ION STABILITY IN BUNCHED ELECTRON BEAMS*

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Accumulation of ions can adversely affect the performance of an electron storage ring. Stability of ion motion under the influence of passing bunches is often used to predict if ion accumulation will occur in a particular storage ring design. This stability question has been previously studied with a linear model [1]. We present a nonlinear numerical tool to study ion motion in a beam of short bi-gaussian electron bunches. Results of ion trapping studies on a model electron storage ring are presented. We find that including nonlinear effects may not significantly alter the conclusions of the linear model, and that in a ring containing many bunches a gap in the bunch train effectively destabilizes ion motion.

A Linear Model

The behavior of residual gas ions near a bunched electron beam may be modeled by considering the passing electron bunches to be thin lenses that repeatedly focus ions toward the bunch train. Thus, the ions oscillate about the equilibrium orbit of the electron beam; if the oscillations are stable, ion accumulation may occur. The ions move with thermal velocities; the magnetic force is therefore negligible and only the effect of the electric field of a passing bunch need be included in the calculation. Given the number n of bunches in the ring, the total circulating current I, the ring radius R and the beam dimensions σ_x and σ_y, an average space charge density within the bunch, and hence the linearized electric field at the center of the bunch, may be determined. If the ions are assumed to move only under the influence of this linear electric field, the focussing effect of each passing bunch may be computed. A stability criterion (equivalent to requiring that the absolute value of the trace of the transfer matrix of thin lens bunches with interleaved drifts be less than 2) may then be established [2]. The motion of an ion parallel to the x(y) axis will be stable if the ion mass is greater than a critical value $A_{x(y)}$ given by the following expression.

$$A_{x(y)} = 2(\pi R/n)^2(m_e/m_p)(I/I_0)(\sigma_{x(y)}(\sigma_x+\sigma_y))^{-1}$$

Here, $I_0=ec/r_e$ is the Alfven current (17045 A), $m_{e(p)}$ the mass of the electron (proton), and $A_{x(y)}$ is in a.m.u. If the ion mass is less than this critical value, the effective "focal length" of the passing electron bunches is small relative to the interbunch spacing, and the ion is thus overfocussed and lost. If, however, the ion mass is greater than this critical value, the effective focal length of the passing bunches is long compared to the interbunch spacing. The ion then oscillates stably in the x (y) plane. Since $\sigma_x>\sigma_y$ for most electron storage rings, typically $A_x<A_y$. That is, ions that stably oscillate in the magnetic midplane may not oscillate stably out of the median plane. The vertical critical value is therefore the figure of merit for certification of ion stability.

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A Nonlinear Model

The above linear model is deficient in that it does not account for the non-uniform (bi-gaussian) transverse density profile of an electron bunch and thus fails to properly describe the nonlinear amplitude dependence of the electric field of the bunch. It is possible to express the electric field of a two dimensional gaussian distribution in closed form [3]. For the charge density function

$$\rho(x,y) = (Q/2\pi\sigma_x\sigma_y)\exp(-x^2/2\sigma_x^2-y^2/2\sigma_y^2)$$

the electric field may be written in terms of the complex error function w(z=x+iy) as follows.

$$E_x-iE_y = (-iQ/2s\epsilon_0\sqrt{\pi})[w(x/s+iy/s) \quad (1)$$

$$-\exp[-(x+iy)^2/s^2+(xr+iy/r)^2/s^2]w(xr/s+iy/sr)]$$

Here, $s=\sqrt{2(\sigma_x^2-\sigma_y^2)}$ and $r=\sigma_y/\sigma_x$. The transverse momentum impulse imparted to an ion by a passing bunch by way of this electric field may then shown to have the following components.

$$\Delta p_x = \mathrm{Re}[(q/c)(E_x-iE_y)] \quad (2)$$

$$\Delta p_y = \mathrm{Im}[(q/c)(E_x-iE_y)]$$

The ion charge is q, and the Q of (1) is to be taken as the total single bunch charge.

Expressions (1) and (2) describe in closed form the effect on an ion of the passing of a single short bunch. During the interval between bunch passings, the ion simply drifts; such motion is described by the following transformation.

$$\begin{aligned} \bar{x} &= x+(p_x/m_i)(\ell/c) \\ \bar{p}_x &= p_x \\ \bar{y} &= y+(p_y/m_i)(\ell/c) \\ \bar{p}_y &= p_y \end{aligned} \quad (3)$$

In this transformation, m_i is the ion mass and ℓ is the bunch-to-bunch spacing (so that ℓ/c is simply the time delay between the bunch passages described by (2)).

Taken together, the explicitly canonical transformations (2) and (3) provide a nonlinear description of the motion of an ion under the influence of passing electron bunches. These expressions are in closed form and in terms of a well known function w(z). They may therefore be used as the basis of a numerical model for studying the stability of ion motion in the presence of a bunched electron beam.

A Modeling Program

We have written a FORTRAN program based on (2) and (3). The program, TRAPION, allows the user to interactively specify electron beam sizes, bunch configurations, the charge per bunch, and input the initial location and transverse momentum of an ion within a vacuum chamber. The program then simulates the motion of an ion with a user-defined mass for a user-specified period of time (given in terms of a

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"number of turns", i.e., the circulation period of the storage ring) and provides a "turn-by-turn" display of the motion of the ion in phase space. This on-line graphics display allows the user to determine if the ion motion is stable.

The heart of the program is a fast complex error function evaluation routine [4], which allows rapid ray tracing using the nonlinear transformation (2). The total program length, including all comments, graphic subroutine calls, and parameter editing facilities is approximately 600 lines.

TRAPION has been installed on both a CDC-6600 (in single precision) and a DEC VAX-Cluster (in G-Floating double precision) at the Lawrence Berkeley Laboratory. Execution times are dependent upon the number of bunches stored and the total time period simulated. In general, for cases with a few tens of bunches and motion simulated for a few thousand circulation periods of the machine, the interactive turn-around time for a single simulation is on the order of one minute. Typical graphics output is shown in Figures I.

Comparison of Methods and Conclusions

A direct comparison of linear and nonlinear models is possible by testing the stability of a variety of ion species using the program TRAPION, and comparing to the predictions of the linear model. Parameters of a model storage ring are given in Table I. Use of these parameters with the linear model indicates that the critical ion mass for stable vertical oscillations is A_y= 19.6 a.m.u. That is, any ion with mass less than 20 a.m.u. will be lost by means of overfocussing in the vertical plane.

Table II displays the results of a simulation test with the program TRAPION. Ions of various masses were launched with initial displacements of $.5\sigma_x$ and $.5\sigma_y$. The motion was then tracked for 2000 machine periods to determine the stability of each species. We find excellent agreement with the prediction of the linear model. This is to be expected, as the electric field within a gaussian bunch is quite linear at $.5\sigma$. However, we generally find the predictions of both models to be consistent, even at relatively large amplitudes. The only significant discrepancy between the models is the loss time observed for an unstable ion. In the linear model, an unstable ion is lost very rapidly. In the nonlinear model, because of the exponential roll-off of the field at large amplitudes, the loss may take place only after hundreds of machine circulation periods.

If the same model storage ring is studied with 10 bunches and a total current of 200mA (but with all other parameters the same) the critical mass for accumulation drops to 7 a.m.u., according to the linear model. TRAPION confirms this limit, finding that ions of mass greater than 8 a.m.u. are stable. If, however, 3 bunches are removed from the bunch train, so that the ring contains 7 bunches followed by a gap of 3 empty buckets (but with the same single bunch current, for a total of 140mA), the nonlinear model predicts that ions of mass as high as 44 a.m.u. are destabilized. This result is observed in simulations using a variety of parameters; it appears that, even in cases where a large number of bunches (and correspondingly short bunch-to-bunch spacing) tends to stabilize ion motion, a gap in the bunch train destablizes ion motion for even those species of relatively high atomic weight. The gap appears to allow the ions to clear the beam, and to drift to large amplitudes and be lost. Figures I illustrate the ion phase space for the case of an ion of mass 44, which is stable with 10 bunches but unstable with a 7 bunch, 3 empty bucket configuration.

We remark that this method does not address the question of ion accumulation rates, but is only a simple test of ion stability in specific instances. Such a method could also be employed to consider the stability of electrons in the vicinity of a bunched proton beam, or to determine if ion accumulation is possible in the vicinity of an antiproton beam.

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Table I Parameters for Model Storage Ring

Horizontal Emittance	1×10^{-8}m-rad
Vertical Emittance	1×10^{-9}m-rad
Average β_x	10 m
Average β_y	10 m
Number of Bunches	3
Total Current	50 mA
Circumference	100 m

Table II Simulation Results Using TRAPION

Ion Mass, a.m.u.	Species	Result of Simulation For 2000 Machine Periods
1	H	unstable
2	H_2	"
12	C	"
14	N	"
16	CH_4	"
17	OH	"
18	H_2O	"
22	(hypothetical)	"
23	"	stable
28	CO	"
40	Ar	"
44	CO_2	"

Figure I a) Phase space plot for ion of mass 44 when all buckets are filled. The ion motion is stable for 2000 machine periods.

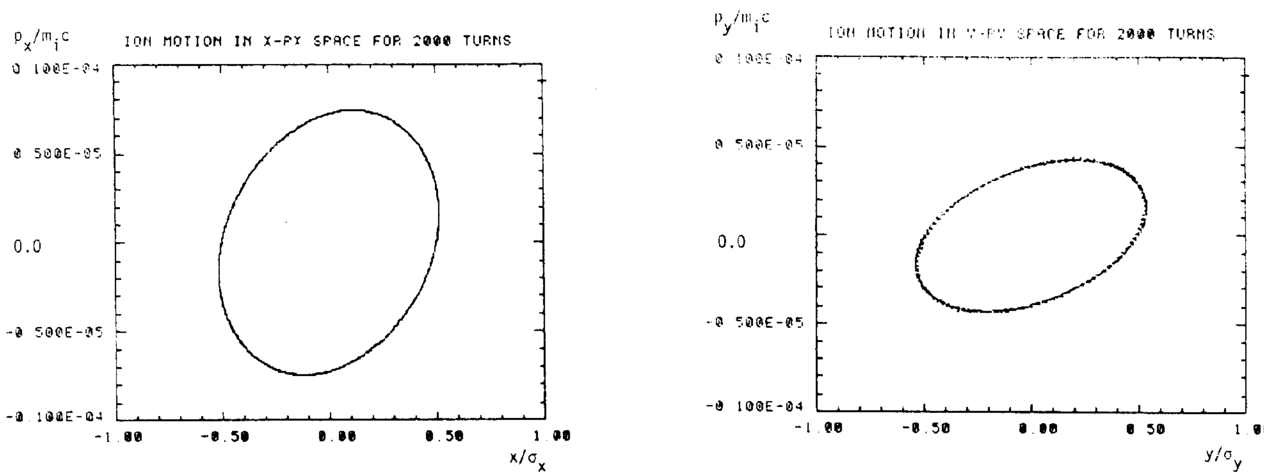

Figure I b) Phase space plot for ion of mass 44 when a 3 bucket gap is left in the bunch train. The ion motion is now unstable prior to reaching the 2000 turn limit.