The associated series inductance is related to the between adjacent walls of the rectangular waveguide.

Finally the TE II waveguide is loaded inductively approximately by the capacitance per unit length shunt capacitance by

The shunt capacitance per unit length, $C_{sh}$, of the usual $TE_{11}$ mode (Fig. 1(b)) is given approximately by the capacitance per unit length between adjacent walls of the rectangular waveguide. The associated series inductance is related to the shunt capacitance by

$$4L_{series}C_{sh} = c^{-2}$$

where $c$ is the speed of light.
\[ \begin{align*}
\frac{\partial i_1}{\partial t} &= (-Y_1 - Y_5) v_1 + (-Y_4 - Y_5) v_2 + (-Y_4) v_3 \\
\frac{\partial i_2}{\partial t} &= (Y_1) v_1 + (Y_2 - Y_6) v_2 + (Y_6) v_3 \\
\frac{\partial i_3}{\partial t} &= (Y_5) v_1 + (Y_2 + Y_5) v_2 + (-Y_3) v_3 \\
\frac{\partial v_1}{\partial t} &= (-Z_1) i_1 + (Z_2) i_2 \\
\frac{\partial v_2}{\partial t} &= (-Z_2) i_2 + Z_3 i_3 \\
\frac{\partial v_3}{\partial t} &= (-Z_4) i_1 + (-Z_4) i_2 + (-Z_3 - Z_4) i_3 \\
V_4 &= -V_1 - V_2 - V_3 \\
i_4 &= -i_1 - i_2 - i_3
\end{align*} \]

Fig. 2 Distributed equivalent circuit, including opposite vane capacitance. The system variables and constants are defined.

Infinite shunt inductance), they describe the TEM mode operation. Note that the currents used in the equations are the longitudinal currents which flow axially, primarily along the vane tips. As there are only three independent voltages, there are only three transverse resonator modes.

The numerical values for the vane-to-adjacent-vane capacitance per unit length and the shunt inductance per unit length are easily derived from quasi-static formulae. The vane-to-opposite-vane capacitance is influenced by the shielding effect of adjacent vanes. A SUPERFISH calculation of the dipole-quadrupole frequency separation is used to determine an approximate value.

A Lumped Equivalent Circuit Description of RFQ End Regions

For a standard RFQ end region configuration (Fig. 3), each tuner gap is modeled as a variable capacitor, and each path from the tuner around the end wall and back up to the vane tip, is modeled as an inductive loop. If the vane ends are cut back, the long "uniform" TE waveguide part of the RFQ is considered to end at approximately the mean position of the cutback. Thus the lumped equivalent circuit for the end region must include the vane-to-vane capacitance for the length between the mean cutback position and the end tuner.

Calculation of the numerical values of the end tuner gap and vane-to-vane capacitances is straightforward, with some uncertainty introduced by fringing fields - an error of 20% might be expected. In contrast, the calculated numerical value of the end loop inductance is uncertain by at least a factor of two, primarily because the geometry is non-standard. Thus a calibrating constant in the end loop inductance formula was determined by fitting a complete RFQ calculation to measured properties.

RFQ configurations in which a vane extends to the end plate are easily modeled by making the appropriate capacitance very large and inductance very small - thus shorting the vane tip to the end wall.

Fig. 3 (a) The end region of a standard four-vane RFQ, showing an end tuner and a cutback vane. (b) The lumped equivalent circuit model of an RFQ end region.

Comparison with the RFQ Low Power Measurements of Nakanishi et al.

The report of the Japanese measurements contains an extensive set of data including longitudinal and azimuthal distributions of field for symmetric and displaced vanes, the effect of end tuner variations on both longitudinal and azimuthal distributions, higher order mode spacings, and quadrupole-dipole mode frequency differences. Comparison with these measurements constitutes a good test of the equivalent circuit model.

The model was found to reproduce the following basic RFQ properties demonstrated by the data.

1. If, starting at a withdrawn position, all end tuners are gradually inserted, the longitudinal vane voltage distribution is first convex to the axis, then flat (at "cutoff"), and then concave - the frequency progressively decreasing.

2. Increasing end tuner capacitance on only one end causes the relative vane voltage to increase at that end.

3. The dipole mode frequencies are lower than the quadrupole mode frequency.

4. The spacing between the two lowest quadrupole modes was measured to be 33.0 ± 0.2 MHz and predicted to be 32 MHz.

The quadrupole-dipole frequency separation of 3.5 MHz was used to determine the equivalent circuit values for the end inductances and cutback calibration.

The Japanese report shows data (reproduced as lines on Fig. 4) for a case in which a single vane was moved transversely by 0.25 mm (Fig. 4(a)). The field imbalance was then corrected by inserting only one set of end tuners (Fig. 4(b)). The equivalent circuit model predictions are shown as individual points on Fig. 4; the agreement is good.
Sensitivity to a Systematic Linear Variation in Mean Bore Radius

J. Potter has suggested an \( L^2 \) dependence of the longitudinal voltage tilt on systematic longitudinal variations. This is important for it suggests a practical limiting length (of an unstrapped RFQ) which is dictated by achievable machining and alignment tolerances.

The present model was applied to 270 MHz RFQ's of various lengths, with approximately 0.46 cm beam hole radius, 0.35 cm vane tip radius, and end regions adjusted to produce an essentially flat longitudinal field. A \( \pm 0.5\% \) linear longitudinal variation in each vane-to-vane distributed shunt capacitance was then introduced for each length, \( L \), as given by the equation on Fig. 5. This is equivalent to a 0.075 mm (1.6\%)

change in mean bore radius along each length. The frequencies were essentially the same for all lengths. The dashed line on Fig. 5 shows a rapid increase of longitudinal field tilt with RFQ length, consistent with an \( L^2 \) dependence. This demonstrates how one may quantitatively determine a maximum practical RFQ length, given machining and assembly tolerances.

Calculation of the Properties of Vane Strapping

Howard and Lancaster have introduced vane strapping as a method of stabilizing the RFQ fields against mechanical errors. The straps consist of semi-circular metal hoops joining opposite vanes by looping through a hole in adjacent vanes.

The straps influence the fields in two ways:
1. opposite vanes are connected to each other through a small inductance, so only a small voltage difference will exist between them, even with opposite vane imbalances.
2. the strap introduces a significant increase in nearest neighbour vane-to-vane shunt capacitance. The latter effect produces a very significant decrease in the RFQ frequency (~1 MHz per ring pair), and, because of the local increase in capacitance, causes the vane voltage to peak at the strap. This effect can be compensated by introducing a local decrease in tube inductance. As an example of the stabilizing effect of compensated straps, a calculation was done for a 270 MHz, 1.6 meter long RFQ with a dielectric perturbation at the left end of one quadrant. The straps in the middle obviously stabilize the azimuthal field (Fig. 6) and appear to divide the system into two essentially independent (as far as tolerances are concerned) RFQ units.

Conclusions

The ability of the present equivalent circuit model to predict basic RFQ properties and give approximate quantitative predictions of mode frequencies, tuning plunger properties, perturbed field distributions, and strap characteristics make it a very useful aid in understanding and designing RFQ structures.

References
1. R.M. Hutcheon, "The Distributed Parameter RF Network Theory for the RFQ Resonator", to be published.