1. \end{cases} \tag{13}$$

Therefore a pair of solutions to Eq. (12) occur for $2A/D < 1.0$, one with $x < 2.1$ and one with $x > 2.1$. Only the solution with $x < 2.1$ is physically obtainable and stable. Assuming small relative displacement, $x_m \ll 1$, Eq. (12) simplifies to

$$N_i = \frac{\gamma^2\beta B_z(R_i^2 + R_e^2)^{1/2}}{Zex_m}. \tag{14}$$

Equation (14) therefore uniquely determines the lower limit on the ion line density, if we do not want the relative separation of the two beam axes to exceed x_m. Note that Eq. (14) applies only when $N_i m_i \gg N_e m_e$ (see Eq. (1)). Otherwise, N_i should be replaced by $N_i + m_e N_e/m_i$.

In order to fully characterize the behavior of the beams motion, we define the center of mass of the two beams as

$$x_c = \frac{N_i m_i x_i + N_e m_e x_e}{N_i m_i + N_e m_e}. \tag{15}$$

This equation together with Equation (2) give us the equation of motion for the center of mass

$$\frac{d^2x_c}{dt^2} = \frac{eB_z\beta}{\gamma(N_i m_i + N_e m_e)}(ZN_i - N_e) \tag{16}$$

$$\simeq -\frac{eB_z\beta N_e}{\gamma N_i m_i},$$

where we have used the constraints in Eq. (1). Since β and γ remain unchanged, Eq. (16) implies that the acceleration of the center of mass is constant and perpendicular to the direction of motion. Thus the center of mass motion bends with radius of curvature ρ_c,

$$\rho_c = \left|\frac{\gamma\beta m_i c^2}{eB_z}\frac{N_i}{N_e}\right|. \tag{17}$$

Equations (14) and (17) constitute the main results of this paper, and can be used together with the constraints in Eq. (1) to calculate the design parameters for various applications, when steering or bending a high energy, high current ion beam is necessary. However, in arriving at these results, we have assumed the beams to be rigid, i.e. neglecting the effects of self fields. While the ion beam is trapped and confined by the electron beam space charge potential $(N_e/N_i \gg 1)$, external field is necessary to confine the electron beam minor cross section from self expansion. Which is especially detrimental for applications, where it is necessary to have the two beams remain together as long as possible[4]. Part of the reason is obvious from Eq. (14), in which the critical current scale as $R_i^2 + R_e^2$. To confine the electron beam minor cross section from self expansion, we can adapt the techniques used in various cyclic accelerators, i.e. magnetic quadrupole field focussing as in the quadrupole accelerator[2] or in the stellatron[3], or an externally applied magnetic field along the direction of propagation as in the modified betatron[5].

In this paper, we consider the use of an applied magnetic field along the trajectory of the center of mass, which can be readily calculated from Eq. (16), in order to focus the electron beam. The effect of this axial magnetic field on the center of mass trajectory is negligible, provided the condition $x_m << 1$ is satisfied. At any rate, any possible electron drifts due to the bending magnetic field error or the presence of the axial magnetic field can be effectively prevented by introducing a strong focussing field, such as the continuous quadrupole field in the Stellatron.[3] A charged particle beam can be transported along an axial magnetic field, if the following condition is satisfied[1]

$$n < \frac{\gamma B_\theta^2}{8\pi mc^2}, \tag{18}$$

where n is the beam density and B_θ is the axial magnetic field strength. It is obvious from the above equation that substantial solenoidal magnetic field energy can be saved by employing the comoving electron beam, due to the fact that $N_i m_i >> N_e m_e$. The axial magnetic field B_θ can be obtained by solenoidal winding around the center of mass trajectory.

In order to bend the beam, a uniform bending magnetic field must be applied transversly to the plane of the beam propagation; this can be achieved either by conventional permanent dipole magnets or by a new winding technique we recently developed[6]. In this technique, parallel conducting wires are formed into a tube (the tube does not have to be straight), so that the cross section of this tube is as shown in Figure 1. In Cartesian coordinates, when the wires are packed close to one another and the current in each wire are given by:

$$I(z,x) = I_{max}\, x/a, \tag{19}$$

where a is the radius of the cross section of the tube, we find that the magnetic field inside the tube can be expressed as

$$\vec{B}(z,x) = \frac{\mu_0 I_{max}}{2a}\hat{z}. \tag{20}$$

Since this field is uniform, it can effectively be used as a bending field for charged particle beams.

In order to illustrate some of the points discussed here, we consider the bending of a proton beam with a current of 200 Ampere, energy of 20 MeV and R_i = 1 cm, by employing a comoving electron with a current of 5kA and R_e = 5 cm. If we demand x_m = 0.1, then from Eq. (14) the bending magnetic field must be B_z = 16.2 Gauss. Thus the radius of curvature is

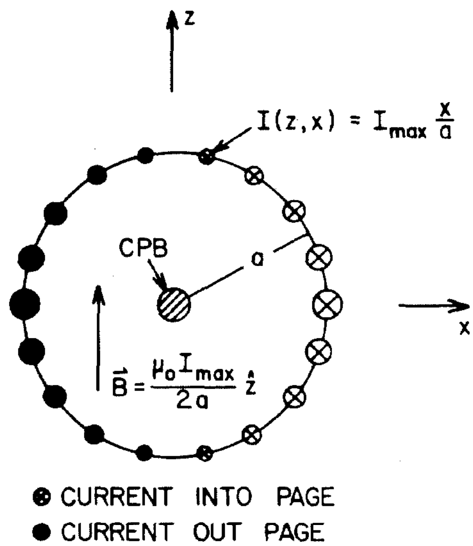

Figure 1. Cross sectional view of a tube to create uniform bending magnetic field.

ρ_c = 10.6 meters from Eq. (17). The axial field strength required is B_θ = 1.3 kG from Eq. (18). For the same configuration without the presence of the electron beam, the magnetic field required for bending and focussing the ion beam would be 405 Gauss and 5.6 Telsa, respectively. Obviously, considerable magnetic energy must be expended to confine and to bend an ion beam without the presence of the electron beam.

Conclusion

We have presented here a new technique to bend and confine a high energy and high current ion beam. It should be noted, however, that the feasibility and usefulness of this technique depends on the objectives one wants to accomplish and the availability of technology. For certain applications, electron cooling of high current ion beam[4], for example, this technique is very advantageous in reducing the physical space required.

References

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