Introduction

The need of very low output impedance rf systems for heavy particle accelerators has led to the development of very sophisticated amplifier configurations. Among them, the use of a common-anode final amplifier seems most promising. The functional relation \( I_p = I_p(V_{gk}, V_{pk}) \), that is normally given in graphical form, renders the problem non-linear and analytically unfeasible, especially since we are interested in the large signal operation. A numerical treatment of the problem was thus needed.

If the output impedance is defined in the Thévenin Theorem sense, then class-A operation seems inavoidable. However, for particle dynamics only the voltage induced by the beam is of real concern and voltage requirements do not necessarily demand a time invariant output impedance.

These considerations led us to investigate the conditions that must be met to obtain control over the beam-induced voltage. It was possible to demonstrate that in the common anode case the induced voltage can be largely independent of the bias applied to the final tube, if the beam loading is very strong.

Operation Description

When the accelerating cavity is driven by a common anode amplifier, the beam current has to flow mainly through the tube, thus tending to turn the tube on when beam current, rf drive, and tube connection to the cavity are properly matched.

In this case, the high value for the tube transconductance (low output impedance) can be reached with the beam current, thus greatly reducing or even suppressing the quiescent current. (The advantages in energy saving and in the use of lower plate dissipative tubes are evident and need not to be discussed).

After the beam pulse has gone by, the amplifier's only function is to reset the initial voltage across the accelerating gap, and in the case of a common anode amplifier this depends mostly on the rf drive and very little on the quiescent point, when reasonably chosen.

In the ideal case, the electric network to be analyzed appears as shown in Fig. 1, where only the cavity fundamental mode is taken into account. \( I_p \) and \( I_b \) are tube and beam current, respectively.

![Fig. 1. Equivalent scheme for the beam-tube interaction.](image)

Numerical Solution

Some values for the parameters of the ISABELLE rf system preliminary design are in Table 1. The assumed grid drive is a sine wave, while the bunch profile is approximated with a raised cosine pulse.

\[
I_b = \frac{q_0}{\Delta t} \left(1 - \cos \frac{2\pi t}{\Delta t}\right)
\]

where \( q_0 \) is the bunch charge and \( \Delta t \) its duration.

Table 1. Tube and Circuit Parameters.

<table>
<thead>
<tr>
<th>Tube ML-7560</th>
<th>Cavity Inductance ( L )</th>
<th>Cavity Shunt resistance ( R )</th>
<th>Cavity rf period ( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_g = 16 \text{ kV} )</td>
<td>( 2 \times 10^{-8} \text{ H} )</td>
<td>( 6.1 \times 10^3 \text{ ohm} )</td>
<td>( \pi/4 \text{ sec} )</td>
</tr>
<tr>
<td>Driving voltage, peak ( V_{do} )</td>
<td>( 12 \text{ kV} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The state equations for the circuit of Fig. 1 have been integrated by means of a numerical predictor - corrector routine. At each step of integration, the plate current \( I_p \) has been calculated by interpolation in the constant current chart following two distinct procedures. In the region far from cut-off, we used a linear interpolation among the four values of \( I_p \) that encompass the working point \((V_{gk}, V_{pk})\). In the region close to the cut-off line a fourth order interpolation polynomial was used.

![Fig. 2. \( I_p \), \( I_b \), \( V_d \) vs phase angle \( \omega t \). High bias: \( E_b = -360 \text{ V} \). Steady state.](image)
The values for L and C are chosen such that the cavity is tuned under no beam loading conditions. When the beam has a strong out-phase component, the cavity should be re-tuned accordingly. To work under the worst conditions that are reached when the cavity tune remains unchanged, the cavity parameters were not readjusted.

Figures 2 and 3 give a detailed picture for normal class-A and hard class-C operation. Some numerical results are given in Table 2. A more extensive discussion can be found in Ref. 4.

Table 2. Numerical Results - steady state beam crossing phase $\phi_0 = 180^\circ$.

<table>
<thead>
<tr>
<th>Bias grid voltage $E$ (V)</th>
<th>-220</th>
<th>-360</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average plate current $I$ (A)</td>
<td>12.117</td>
<td>9.3707</td>
</tr>
<tr>
<td>Input power $P_{in}$ (kW)</td>
<td>193.87</td>
<td>149.93</td>
</tr>
<tr>
<td>Output power $P_{out}$ (kW)</td>
<td>11.716</td>
<td>12.833</td>
</tr>
</tbody>
</table>

The power $P_{out}$ is calculated taking into account only the first Fourier harmonic for both voltage and current, while the power $P_{out}$ is the power that a sine wave voltage would develop across the cavity shunt impedance.

It appears that $P_{out}$ and $P_{out}$ are different, as they should. The true output could be calculated by adding, with their signs, all the power partial components.

Since a small percent distortion in the voltage results in a twice larger error in the power calculation, we can conclude from numerical results, that the total voltage distortion appears smaller than 5%.

**Conclusions**

From the numerical analysis performed with two widely different values for tube dc grid bias, the following conclusions can be drawn:

a) The percent difference between tube current peak amplitudes is less than 5% while the percent bias difference was set up to more than 45%. This demonstrates that under heavy beam loading the peak tube current is practically independent of the quiescent point chosen.

b) The cathode voltage is nearly unaffected by grid bias, the percent different for the two cases being less than 1%.

c) Cathode voltage waveforms remain very nearly sinusoidal in both cases. This is due to the high value of the cavity quality factor ($Q = 177$). However, it can be shown that with an unloaded $Q$ factor as low as 20, a reasonably good waveform can also be obtained.

d) The difference between the dissipated power is, as expected, very large, depending upon the quiescent point chosen. The rf power amplifier general theory holds also in this case, and the predictions based on the total circulation angle are fairly accurate.

From the above considerations, valid for the steady state, we can state that the outlined design operation can be safely adopted. However, transient operations do not show any unusual or unpredictable behavior and will not be discussed here. Instead, if we consider that with the outlined procedure the behavior of the tube is controlled both by the driver and by the beam, it is immediately obvious that some slow feedback between the tube dc-current and dc-bias would greatly improve the overall amplifier operation. This feedback would automatically provide the correct bias required when the amplifiers operate with a partially bunched or totally debunched beam.

**Acknowledgments**

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**References**

2. A. Luccio and M. Puglisi, Behavior of Single Ended vs Push-Pull Amplifiers for the Accelerating System of High Current Beam Storage Rings, Report BNL 27900 to be published by "Particle Accelerators."