TRANSPORTABLE CHARGE IN A PERIODIC ALTERNATING GRADIENT SYSTEM*

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Abstract

A simple set of formulas is derived which relate emittance, line charge density, matched maximum and average envelope radii, occupancy factors, and the (space charge) depressed and vacuum values of tune. This formulation is an improvement on the smooth limit approximation; deviations from exact (numerically determined) relations are on the order of ±2%, while the smooth limit values are in error by up to ±30%. This transport formalism is used to determine the limits of transportable line charge density in an electrostatic quadrupole array, with specific application to the low energy portion of the High Temperature Experiment of Heavy Ion Fusion Accelerator Research. The line charge density limit is found to be essentially proportional to the voltage on the pole faces and the fraction of occupied aperture area. A finite injection energy ($\gtrsim$ 2 MeV) is required to realize this limit, independent of particle mass.

Introduction

For the design of the Multiple Beam Experiment (MBE-4) [1], the High Temperature Experiment (HTE) [2] and a heavy ion ICF driver it is not only necessary to determine the matched beam envelope functions for particular parameters, but also to provide accurate scale relations, particularly for those parameters relating to physically limiting features of the transport lattice. In general, an exact evaluation of the envelope functions may be obtained from the solution of the nonlinear envelope equation (including space charge) if the Kapchinski-Vladimirskij distribution [3] in transverse phase space is assumed. This has been found to be an adequate theoretical basis for design of a transport system, but it is too cumbersome for parametric display and scaling. The well-known continuous limit formulation [4] does yield useful relations between emittance, line charge density and mean radius but they are inaccurate, contain no detail of the focal lenses, and make no prediction of the maximum beam radius. These deficiencies of the continuous limit formulas are remedied in an improved approximate calculation described here, while retaining the desired simplicity of form and explicit scaling of the continuous limit.

Method of Calculation

The improved approximation of the envelope functions is based on an evaluation of the transfer matrix [M] for a full lattice period of length 2L. This matrix is the product of the simple matrices which represent lenses and drifts in the absence of space charge forces. Additional defocussing (thin) lenses representing the effect of space charge are placed in the middle of the drifts. At these mid-points the space charge force is close to its average for the full lattice period and can be evaluated using the matched envelope radius ($\bar{a}$) at the midpoints, which is the same for x and y. Thus there is a kick representing the charge effect for a half period:

$$\Delta x' = \frac{QL}{\bar{a}^2}\, x \,, \tag{1}$$

where Q is a dimensionless measure of the particle line charge λ_0:

$$Q = \frac{2eZ^2}{\beta^2\gamma^3 M_0{}^2}\left(\frac{\lambda_0}{4\pi\epsilon_0}\right) \to \frac{eZ^2}{T}\left(\frac{\lambda_0}{4\pi\epsilon_0}\right)(\mathrm{NR}) \,. \tag{2}$$

Standard relations between envelope functions and elements of [M] are employed [5]. The cosines of the normal and depressed tunes (σ_0 and σ) are determined from the trace of [M], and the ratio $\bar{a}^2\,(\sin\sigma)/\epsilon$, where $\pi\epsilon$ is unnormalized emittance, is given by the component M_{12}. The maximum radius (a_m) appears at the center of a focal quadrupole and is determined using the components m_{11} and m_{12} from the transfer matrix [m] evaluated between the lens center and drift center. In the consideration of finite lens length (ηL) and symmetric treatment of charge this formulation improves on the similar results derived by Keefe [6]. The four derived relations essentially relate $\bar{a}$, a_m, σ and σ_0 to the fundamental quantities η, L, ϵ, Q, and quadrupole strength K:

$$K = \frac{|B' \text{ or } E'/v|}{[B\rho]} \to \frac{E'Ze}{2T}\;(\mathrm{NR,ES}) \,. \tag{3}$$

In order to simplify the derived relations all trigonometric functions are expanded in the dimensionless lens strength ($\eta\sqrt{K}\,L$), keeping only lowest order non-trivial contributions to the envelope functions. A highly accurate value of σ_0 ($\lesssim 1\%$ error) and reasonably accurate expressions for the other quantities ($\lesssim 5\%$ error) are thereby obtained. To improve accuracy two further steps are taken. First, in the evaluation of [m], a space charge kick of one quarter the magnitude of Eq. (1) is inserted at both the lens center and drift center, this being a more symmetric application. Second, the coefficient of Γ^2 in the formula for $\cos\sigma$ [Eq. (5)] is corrected (from 1/8 to 1/6) to agree with the exact envelope results which can be derived in the limit $\epsilon \to 0$, $\sigma \to 0$. This is a natural modification of the basic calculation since the use of thin lenses to represent charge inevitably lead to errors in terms of second order in line charge ($\propto \Gamma$).

Summary of Relations

The four lattice and envelope relations we have derived are conveniently written as follows:

$$\cos\sigma_0 = 1 - \frac{(3-2\eta)\eta^2}{6}K^2L^4 \,, \tag{4}$$

$$\cos\sigma - \cos\sigma_0 = \Gamma\left[1 - \frac{5-\eta}{20}(1-\cos\sigma_0)\right] + \frac{\Gamma^2}{6}\left[1 - \frac{15-7\eta}{120}(1-\cos\sigma_0)\right] \tag{5}$$

$$\frac{\bar{a}^2\sin\sigma}{2\epsilon L} = \frac{\partial}{\partial\Gamma}\left(\cos\sigma - \cos\sigma_0\right) , \tag{6}$$

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$$\left(\frac{\bar{a}}{a_m}\right)^2 = \left[1 - \sqrt{\frac{(2-\eta)}{8}}\sqrt{\frac{6(1-\cos\sigma_0)}{3-2\eta}} + \frac{\Gamma}{16}\right]^2 + \left(\frac{L\varepsilon}{2a_m^{\,2}}\right)^2 . \qquad (7)$$

Here we have defined

$$\Gamma = \frac{2L^2Q}{\bar{a}^2} . \qquad (8)$$

The predictions of these formulas are compared with exact results in Table 1.

Typically, in using these relations a set of constraints is specified. For example the ratio of maximum beam radius to aperture radius (a_m/b) and ratio of half period to aperture (L/b) may be bounded. For the example given below we also specify a maximum voltage on pole faces

$$\phi(b) = \frac{E'b^2}{2} , \qquad (9)$$

which must be less than or on the order of 50 kV to avoid electrical breakdown.

Space Charge Limit

Equations (4)-(7) are readily solved for Γ with $\sigma = 0$, $\varepsilon = 0$, and a given value of σ_0. For the conservative value $\sigma_0 = 60°$, which avoids space charge induced instabilities [7], and $\eta = 1/2$ we find

$$\Gamma = .516 , \qquad (10a)$$

$$a_m/\bar{a} = 1.25 , \qquad (10b)$$

$$KL^2 = 2.45 . \qquad (10c)$$

Using these values, the particle energy and particle line charge may be written

TABLE 1. Comparison of approximate and exact relations. The rhs of Eqs. (4)-(7) is computed for a range of depressed tunes. The value paired immediately below is the exact value. For this study we have set $\eta = 1/2$ and $\sigma_0 = 60°, 90°$. The values of σ given in column 1 are exact (numerical), corresponding to the given values of Γ.

Given Parameters	σ_0	$\cos\sigma - \cos\sigma_0$	$\frac{\bar{a}^2 \sin\sigma}{2\varepsilon L}$	$a_m/\bar{a}$
$\sqrt{K}\,L = 1.5661$	60.085 60.000			
$\Gamma = .00000$ ($\sigma = 60°$)		.0000 .0000	.8875 .8921	1.274 1.269
$\Gamma = .43003$ ($\sigma = 24°$)		.4110 .4135	1.024 1.034	1.250 1.253
$\Gamma = .50741$ ($\sigma = 6°$)		.4912 .4945	1.049 1.061	1.246 1.250
$\sqrt{K}\,L = 1.8636$	90.297 90.000			
$\Gamma = .0000$ ($\sigma = 90°$)		.0000 .0000	.77500 .7841	1.443 1.430
$\Gamma = .57337$ ($\sigma = 60°$)		.4939 .5000	.9478 .9661	1.388 1.395
$\Gamma = 1.0323$ ($\sigma = 12°$)		.9606 .9781	1.086 1.125	1.355 1.371

$$T = (.408)\left(\frac{L}{b}\right)^2 Ze\phi(b) \qquad (\text{eV}) \qquad (11)$$

$$\lambda_0 = 1.85\times10^{-11}\,\frac{T}{Z^2e}\left(\frac{b}{L}\right)^2\left(\frac{a_m}{b}\right)^2 \qquad (\text{C/m}) \qquad (12)$$

If we allow as a maximum quadrupole voltage $\phi(b) = 50$ kV and aperture fill factor $a_m/b = .5$, then $\lambda_0 = .0943$ μC/m. More generally

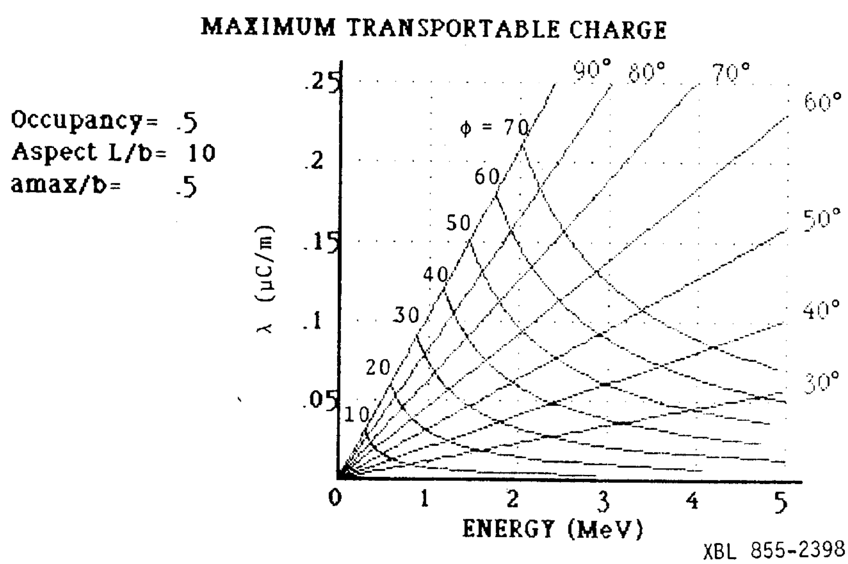

Fig. 1. Dependence of the maximum transportable charge per meter on beam energy for various lattice tunes σ_0 (in degrees) and quadrupole voltages ϕ (in kV) for $\eta = 1/2$. $L/b = 10$, and $a_m/b = .5$.

$$\lambda_0 \leq \frac{(.0943\ \mu C/m)}{Z} \left(\frac{a_m}{.5b}\right)^2 \frac{\phi(b)}{50\ kV} . \qquad (13)$$

The particle energy required to realize this limit is

$$T = (2.04\ MeV) \left(\frac{L}{10b}\right)^2 Z \frac{\phi(b)}{50\ kV} . \qquad (14)$$

Since aspect ratio L/b conservatively is taken greater than ~10 to avoid field nonlinearities, an injection energy of at least 2 MeV is required to realize the space charge limit of Eq. (13). Higher injection energy requires increased L/b to avoid a reduction of σ_0. Lower injection energy requires reduced $\phi(b)$ and λ_0 is reduced from the given limit. Specific examples are the HTE injection scheme, which approaches the space charge limit (T = 2 MeV, λ_0 = .075 μC/m), and the MBE-4 injector which operates with λ_0, $\phi(b)$, and T about one order of magnitude lower. The relationship of these quantities is displayed in Fig. 1.

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