SIMULATION STUDIES OF INSTABILITIES AND EMITTANCE GROWTH IN A PERIODIC SOLENOIDAL CHANNEL WITH LINEAR AND NONLINEAR FOCUSING+

H. Dantsker-Rudd*, I. Haber, M. Reiser**
Naval Research Laboratory
Washington, DC 20375

I. Introduction

While a carefully designed electron beam and its transport system are interesting on their own because of the very low emittances which can be achieved, the behavior of such a beam can also be of more general applicability. If the electron current is much less than the Alfven current, so that the space charge potential depression across the beam is negligible compared with beam energy, the equations which describe this system are identical in form to other such systems. The fully nonlinear beam dynamics can then be used to mimic the behavior of other appropriately scaled electron or ion beams which are neither so economical to construct nor as easy to employ in detailed experiments as the low energy electron analog.

This scaleability and the ease of construction of a low energy electron transport system are advantages which the University of Maryland Intense Beam Transport Experiment[1,2] has been designed to exploit. This 36 cell long channel with discrete solenoidal focusing magnets is a suitable test bed for the basic physics of a periodic transport system because of the simple cylindrically-symmetric geometry. At the same time, the cylindrical geometry can be exploited to examine the differences as well as similarities to the more complicated A-G geometry. In this way the influence of details of geometry can be explicitly examined by comparing the two systems, especially with respect to behavior of those phenomena for which theory predicts strong similarities.

At the very low emittances obtained in the Maryland experiment, the particle beam can be strongly influenced by a complex set of nonlinear phenomena as it propagates down the channel. In this regime, simulations can be a particularly useful tool in interpreting the experimental data and relating beam behavior to analytic calculations which, to be tractable, generally require substantial simplifying assumptions. The simulations described here are designed to explore the range of parameters for which interesting nonlinear physics is important and which are also relevant to the experiment, so that meaningful comparisons to the simulations will be possible.

II. Space Charge Instabilities in an Interrupted Solenoidal System with 90° Phase Advance

From a combination of theoretical[3] and experimental[4,5] evidence, as well as simulations[6], an alternate-gradient (A-G) transport system with 90° phase advance is known to be subject to space charge driven instabilities when the beam is sufficiently intense. The 90° phase advance is below the region where the beam envelope is itself unstable and above the region where the instabilities have been found to saturate and stop growing before any change in rms emittance results.[3,6] In many of these cases no clues have been found within the linear theory as to whether rms emittance will or will not grow. In order to examine the nonlinear behavior of a solenoidal system with 90° phase advance the SHIFT-XY code used in many of the alternate-gradient system simulations, was used to examine the behavior of the third order and fourth order instabilities which were extensively studied in the A-G case. Instead of the approximately factor of two emittance growth observed in the A-G system, virtually no growth in the rms emittance was observed in the 90° solenoidal system.

In order to investigate a broader parameter regime, the current can be swept over a range of values. This is analogous to what would occur in the center of a long beam during longitudinal compression.[6,7]

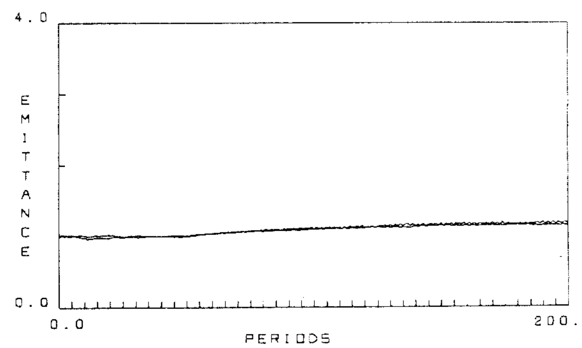

Fig. 1 Evolution of the x and y rms emittances of an initial K-V distribution in a 90° solenoidal transport system as the current is linearly increased tenfold.

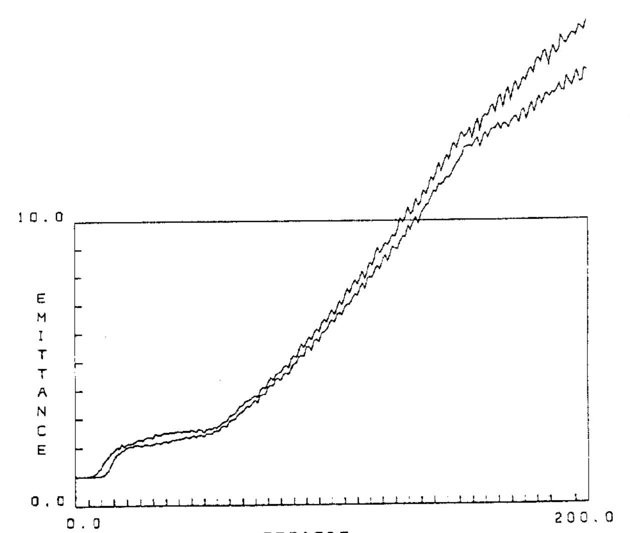

Fig. 2 Evolution of the x and y rms emittance in an alternate-gradient transport system with the same current increase as in Fig. 1.

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Figure 1 shows the behavior of the rms emittance of a thin lens solenoidal system with an initial K-V distribution and enough current to depress the 90° phase advance to 15°. The current is held constant for 50 periods, linearly increased by a factor of ten over the next 100 periods, and then held constant for another 50 periods. The rms emittance does not increase substantially. This is in contrast to the behavior of a similar alternate-gradient transport system, as shown in Fig. 2, with the same phase advances and current variation.

From these simulations, it appears that a solenoidal system is less subject to rms emittance growth from space charge driven instabilities, than a similar A-G system. Without a detailed theoretical model of instability saturation, it is difficult to explain the differences between the two systems. However, both the characteristic frequencies and the way in which the beam varies within each cell of the focusing system, are different. In an alternate gradient system as the beam propagates between cells, the profile distorts in the transverse plane but the beam density is approximately a constant. In a solenoidal system, on the other hand, the beam density changes as the beam compresses in both x and y at the same time. Therefore the density is changing during each period. Any resonances associated with plasma frequency phenomena may not be as sharp because the plasma frequency is rapidly changing in a time which is shorter than the plasma period. In addition, the relationship between the lens transit time and the average plasma period is different in the two cases. Plasma resonances which may be present in the alternate gradient transport may be absent in the solenoidal system.

A major conclusion, however, is that the details of the nonlinear behavior appear to have a subtantial influence on the evolution of beam emittance. This is an argument for examining the details of any nonlinearities which appear likely to affect the beam emittance. In the present case, this examination leads to much more optimistic conclusions on emittance growth than are predicted by the linear stability theory.

III. Transport with Lens Nonlinearity but without Space Charge

In view of the absence of instability caused emittance growth over a wide range of parameters, a more realistic representation of the experimental focusing system has been incorporated into the numerical model to examine the effect on emittance of the focusing nonlinearities present in the actual experimental lens system.

When an electron trajectory radius becomes comparable to the lens aperture, the focusing forces become nonlinear. These nonlinear forces overfocus the outer trajectories relative to the inner ones. A theoretical and experimental investigation of the space charge as well as the nonlinear forces of the actual transport system lenses and their effects on beam quality has been done recently by Loschialpo.[8] His study of various lens aberrations concluded that the spherical aberration associated with the third order (r^3) term of the nonlinear focusing force expansion dominate the behavior of the beam.

Therefore, the SHIFT-XY code has been modified to incorporate a third order nonlinear component in the external focusing force expansion. In the thin-lens approximation used, the focusing is being performed as

$$r'_{out} - r'_{in} = - a_1 r - a_3 r^3 = F_1 + F_2, \qquad (1)$$

where r'_{in} is the incident slope of an electron and r'_{out} is the final electron trajectory slope after the lens. We specify the strength of the nonlinear force by the ratio of the cubic term in (1) to the linear one (F_3/F_1).

When both space charge nonlinearities and external nonlinearities are included, the beam behavior is extremely complex. In order to separate the effects of external nonlinearity from those of space charge, numerical simulations were performed in which the current is reduced sufficiently so that the space charge forces can be ignored.

Several problems of this category have been run with 90° and 60° phase advance periodic channels. The nonlinearity ratio has been varied from 0.1 to 0.8.

For runs which exceed a threshold nonlinearity, particles are lost until the remaining particles lie on orbits in phase space which do not cross this threshold. Figure 3 shows the fractional beam loss for a 90° system with an initial nonlinearity ratio of 0.4. Since particles near the threshold are lost very slowly, the long time saturation of particle loss and the location of a threshold nonlinearity below which orbits are stable can require very long runs -- which exceed what would be appropriate to the linear accelerator system of interest. From the percentage of particles remaining after 1000 periods, however, it is possible to estimate the threshold nonlinearity. For a 90° system the appropriate ratio of 3rd order to first order term at the beam edge is estimated to be .35, while for a 60° system no beam loss is expected until a nonlinearity ratio of 0.5.

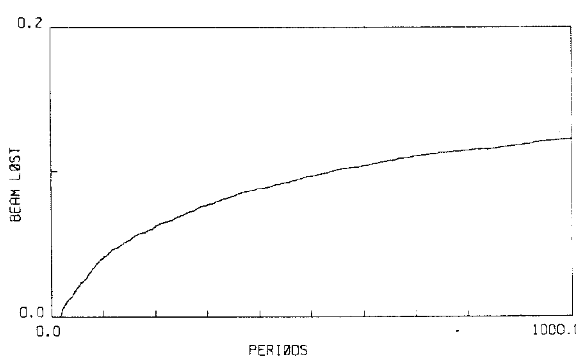

Fig. 3 Fractional beam loss for a low current system with 90° phase advance and nonlinearity ratio of 0.4.

IV. Particle Loss and Emittance Growth with Both Space Charge and Lens Nonlinearity

Inclusion of the self-consistent space charge forces complicates matters considerably. Lacking a prescription for establishing detailed equilibria in a periodic system with nonlinear focusing, simulations were run with both initial K-V and thermal (Gaussian in velocity space, uniform in configuration space) distributions which would be rms matched in the absence of the nonlinear terms. Since the distributions are not in detailed equilibrium with the external focusing, the beam responds by growing in emittance (although not necessarily rms emittance). Because the beam current distribution can redistribute itself so as to cancel the external nonlinearity, it is possible that the total nonlinearity is substantially reduced in the body of the beam, but this reduction is often out-weighed at the beam edge.

Figure 3 is a plot of fractional beam loss after 50 and 200 periods for a 90° transport system. Two currents, enough to depress the tune to 15° and to 6° are shown for an initial thermal distribution. Results for an initial K-V distribution behave quite similarly. Figure 4 shows the rms emittance growth after 50 periods for the same runs. Since the contribution from lost particles is taken out of the emittance calculation, these curves substantially understate the emittance growth, because once the particle orbits go unstable, they increase rapidly in radius. While not as many simulations of 60° systems have been run, the behavior of these systems is considerably better. For example, a periodic channel with 60° phase advance depressed to 12° loses fewer particles after 1000 timesteps, than a 90° system depressed to 15° after 200 timesteps. This behavior is typical for all cases run.

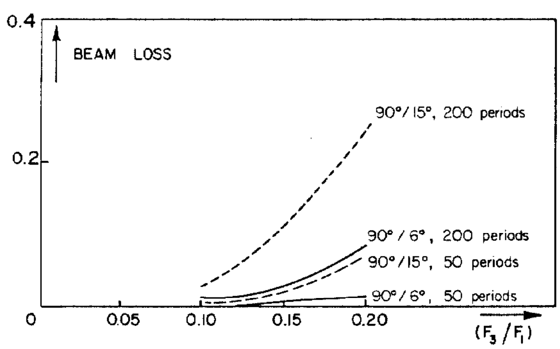

Fig. 4 Fractional beam loss of an initial thermal distribution in a 90° solenoidal transport system after traversing 50 and 200 cells.

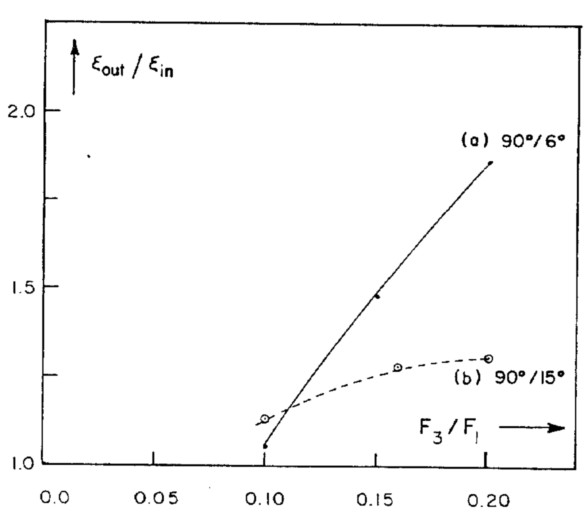

Fig. 5 Emittance growth of an initial thermal distribution after 50 cells.

V. Conclusions

Simulations have been run which examine the parameter range expected in the University of Maryland transport experiment. In view of the findings on both space charge instabilities and external-nonlinearity-caused particle loss, it appears likely that, even in a 90° system, with the approximately 0.1 nonlinearity ratio expected in the experiment only small beam loss is predicted. Further simulations may reveal a parameter regime, e.g. for greater phase advances, for which more easily measureable effects are expected.

On a more general level, however, it appears possible to conclude that, at least for 90° phase advance, solenoidal systems are somewhat better behaved than alternate-gradient systems in the sense that they suffer less space-charge-instability caused emittance growth. Also 60° solenoidal transport systems are somewhat more tolerant of external-field nonlinearities than the 90° systems.

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* Berkeley Scholars Inc., Springfield, VA 22150
** University of Maryland, College Park, MD 20742