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# EM WAVE ELECTRON ACCELERATION

D. J. Sullivan and B. B. Godfrey Mission Research Corporation 1400 San Mateo Boulevard, S. E., Suite A Albuquerque, New Mexico 87108

## Abstract

Several aspects of the laser electron accelerator concept are studied. The minimum laser intensity necessary for electron trapping is strongly dependent on frequency. A pulse length equal to the plasmon wavelength  $(2\pi \omega_p^{-1})$  produces significantly better acceleration than one with half that length. Relativistic plasma effects enhance acceleration and increase the maximum accelerating field. If the laser risetime or intensity is such that electron trapping does not take place, plasma heating by the nonlinear ponderomotive force still occurs producing suprathermal electrons. Application of this mechanism to microwaves has severe drawbacks.

#### Introduction

The concept of laser electron acceleration advanced by Tajima and Dawson<sup>1</sup> has been examined both numerically and analytically. The mechanism depends upon the interaction between an intense electromagnetic wave packet and the electrons of an underdense plasma. The nonlinear ponderomotive force associated with the light wave's propagation in the plasma displaces the electrons. This leads to a charge separation and coincident restoring force producing a train of plasma oscillations. The phase velocity of the wake plasma wave is equal to the group velocity of the EM wave, which is derived from the dispersion relation  $\omega^2 = k^2c^2 + \omega_p^2$  to be

$$\omega_{\rm p}/k_{\rm p} = v_{\rm p} = v_{\rm g} = (1 - \omega_{\rm p}^2/\omega^2)^{\frac{1}{2}}c,$$
 (1)

where  $\omega_{p}$  is the electron plasma frequency,  $k_{p}$  the plasma wave number,  $v_{p}$  the plasma wave phase velocity,  $v_{g}$  the EM wave group velocity,  $\omega$  the EM wave frequency and c the speed of light. Because of the mobility of electrons, and the fact that large changes in energy for relativistic electrons translate into small velocity changes, the electrons are synchronous with the wave front for long periods.

In the wave frame the electrostatic field associated with the plasma can be viewed as a particle mirror with the maximum electron acceleration taking place when the electron experiences a momentum change of  $2\gamma\beta mc$ , where m is the electron rest mass,  $\beta$  is the wave velocity normalized to c and  $\gamma$  is the usual relativistic factor given by  $(1 - \beta^2)^{-1}$ . Transforming back to the laboratory frame yields a maximum electron energy of  $\gamma^{max}mc^2$  where  $\gamma^{max} = 2 \ \omega^2/\omega_p^2$ . The critical plasmon electric field derived from wave-breaking arguments is  $E_z = mc\omega_p/e$ , which implies an accelerating field on the order of a GeV/cm for a  $10^{18} \ cm^{-3}$  plasma density. The optimal pulse length was reported to be half a plasma wavelength or  $\pi \ \omega_p^{-1}$ . This is a severe constraint on the mechanism, because it implies a 0.056 picosecond laser pulse for a  $10^{18} \ cm^{-3}$  plasma. However, a proposed alternate method<sup>1</sup> of using two lasers with a frequency difference of  $\omega_p$  to produce a beat wave has been shown to be feasible<sup>2</sup>.

The purpose of this paper is to report on certain important aspects of the acceleration mechanism which were not addressed in the original paper<sup>1</sup>. These include the minimum laser intensity necessary for electron trapping, relativistic plasma effects, injection of the pulse through the plasma boundary, and effects of temperature, plasma gradient, and intensity on the acceleration. The role of pulse length was also studied. Finally, microwaves were looked at as a possible driver for acceleration, since particle energy depends on  $\omega/\omega_p$  and the analysis is valid for any EM radiation.<sup>3</sup>

#### Minimum Intensity

One can determine a relationship for the minimum wave E-field amplitude,  $E_0^{\min}$ , from trapping arguments and the nonlinear ponderomotive force equations. In the wave frame trapping requires that the potential be large enough to stop the electrons moving in the negative direction. Therefore,  $e\phi^{WaVe} > \gamma mc^2$ . Transforming back to the laboratory frame yields that  $e\phi^{lab} > mc^2$ . Using the relation that  $E_z = k_p \phi^{lab}$  and  $v_p = v_g$ , one obtains for the minimum plasma wave electric field

$$E_{z}^{\min} = \frac{mc\omega_{p}}{e} \frac{c}{v_{g}} > \frac{mc\omega_{p}}{e}$$
(2)

This is an interesting result since it appears that the minimum E required for trapping exceeds the wave breaking limit  $^{z}$  noted earlier  $^{1}$ .

If one neglects ion motion due to the high frequency nature of the phenomena, the restoring force on the electrons is due solely to space charge. Thus, the nonlinear ponderomotive force per  $\rm cm^2$  can be equated to the energy density gradient of the resulting plasma wave, or

$$\frac{\nabla |E_z|^2}{8\pi} = \frac{\omega_p^2}{\omega^2} - \frac{\nabla E_o^2}{16\pi}.$$
 (3)

where E is the EM wave electric field. Solving for E and substituting  $E_z^{min}$  from equation (2) yields

$$E_{o}^{\min} = \sqrt{2} \quad \frac{mc}{e} \quad \frac{\omega^{2}}{(\omega^{2} - \omega_{p}^{2})^{\frac{1}{2}}} > \sqrt{2} \quad \frac{mc\omega}{e}$$
(4)

This is valid provided the laser pulse is shorter than the plasma period,  $2\pi~\omega_{\rm p}^{-1}.$ 

The minimum laser intensity is readily calculated from equation (4) to be

$$I^{\min} = \frac{m^2 c^2}{4\pi e^2} \frac{\omega^4}{\omega^2 - \omega_p^2}$$
(5)

This is an absolute minimum, since the analysis assumed an effectively instantaneous risetime. Nevertheless, the strong dependence on laser frequency must be noted. For example, a 1.06  $\mu m$  Nd-Glass laser requires an intensity 100 times greater than a 10.6  $\mu m$  CO $_2$  laser or 2.4×10<sup>18</sup> W/cm<sup>2</sup> assuming  $\omega/\omega_p >> 1$ .

<sup>\*</sup>This work supported by the Air Force Weapons Laboratory, Kirtland Air Force Base, New Mexico 87117.

An absolute minimum power can also be obtained assuming  $E_0^{\min} = \sqrt{2} \mod e$  and the focal spot size is the laser wavelength. Then

$$P^{\min} = \left(\frac{\pi}{2}\right)^2 \frac{mc^2}{e} \cdot \frac{mc^3}{e}$$
(6)

or approximately 21.5 Gigawatts. Since such a tight focus can only be maintained over a few laser wavelengths, this is not the best configuration for electron acceleration. Rather, a more powerful laser beam over a larger area is necessary, which will assure the minimum intensity over the acceleration saturation length<sup>1</sup>.

### Simulations

Simulations were carried out in conjunction with this analysis using a two-dimensional, fully relativistic and electromagnetic, particle-in-cell code, CCUBE. The code can solve self-consistently for the time dependent trajectories of tens of thousands of plasma particles over thousands of plasma periods. All variables are expressed in dimensionless terms. Therefore, length is in units of  $c/\omega_p$ ; time is measured in units of  $\omega_p^{-1}$ , and particle velocity is given by  $v_i = \beta_i \gamma$  (i = 1,2,3), where  $\omega_p$  is the initial electron plasma frequency.

In the laser simulations a plane polarized electromagnetic wave is launched into a Cartesian geometry. Periodic boundary conditions in the transverse (y) direction make configuration space effectively onedimensional. In general, the simulation had 1250 cells in the longitudinal (z) direction modelling a length of 100 c/ $\omega_{\rm p}.$  The cells in the y direction appeared uniform because of periodicity. Each of these macrocells initially contained 24 particles. The ions were taken to be an infinitely massive neutralizing background. A vacuum region  $10\pi$  c/ $\omega_p$  long was left between the left hand boundary and the plasma in order to accurately determine the dynamics of laser injection into the plasma. Different runs were made in which the values of  $\omega/\omega_p$ ,  $E_o$ , plasma gradient length,  $\nabla_n z$ , electron temperature,  $T_e$ , and pulse length,  $\tau_p$ , were varied. The canonical simulation has values:  $\omega = 3\omega_p$ ,  $E_o = E_o^{min}$ ,  $\nabla_n z = 0.01 \text{ c}/\omega_p$ ,  $T_e = 0$  and  $\tau_p = \pi\omega_p^{-1}$ .  $\nabla_n z$  = 0.01 c/ $\omega_p$ , T<sub>e</sub> = 0 and  $\tau_p$  =  $\pi \omega_p$ 

When the laser pulse encounters the plasma, the nonlinear ponderomotive force resulting from the intensity gradient causes the electrons to snow plow. This continues until the force arising from charge separation is greater than the ponderomotive force and the electrons attempt to restore the charge imbalance by moving in the negative z direction. This motion initiates a train of large amplitude plasma waves. The electrons are trapped by the waves and accelerated. In several code runs the electron density momentarily exceeded the critical density for the laser pulse and part of the wave was reflected. The plasma wave train and resultant electron bunching are shown in Figure 1. The wave steepening evident in the electric field profile indicates the nonlinear nature of the mechanism even at minimum intensity.

If the laser intensity exceeds the minimum required by a factor of two, the plasma becomes so turbulent that a discernible plasma wave train cannot be established. Coherent acceleration takes place primarily at the pulse front. This is depicted in Figure 2a). More importantly, both the particle acceleration and plasma wave accelerating field exceed their anticipated maximums<sup>1</sup>. This is the result of relativistic plasma effects which must be accounted for as seen in Figure 2b).



Figure 1. CCUBE diagnostic of the plasma wave electric field and normalized electron charge density versus axial distance z.



Figure 2. Particle plots of the longitudinal and transverse electron acceleration versus z.

The laser pulse is coupled to plasma motion in the transverse direction. In contrast to acceleration in the longitudinal direction where only a fraction of the electrons at any given position are trapped and accelerated, all electrons experience the same transverse acceleration for a set value of z and time. It is, therefore, appropriate to quantify the motion by a plasma relativistic factor,  $\gamma_p$ . An estimate for  $\gamma_p$  can be obtained from the relativistic power equation

$$\frac{d}{dt} (\gamma - 1) mc^2 = e \vec{E} \cdot \vec{v}$$
(7)

If we assume that the electron velocity approaches the speed of light, and E has the form E =  $E_{\rm O}$  cos  $\omega t$  then

$$\gamma_p = \sqrt{2} \quad \alpha \sin \omega t \quad \frac{c}{v_g}$$
 (8)

where  $\alpha = E_{\alpha}/E_{0}^{min}$ . The effect of the relativistic plasma enters into the wave breaking limit for the plasma electric field and the maximum electron energy, because

$$\left(\omega_{p}^{rel}\right)^{2} = \frac{4\pi ne^{2}}{\gamma_{p}^{m}} = \frac{\omega_{p}^{2}}{\gamma_{p}}$$
(9)

where  $\omega_p^{\mbox{rel}}$  is the relativistic plasma frequency. The critical electric field is then modified to be

$$E_{z}^{cr} \approx \gamma_{p}^{3/2} \frac{mc\omega_{p}}{e}$$
(10)

which explains the simulation results where the magnitude of  $E_z$  has attained values several times  $mc\omega_p/e$ . Similarly the maximum energy becomes

$$\gamma^{\max} = 2 \gamma_p^{\max} (\omega/\omega_p)^2$$
(11)

where  $\gamma_p^{\text{max}} = \sqrt{2} \alpha c/v_g$ .

A plot of  $\gamma^{max}$  and energy conversion efficiency,  $\eta$ , versus the laser wave electric field is given in Figure 3. The horizontal line indicates  $\gamma^{max} = 2(\omega/\omega_p)^2$  or 18. The sloped line is given by equation (11). The symbols refer to simulations with various physical parameters. It is clear that laser intensity plays a role in determining the maximum electron energy. The graph also shows the results of varying T<sub>e</sub>,  $\nabla_n z$ , and T<sub>p</sub> from the standard values. Increasing the electron temperature to 10 keV and the plasma gradient length by three orders of magnitude had minimal effect on  $\gamma^{max}$ . In contrast, using a laser pulse length equal to the plasma wavelength, rather than one half that value, significantly increases both the maximum electron energy and the efficiency. Code runs with  $\omega = 5 \omega_p$  produced similar results verifying the scaling of equation (11).

The simulations discussed thus far were for short laser pulses  $(\tau_p \leq 2\pi \omega_p^{-1})$  and instantaneous risetimes. Two long pulse simulations were made. In the first, a laser pulse of length greater than 100 c/ $\omega_p$  (the length of the simulation) and an instantaneous risetime was injected into the plasma. The maximum acceleration at the wavefront was the same as the equivalent simulation with  $\tau_p = 2\pi \omega_p^{-1}$ . In long pulses, it is pulse shape and not length which is important for the acceleration mechanism. The plasma behind the front was heated to a temperature of several MeV. In the second run a Gaussian shaped laser pulse of  $72\pi \omega_p^{-1}$  duration was injected with a peak wave field of E<sup>min</sup>. No electron trapping and subsequent coherent acceleration were observed, because the gradient of the laser wave (equation (3)) was insufficient to produce the minimum plasma wave fields necessary for electron trapping.



Figure 3. Graphs of  $\gamma^{max}$  and energy conversion efficiency,  $\eta$ , as a function of incident EM wave electric field amplitude,  $E_0$ . The horizontal line is  $\gamma^{max} = 2(\omega/\omega_p)^2 = 18$ . The sloped line is given by equation (11). The dots indicate simulation results with  $\nabla_n z = 0.01 \ c/\omega_p$ ,  $T_e = 0$ , and  $\tau_p = \pi \ \omega_p^{-1}$ . The other symbols are as noted.

plasma interaction again produced a thermal electron distribution in the direction of propagation with electron energies up to 4 MeV. The process of producing hot electrons during the laser pulse risetime has implications for laser pellet fusion which will be addressed in a separate paper. This effect may already have been observed experimentally<sup>4</sup>.

#### Power Enhancement

The electron bunching evident in Figure 1 is even more pronounced when wave intensity exceeds the minimum required. In this case the electron bunch at the pulse front can be seven to eight times as dense as the initial plasma. The electron packet's spatial extent is a small fraction of a plasma wavelength. The particle energy spectrum is basically exponential with a lowdensity, high-energy tail out to the maximum value attained. This single pulse contains from 25 to 50 percent of the total particle energy with a power as high as 50 times the injected wave power.

#### References

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