IEEE Transactions on Nuclear Science, Vol. NS-32, No. 5, October 1985

LONGITUDINAL EQUILIBRIUM DISTRIBUTIONS OF ION BEAMS IN STORAGE RINGS WITH INTERNAL TARGETS AND ELECTRON COOLING*

T. Ellison

Indiana University Cyclotron Facility, Bloomington, IN 47405

Summary

A derivation of the model used to describe the longitudinal drag rate is presented along with analytic equations to describe various aspects of the longitudinal equilibrium of an ion beam in a storage ring with internal targets and electron cooling. The effects of transverse and longitudinal heating, cathode power supply ripple voltage, and the electron beam space charge depression are discussed.

Introduction

A light ion storage ring, with electron cooling and three internal target areas, is under contruction at IUCF[1,2,3,4,5,6] as a research tool to study nuclear physics using very thin targets ($\sim 10^{-8}$g/cm^2) and stored beams with very high spatial and energy resolution. Parameters for the IUCF Cooler are listed in Table I. Knowledge of the stored beam properties is important for planning experiments for this new facility. A Monte-Carlo program[7,8] has been written and used to study and provide insights into various effects of the interaction between ion beam heating by internal targets and ion beam cooling with electrons. Many of these effects can also be described well by analytic techniques. Some of these effects are described here in a general form, and are compared where possible to the results predicted by the Monte-Carlo program.

Model for Longitudinal Cooling

The Rest Frame Friction Force

The expression for the electron cooling friction force, F, on an ion of charge ze is given in terms of rest frame quantities in the MKS system of units for the nonmagnetized case as:[9]

$$\vec{F} = \frac{-4\pi z^2 e^4 n}{(4\pi\varepsilon_o)^2 m} \int d^3v\ L_c(u) f(\vec{v}) \frac{\vec{u}}{|u|^3} = \frac{d\vec{p}}{dt} \qquad (1)$$

where: e = electron charge, n = electron density, m = electron mass, v = electron velocity, v_I = ion velocity, p = ion momentum, L_c = Coulomb logarithm, $u = v_I - v$, f(v) = electron velocity distribution, normalized to 1

To simplify this expression we remove L_c, which depends logarithmically on u, from the integral as in ref. 10, where we believe the subscript "p" in the denominator of eq. (43) is a misprint.

Evaluation of Velocity Integral. Two reasonable disk-shaped electron velocity distributions are:

$$f(v) = (\pi\Delta_\parallel \Delta^2_\perp)^{-1}, \quad v_\parallel < \Delta_\parallel/2 \text{ and } v_\perp < \Delta_\perp$$
$$= 0 \quad , \quad v_\parallel > \Delta_\parallel/2 \text{ or } v_\perp > \Delta_\perp$$

or

$$f(v) = \left(2\pi\Delta_\parallel \Delta^2_\perp \exp(v^2_\perp/2\Delta^2_\perp)\right)^{-1}, \quad v_\parallel < \Delta_\parallel/2$$
$$= 0 \quad , \quad v_\parallel > \Delta_\parallel/2$$

where $\Delta_\parallel \ll \Delta_\perp$. Using the electrostatic analogy[10] we find, for $v_{I\perp} \ll \Delta_\perp$, that both of the above distributions yield:

Table I. IUCF Cooler Parameters

PARAMETER	SYMBOL	VALUE	UNITS
Magnetic Rigidity	K	3.6	T-m
Electron Beam Kinetic Energy	E_e	6-270	keV
Electron Current: Operation	I	0-2	A
Power Supply Limit		4.8	
Electron Beam Radius	r_b	1.27	cm
Electron Gun Perveance	k	0.7	$\mu A \cdot V^{-3/2}$
Electron Beam Transverse Temperature (design goal)	T	<0.2	eV
Cathode Power Supply Regulation	$\Delta V_{PP}/V_{MAX}$	$<2.5\times10^{-5}$	
Solenoid Guide Field	B	1.5	kG
Electron Interaction Length	l_c	3	m
Ring Circumference	C	86.83	m
Transverse Ring Acceptance	A	25π	mm-mrad
Longitudinal Ring Acceptance	$\Delta p/p$	$\pm 0.2\%$	
Beta Functions in Cooling Region	β_c	3 - 13	m
Dispersion in Cooling Region	η_c	0	m

$$I_\parallel = \int d^3v\ f(v) \frac{u}{|u|^3} = \langle v^2_\perp \rangle^{-1}, \qquad v_{I\parallel} = \Delta_\parallel/2$$
$$= 2v_{I\parallel}/\langle v^2_\perp\rangle \Delta_{e\parallel}, \qquad v_{I\parallel} < \Delta_\parallel/2$$

For $v_{I\parallel} > \Delta_\parallel/2$, we approximate the integral I as:

$$I_\parallel = \left(\langle v^2_\perp\rangle + (v_{I\parallel})^2\right)^{-1}$$

Laboratory Drag Rate

In order to express F in terms of laboratory frame quantities, we note that:

$$F^*_\parallel = \frac{dp_\parallel^*}{dt^*} = \frac{\gamma dp_\parallel^*}{dt} \cong \frac{dp_\parallel}{dt} \cong \frac{dE}{\beta c dt}$$

$$\text{and} \qquad n^* = \frac{I\eta}{\gamma\beta c e\pi(r_b)^2}$$

Where $\eta = l_c/C$. The definition of the other symbols may be found in Table I. The electron temperature, T, is defined in units of eV as:

$$T \equiv m\langle v^2_\perp{}^*\rangle = mc^2\beta^2\gamma^2(\langle\theta^2\rangle_H + \langle\theta^2\rangle_V)$$

The longitudinal cooling effect can be expressed as a drag rate, $R(\Delta) = \beta c F^*(\Delta)$, the rate at which the electron beam can change the energy of an ion with an energy deviation, Δ. The maximum drag rate, which we denote as R_M, occurs when the energy deviation, Δ, of an ion is such that the longitudinal rest frame velocity of the ion is equal to half the longitudinal rest frame velocity spread of the electron beam, $\Delta_{e\parallel}/2$. We denote this energy deviation as Δ_1.

Using the above relations, $R(\Delta)$ can be expressed as:

$$R_M \cong R(\Delta_1) = \left.\frac{dE}{dt}\right|_{max} = \frac{4z^2Mc^2 r_e r_p I\eta L_c}{\gamma e (r_b)^2 (T/mc^2)} \qquad (2a)$$

$$R(\Delta) \cong R_M \cdot (\Delta/\Delta_1), \qquad \Delta < \Delta_1 \qquad (2b)$$

$$\cong R_M \cdot \left(1 + \Delta^2 m/p^2 T\right)^{-1}, \qquad \Delta > \Delta_1 \qquad (2c)$$

where M is the proton rest mass, and p is the ion beam lab frame momentum. We use equation (2) for estimating

*Research supported by the National Science Foundation under grants NSF PHY 81-14339 and NSF PHY 82-11347.

the longitudinal drag rates for beams in the IUCF Cooler. This equation predicts a maximum longitudinal drag rate of 1.8 MeV/s for a 644 MeV/c proton beam cooled with a 4 Amp electron beam using the IUCF parameters from Table I. In the calculations that follow, the ion beam has z = A = 1.

Equilibrium Energy Distributions:

The probability that a particle will interact with the thin target on any given passage is very small. In addition, the most probable energy loss rate of a particle is much less than the drag rate for target thicknesses practical for use as internal targets. Ions in the beam, however, can lose as much as 0.5 MeV on a single traversal through the target. The combination of these two effects leads to an energy distribution which has a narrow peak, with a width given by Δ_1, and a very long tail.[7]

Fraction of the Ion Beam in the Narrow Peak.

$P(\Delta)d\Delta$ is the probability that a particle will suffer an energy loss between Δ and $(\Delta + d\Delta)$, when traversing the thin internal target. Using the formalism developed by Landau, this quantity can be expressed asymptotically, for $\Delta \gg \Delta_o$ (where Δ_o is the most probable energy loss) as:[11]

$$P(\Delta)d\Delta \cong (\xi/\Delta^2)d\Delta; \qquad \xi = \frac{2\pi N x \rho e^4}{(4\pi\varepsilon_o)^2 m\beta^2 c^2}\,\frac{Z}{A} \tag{3}$$

N is Avogadro's number, ρ is the target density, x is the target thickness, and Z and A are the target nuclei charge and atomic weight.

The time it takes to cool an ion back to the narrow peak after an energy loss, Δ, is given by:

$$T(\Delta) \cong \int_{\Delta}^{\Delta_1} \frac{d\Delta}{R(\Delta)} = \left((\Delta-\Delta_1) + (\Delta^3-\Delta_1^3)m/3p^2T\right)/R_M \tag{4}$$

assuming that there is mechanism present other than cooling to compensate for the average energy loss, such as the use of rf which will randomize subsequent energy changes through synchrotron oscillations. The fraction of the ion beam which is contained within the narrow peak, f_p, can then be approximated by:

$$f_p \cong 1 - f_o\int_{\Delta_2}^{\Delta_1} P(\Delta)T(\Delta)d\Delta \tag{5}$$

$$= 1 - \frac{f_o\xi}{R_M}\left(\ln(\Delta_2/\Delta_1) - 1 + \frac{\Delta_1}{\Delta_2} + \frac{m(\Delta_2^2 - \Delta_1^2)}{6p^2T}\right)$$

where Δ_2 is the maximum energy the ion can impart to an electron at rest $(\Delta_2 \cong 2m\beta^2\gamma^2c^2)$ and f_o is the ion beam revolution frequency.

In order to compare the predictions of this simple model with the results of the Monte-Carlo program, we set R_M = 2 Mev/s, and set T = 0.04 eV in eqs. (2c) and (5) to give the same rapid fall off of $R(\Delta)$ with Δ as in ref. 7. These two models of the longitudinal drag rate are shown in Figure 1. Figure 2 shows the prediction of f_p using equation (5) along with the results of the Monte-Carlo calculations. The fraction of the beam located within the tail region is proportional to the target thickness and inversely proportional to the maximum drag rate. The predominance of the logarithmic term in eq. (5) suggest a $1/\Delta$ fall off of the particle density per unit energy deviation.

Width of the Peak in the Energy Distribution

Width Due to Cathode Power Supply Regulation.

Due to the very high longitudinal drag rates and the fast

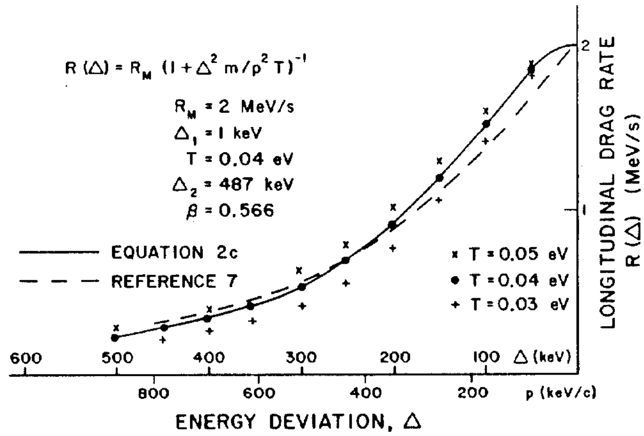

Figure 1: Models of $R(\Delta)$ used in calculations

cooling times within the peak of the energy distribution, the peak will coherently track electron beam energy changes due to the cathode power supply ripple with an energy deviation of $e(V_{ripple,pp})(M/m)$ provided the slew rate of the power supply does not exceed $R_M(m/eM)$. The ion beam will be unable to track ripples of higher slew rates, and the energy deviations of the ion beam will be limited to about $R_M/2\pi f$, where f is the ripple frequency. Very fast ripples will cause minimal coherent shifting of the ion beam energy, but will increase Δ_1. The energy deviation of ions with deviations less than Δ_1 decreases exponentially, with a time constant given by Δ_1/R_M. Increased ripple will increase the range of energies in which the proton beam cools exponentially, increase the cooling time, and extend the range of energies in which the ion experiences a lower drag rate. The electron cooling system high voltage system design, and the regulation of the cathode power supply are discussed in ref. 12.

Width Due to the Electron Beam Space Charge Depression.

The electron beam space charge depression causes a variation in the electron energy which is quadratic with distance from the axis of the electron beam. This effect is negligible in rings without internal targets due to the very small equilibrium emittance of the ion beam, but may cause a relatively large energy spread of the ions in a storage ring with an internal target, due to the larger transverse equilibrium emittances.[7] Ions with large betatron amplitudes will sample portions of the electron beam with higher lab frame velocities and their equilibrium

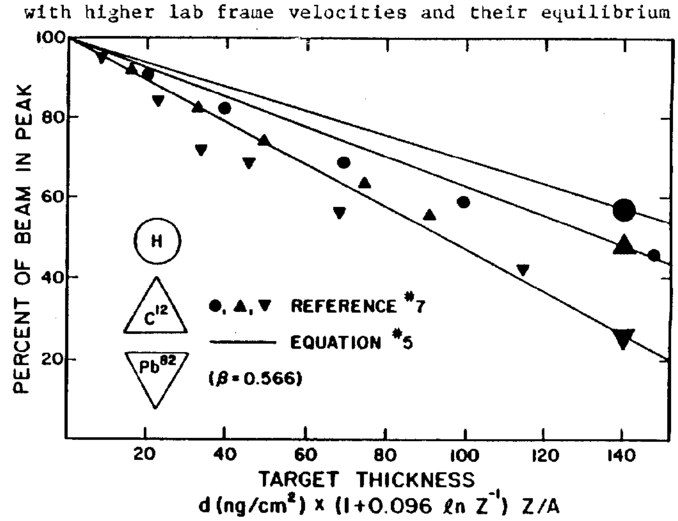

Figure 2. Percent of ion beam contained in peak.

energy will be shifted upwards.

The *transverse cooling time is given approximately* by[13]:

$$\tau_c = \frac{3}{2(2\pi)^{1/2}} \frac{\beta\gamma^2 e\pi(r_b)^2}{r_e r_p \eta I L_c} (T/mc^2)^{3/2} \qquad (6)$$

for a spatially symmetric Gaussian electron velocity distribution. Here we use this formula for an order of magnitude estimate of the ratio of the transverse cooling time to the longitudinal cooling time:

$$\tau_c/\tau_\parallel = \tau_c/(\Delta_1/R_M) = (2\beta\gamma Mc^2/\Delta_1)(T/mc^2)^{1/2} \sim 10^2$$

The longitudinal equilibrium energy distribution can be treated as a function of the transverse equilibrium distribution of the ion beam due to this great difference in cooling time constants.

We define the ion beam transverse emittance at the waist in the cooling region to be a Gaussian distribution where both $\pi\varepsilon_x$ and $\pi\varepsilon_y$ are the phase space areas which contain 90% of the particles:

$$d^4N = \frac{4}{\pi^2\varepsilon_x\varepsilon_y} \exp\left(-2\left(\frac{x^2}{\beta_x\varepsilon_x} + \frac{x'^2\beta_x}{\varepsilon_x} + \frac{y^2}{\beta_y\varepsilon_y} + \frac{y'^2\beta_y}{\varepsilon_y}\right)\right) dx\,dx'\,dy\,dy' \qquad (7)$$

This distribution can be rewritten in terms of the betatron amplitudes (X, Y) in the z-x and z-y planes as:

$$d^2N = \frac{16XYdXdY}{\varepsilon_x\varepsilon_y\beta_x\beta_y} \exp\left(-2(X^2/\beta_x\varepsilon_x + Y^2/\beta_y\varepsilon_y)\right) \qquad (8)$$

The energy shift of a particle is sensitive to the variable A, the "sum betatron amplitude", which is given by: $A^2 = X^2 + Y^2$. *For simplicity we set* $\beta_x = \beta_y = \beta_c$. In addition, we set $\varepsilon_x = \varepsilon_y = \varepsilon$ which is reasonable due to the coupling of horizontal and vertical betatron oscillations by the cooling system solenoid and compensating solenoids which will cause mixing in about 10^2 to 10^3 turns. The distribution of A can then be expressed as:

$$dN = \frac{8A^3}{\varepsilon^2\beta_c^2} \exp(-2A^2/\beta_c\varepsilon)\, dA \qquad (9)$$

The energy shift, Δ_{sq}, due to the electron beam space charge depression is a function of A and is given by the relation:

$$\Delta_{sq} = KA^2; \qquad K = \frac{(1 + l_c^2/12\beta_c^2)eI}{8\pi\varepsilon_o\beta c(r_b)^2}\frac{M}{m} \qquad (10)$$

This relation is obtained by averaging Δ_{sq} for a particle as it moves through the cooling region, and then averaging over random betatron phases. The equilibrium energy distribution can then be expressed as:

$$\frac{dN}{d\Delta} = \frac{4\Delta}{\varepsilon^2\beta_c^2K^2} \exp(-2\Delta/K\beta_c\varepsilon) \qquad (11)$$

This distribution function is plotted in Figure 3 along with the results of a similar Monte-Carlo calculation[8]. This distribution is expected for a fixed-emittance (due to the balance between cooling and transverse heating), with the effects of longitudinal cooling turned off. We expect this energy spread to approximate the energy distribution of beam within the peak when longitudinal heating effects are present since the most probable energy loss per turn, Δ_o,[11] is so much smaller than R_M/f_o. We note the equilibrium emittance varies with the product of the transverse cooling time, the beta functions at the target waist, and the target thickness.[7] (This

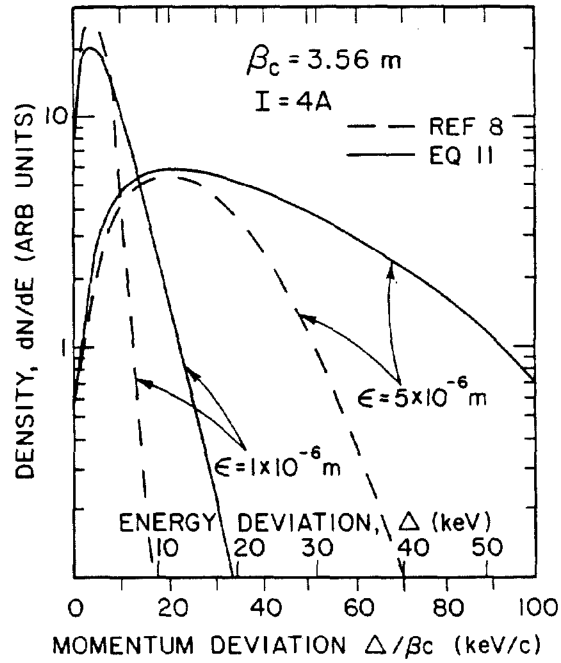

Figure 3. Ion beam energy distribution due to a finite emittance and the electron space charge.

quantity is found by setting the increment in the emittance due to multiple scattering equal to the decrement due to cooling). Using these relations, we find that the energy spread due to the electron beam space charge effect is proportional to the target thickness, the beta functions in the cooling region and at the target location, and the three halves power of T; it is independent of I since ε_{eq} is inversely proportional to I, and K is proportional to I.

References

1. R.E. Pollock, IEEE Trans. Nucl. Sci. NS-30,(1983), 2056-2060.
2. R.E. Pollock, Tenth Int. Conf. on Cycl. and their Appl., East Lansing, Mich. (1984), 583-587.
3. R.E. Pollock, Proc. Workshop on Nucl. Phys. With Stored Cooled Beams, ed. P. Schwandt, AIP Conf. Proc., to be published.
4. H.O. Meyer, Indiana Cooler User Guide, IUCF, Internal Report (1983).
5. H.O. Meyer, Proc. Symp. on Nucl. Shell Models, Philadelphia 1984, World Scientific, Singapore, to be published and IUCF Preprint
6. D.L. Friesel, T. Ellison, and W.P. Jones "Electron Gun Design Study for the IUCF Beam Cooling System", these proceedings.
7. H.O. Meyer, Proc. 8th Int. Conf. on Appl. of Acc. in Res. and Ind., Denton, Texas (1984); Nucl. Inst. and Meth., to be published.
8. H.O. Meyer, Proc. Workshop on Nucl. Phys. With Stored Cooled Beams, ed. P. Schwandt, AIP Conf. Proc., to be published.
9. Ya.S. Derbenev and A.N. Skrinsky, Particle Accelerators 8, 235-243 (1978).
10. F.T. Cole and F.E. Mills, Ann. Rev. Nucl. Part. Sci. 31 (1981) 295-335.
11. L. Landau, J. Phys., USSR 8. (1944) (English translation).
12. T. Bertuccio, B. Brown, G. Donica, T. Ellison, D.L. Friesel, "High Voltage System Design for the IUCF 300 kV Electron Cooling System, these proc. (Abstract H-37)
13. C. Rubbia, Proc. of the Workshop on Prod. High Lum. High Energy Proton-Antiproton Collisions, 98-101 (1978) Berkley.