The hydrogen atoms encounter $N_Y$ photons in the time $t = L/c$ spent in the interaction region; therefore, the reaction cross section can be written as

$$\sigma(\omega) = \frac{\hbar^2 N_Y(\omega)}{AE}. \quad (6)$$

Thus, the laser power for $n$-photon ionization is

$$P_n = \frac{n}{n(n+1)} \frac{I}{c^2} \left( \frac{\hbar^2}{\sigma(\omega) A} \right)^{1/n}. \quad (7)$$

The intensity $I'$ in the lab frame is related to $I$ by

$$I' = \gamma^2 (1 + B)^2 I. \quad (8)$$

**Applications and Discussion**

We are interested in the laser powers necessary to ionize an actual beam of hydrogen atoms by $n$-photon processes, where $n = 1, 2, 3, \ldots$.

The laser powers for monoenergetic beams are given by Eq. (7), which holds for all $n$ because $C_n(\omega) \sigma(\omega)$. Actual beams are not monoenergetic; they have a momentum spread and thus contain particles with velocities in the range $\vec{v} - \Delta \vec{v} \ldots \vec{v} + \Delta \vec{v}$. We continue to assume a monochromatic laser of frequency $\omega'$. The frequency $\omega$ of the photons encountered by each individual particle depends on that particle's velocity. However, all photon frequencies are in the range $\vec{w} - \Delta \omega \ldots \vec{w} + \Delta \omega$, where $\Delta \omega = 2\gamma^2 \Delta v / c$. The beam's momentum spread has to be considered when $\sigma(\omega)$ varies significantly over this frequency range. To properly determine the laser power in such a case, we choose the smallest $C_n(\omega)$ in the frequency range to calculate $P_n$. This assures 100% ionization, even for those particles with the smallest cross section.

We now replace $\omega$ by the wavelength $\lambda = 2\pi c / \omega$ and stress the dependence of the laser power on the interaction-region geometry by rewriting Eq. (7) as

$$P_n(\lambda) \left[ \text{Watts} \right] = \frac{P_n(\lambda)}{n} \left[ \text{Watts} \right] \left( \frac{N/Y}{cm^2} \right) \left( \frac{\sigma(\lambda)}{cm^2} \right) \left( \frac{1/n}{\lambda} \right). \quad (9)$$

We produce plots of $P_n(\lambda)$ versus $\lambda$ for 100% ionization; from these we can obtain the laser powers for any interaction-region geometry. All numerical examples are for the PSR, where $\vec{v} = 2.52 \times 10^8$ m/s and $\Delta v = 0.001 \vec{v}$.

Most calculations of single- and multiphoton ionization make use of perturbation theory and result in values of $C_n(\lambda)$, which are independent of $I$. At high field intensities, perturbation theory ceases to be valid, and $C_n(\lambda)$ depends on $I$. To check the validity of our results (computed from the results of perturbation theory), we calculate $I$ for reasonable $L$ and compare the resulting values with the "critical" values, for which perturbation theory ceases to be valid.

**One-Photon Process**

Both theoretical calculations and actual measurements of the cross section $C_1(\lambda)$ exist. The results of all those calculations, which use perturbation theory, are essentially identical; also, they are in good agreement with experimental results (see Ref. 2).

Figure 1 is a plot of $P_1(\lambda)$ versus $\lambda$. The function $P_1(\lambda)$ decreases monotonically with increasing $\lambda$. **Summary**

The two-step charge-changing injection used in the Los Alamos Proton Storage Ring (PSR) requires stripping of $H^0$ to $H^+$ by high magnetic fields and subsequent stripping of $H^+$ to $H^0$ by a carbon foil. We consider single- and multiphoton laser ionization as alternatives to using a fragile foil. The multiphoton case is of possible interest for selection of practical lasers, which tend to have increased power output at higher wavelengths. The formulas derived express the necessary laser powers for ionization of monoenergetic $H^0$ beams; they also hold for beams of particles other than atomic hydrogen. The numerical examples are for the RNN-NeV PSR beam with momentum spread taken into account. Additionally, we discuss selective stripping as an implication of the inherent energy selectivity of the photoionization process.

**Formulas for a Monoenergetic Beam**

We consider a monoenergetic beam of relativistic hydrogen atoms, which collide head-on with the photons of a laser beam. In the lab frame the particles have velocity $v$, the photons have frequency $\omega'$, and the interaction region has cross-sectional area $A$ and length $L$. Because of the relativistic Doppler shift, the photon frequency in the rest frame of the atoms is $\omega = \gamma(1 + B) \omega'$, where $B = v/c$ and $\gamma = (1 - \beta^2)^{-1/2}$.

For the photoelectric effect $H^0 + \gamma(\omega) + H^+ + e^-$; $\gamma(\omega)$ is a photon of frequency $\omega$. The cross section of this reaction is $\sigma(\omega)$. A section of the interaction region with $N_{Ho}$ atoms is exposed to $N_Y(\omega)$ photons; of these, $N_Y(\omega)$ [1 - $\exp (-\sigma(\omega) N_{Ho} / A)]$ are annihilated. Generally $(\sigma/\omega) N_{Ho} / A << 1$. Thus, the number of photons is not significantly depleted by the $H^0$ atoms. In this approximation the number of photons necessary for 100% ionization is

$$N_Y(\omega) = \frac{\lambda}{\sigma(\omega)}. \quad (1)$$

The atoms encounter $N_Y(\omega)$ photons for

$$N_Y(\omega) = \frac{\lambda}{\sigma(\omega)} N_Y(\omega) \quad (2)$$

photons in the interaction region. The laser power is

$$P = \frac{\hbar^2 N_Y(\omega)}{I'(1 + B)^2 \sigma(\omega) A}. \quad (3)$$

with $I = L/c$. Thus, for the photoelectric effect,

$$P = \frac{\hbar^2 N_Y(\omega)}{\gamma(1 + B) \sigma(\omega) A}. \quad (4)$$

Multiphoton processes are described by the reaction $H^0 + n_\gamma(\omega) + H^+ + e^-$, where $n$ is the number of photons that are annihilated to ionize one atom. The reaction cross section can be written as

$$\sigma(\omega) = C_n(\omega) t^{n-1}, \quad (5)$$

where $t$ is the laser beam intensity in the rest frame. The hydrogen atoms encounter $N_Y(\omega)$ photons in the time $t = L/c$ spent in the interaction region; therefore,
The reaction threshold is at $\lambda \approx 911.76 \text{Å}$. To ionize all the particles in the beam, we must keep $\lambda \leq 909.15 \text{Å}$ because $\Delta \lambda \approx 2.61 \text{Å}$.

The perturbation-theory results are identical with the correct values of $C_1(\lambda)$ for an intensity of $7 \times 10^8 \text{ W/cm}^2$ and are off by less than 2% for an intensity of $1.7 \times 10^{14} \text{ W/cm}^2$. Laser intensities for $\lambda = 907.0 \text{Å}$ and $L = 1 \text{ m}$ are $I' = 1.40 \times 10^7 \text{ W/cm}^2$ and $I = 1.63 \times 10^8 \text{ W/cm}^2$, respectively. Figure 1 gives the correct laser powers for all reasonable $L$.

Two-Photon Process

To produce a plot of $P_2^0(\bar{\lambda})$ versus $\bar{\lambda}$, we need a plot of $C_2(\lambda)$ versus $\lambda$ or sufficient data to draw such a plot. References 4 and 5 contain enough data for this purpose. We use values of $C_2(\lambda)$ in Ref. 4 because they agree with the values in several other papers. The solid curve in Fig. 2 represents $P_2^0(\bar{\lambda})$ for 100% ionization at all $\lambda$. The minima in the curve correspond to resonant transitions to the intermediate atomic states with principal quantum numbers 2, 3, 4, 5, and 6. The values of $C_2(\lambda)$ in Ref. 5 generally are not in good agreement with those in the other papers; they should be more accurate near the resonances because of the approximations made. Values of $P_2^0(\bar{\lambda})$, computed with data in Ref. 5, are plotted as crosses in Fig. 2. The separation of the crosses from the solid curve gives us an idea of the size of the possible errors near the resonances.

For an intensity of $1.7 \times 10^{10} \text{ W/cm}^2$, actual values of $C_2(\lambda)$ agree with perturbation-theory results for all wavelengths; however, for an intensity of $7 \times 10^{12} \text{ W/cm}^2$, there are large discrepancies. This is especially true for $\lambda < 1250 \text{Å}$. At and near the resonances, actual values of $C_2(\lambda)$ are smaller (by several orders of magnitude) than those obtained from perturbation theory; between resonances they are larger.

The required laser powers for the three-photon process generally are larger than those for the two-photon process. We expect $C_3(\lambda)$ to change with intensity as $C_2(\lambda)$, above. Thus, Fig. 3 does not lead to the correct laser powers for all wavelengths.

Three-Photon Process

References 4 and 5 contain enough data to produce a plot of $P_3^0(\bar{\lambda})$ versus $\bar{\lambda}$. We use the values of $C_3(\lambda)$ in Ref. 4, because they agree with most other results obtained by perturbation theory. The solid curve in Fig. 3 represents $P_3^0(\bar{\lambda})$ calculated for 100% ionization at all $\lambda$. The minima in the curve correspond to resonant transitions to the atomic states with principal quantum numbers 2, 3, and 4. As above, the values of Ref. 5, plotted as crosses, can give us an idea of the size of the possible errors near the resonances.

The required laser powers for the three-photon process generally are larger than those for the two-photon process. We expect $C_3(\lambda)$ to change with intensity as $C_2(\lambda)$, above. Thus, Fig. 3 does not lead to the correct laser powers for all wavelengths.

Selective Stripping

We consider using the narrow resonance peaks in the scattering cross sections of multiphoton processes to strip a selected momentum range of a beam. When directly on resonance such selective stripping can control momentum spread, with the range set by the laser power; off resonance it can be used to monitor drifts in beam energy. Near the resonances, $C_n(\omega)$ (and thus also $\sigma(\omega)$) is described by a Lorentzian of half-width $\Gamma / 2 \leq 10^{-5} \text{ eV}$. The beam's velocity distribution is assumed parabolic. There are $N_1$ particles in the beam; of these, $X N_1 (0 < X < 1)$ are stripped.
THREE-PHOTON PROCESS

We assume that the laser is directly on resonance and is sufficiently intense so that the particles that see photons in the frequency range \( \omega - 6\omega \ldots \omega + 6\omega \) (\( 6\omega < \Delta \omega \)) have an ionization probability of \( p(\omega) = 1 \). Assuming \( \delta \omega \gg \Gamma/2 \), we find the relations

\[
x = 1 - \left( 1 - \frac{\delta \omega}{\omega} \right)^2, \tag{10}
\]
from which \( \delta \omega \) can be calculated, and

\[
p(\omega) = \left( \frac{\delta \omega}{\omega} \right)^2 \quad \text{for} \quad \delta \omega < \omega - \omega < \delta \omega. \tag{11}
\]

For \( X = 0.5 \), we find \( \delta \omega \approx 0.206 \omega \) (so \( \delta \omega \gg \Gamma/2 \) for the PSR beam), and \( C_n(\omega \pm \delta \omega) = 23.5 \). Thus, if only 50% of the particles are ionized, the value of \( C_n \) in Eq. (7) is greater by a factor of 23.5, and the value of \( P_n \) is less by a factor of \( \sqrt{23.5} \) compared to the numbers for 100% ionization. Note that \( p(\omega \pm \delta \omega) \approx 0.0426 \). Most ionized particles are from the central region of the velocity distribution [thus having interacted with photons from the central region of the frequency distribution, where \( p(\omega) = 1 \)]; the ionization probability outside the range \( \omega - 6\omega \ldots \omega + 6\omega \) drops off rapidly, and is only 4.26% at the distribution edges. Also, for \( X = 0.98 \), \( \delta \omega = 0.729 \omega \), \( C_n(\omega \pm \delta \omega) = 1.684 \), \( C_n(\omega - \omega) = 0.531 \), and \( p(\omega \pm \delta \omega) = 0.051 \).

Figure 4a shows the ionization probability \( p(\omega) \) as a function of photon frequency for \( X = 0.5 \) and for \( X = 0.98 \). Figure 4b shows the initial parabolic velocity distribution (solid curve) and the ionized beams' velocity distributions for \( X = 0.5 \) and for \( X = 0.98 \) (dashed curves).

Practical Considerations

There exist lasers with sufficient power densities to completely strip the \( \text{H}^0 \) beam. For example, a Los Alamos XeCl laser (at 3080 Å lab, 907 Å beam just above threshold for single-photon ionization) can provide intensities of \( >10^9 \text{ W/cm}^2 \) over a transverse area with 1-mm radius. This is to be compared to the

1.4 x 10^7 W/cm^2, for a 1-m interaction length, necessary for complete stripping. Pulse repetition rate is ~100 Hz. An arrangement of multiply reflecting mirrors could extend the pulse length to several microseconds before the photon beam is attenuated to 1.4 x 10^7 W/cm^2 by the reflections. A laser array could further extend the ionizable pulse length.

While laser stripping is marginally possible for one of the PSR modes, the technique may prove useful in other applications involving short beam pulses, particularly as the frequency range of high-intensity lasers is increased by future development.

References