HIGH ENERGY ELECTRON COOLING TO IMPROVE THE LUMINOSITY AND LIFETIME IN COLLIDING BEAM MACHINES*

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Abstract

Electron cooling can be applied to improve the performance of high energy colliding beams of hadrons and e−p storage rings. Normal beam excitations such as multiple scattering, resonance growth, or beam-beam interaction can be controlled, leading to longer beam lifetimes, and in some situations to higher luminosities and larger tolerable tune shifts. The electrons, in a small storage ring, are cooled by radiation and heated by the hadron beam. An equilibrium is reached in which the hadron beam is cooled. The electron beam requires strong cooling by "wigglers"3. We have designed a simple cooling experiment for the Fermilab synchrotron.

Relativistic proton or antiproton beams in circular accelerators or storage rings can be damped by interactions with electrons (y_p=y_e) circulating in a storage ring.1,2 The proton or antiproton beam is damped and the electron beam is heated in the interaction. In order to reduce the blowup of the electron beam, synchrotron radiation in bending magnets or wiggler magnets and subsequent acceleration in an RF system is required, thus the electron beam is cooled. This is schematically shown in Fig. 1.

Fig. 1. High energy electron cooling plan.

There are three useful features of relativistic electron cooling for high energy colliding beam devices. (1) The damping increases the luminosity lifetime of the colliding beams and should make the beams more stable to higher order resonances, etc. (2) The damping decreases the beam size and under certain circumstances should lead to an increased luminosity for a given number of particles in each beam. (3) The damping may increase the tolerable beam-beam tune shift limit (Δv).

For proton-antiproton storage rings where N_p, the number of antiprotons, is limited the addition of electron cooling can result in a dramatic improvement in the machine performance. To illustrate this point we consider the collision of equal number of protons (N_p) and antiprotons (N_p) in a storage ring with 1 TeV (the energy doubler at Fermilab). The available luminosity is given by

\[ \mathcal{L} = 5.6 \times 10^{32} \frac{(\Delta \nu)^2}{\beta^4} \text{cm}^{-2} \text{s}^{-1} \]

where \(\beta\) is the beta at the interaction point. Relativistic electron cooling results in a damping of the size of the beam so that for a given number of anti-protons the maximum tolerable tune shift limit is reached. The luminosity therefore depends only on N_p, \(\Delta \nu\) and \(\beta^4\). Both N_p and \(\Delta \nu\) may be further increased thus resulting in an increase in \(\mathcal{L}\). N_p is increased because the \(\beta\) refilling time of a p-p storage ring is increased if the luminosity lifetime is increased. We believe that \(\Delta \nu\) may also be increased in analogy to the larger \(\Delta \nu\) that can be tolerated for an e^+e^- machine (0.06) which has a damping mechanism (synchrotron radiation damping) compared to the maximum \(\Delta \nu\) of 0.003 usually assumed for proton-proton storage rings. Figure 2 shows the luminosity that can be achieved as a function of these parameters for p-p machines.3

A similar gain in luminosity might be realized for e−p storage rings by damping the proton beam emittance and increasing the tolerable tune shift limits. The proton beam should be damped until its dimensions are comparable to the electron beam in the storage ring. The idea of using relativistic electron cooling was originally considered and discarded by Budker and his colleagues because of the constraints it implied an electron storage ring.3 Recently this subject was reconsidered at meetings held at Lawrence Berkeley Laboratory and the University of Wisconsin.1,2 For the early work see the proceedings of these meetings. We present here two additional developments. (1) Refinements in the thermodynamical calculations of the relaxation of the hadron beam and the electron beam. (2) The conceptual design of a simple storage ring that can be used to carry out an experiment on relativistic electron cooling.

In particular such an experiment would be used to extend our knowledge of the performance of high current electron storage rings in the few hundred MeV energy range.

We briefly outline the thermodynamic equilibrium of this system. In the absence of interactions between the two beams, we can write the following equations for the rms beam emittance (\(\epsilon = \sigma / \beta\))

\[ \frac{d\epsilon_p}{dt} = \frac{\beta}{\epsilon_p} \] (1)

\[ \frac{d\epsilon_e}{dt} = -\gamma \epsilon_e + \frac{\beta}{\epsilon_e} . \] (2)

We assume both beams are round, namely, that they have the same horizontal and vertical emittance.

In the absence of diffusion-like processes and of damping effects, the emittances are normally considered.
invariants. The diffusion coefficient $D_p$ on the right-hand side of (1) is primarily given by gas scattering and similar effects. This diffusion is not compensated by damping and will cause a linear increase of the beam emittance with time. In Eq. (2), $\tau$ is the synchrotron radiation-damping time and $D_p$ the quantum-fluctuation diffusion coefficient. The electron beam would have an equilibrium emittance which is given by

$$\epsilon_e = \frac{1}{2} \tau \epsilon_e .$$  

(3)

This equilibrium value is reached in the e-folding time $\tau/2$.

Observe that $\tau$ and $D_p$ depend strongly not only on the beam energy but also on the electron-beam storage ring lattice.

We now modify Eqs. (1) and (2) to include the beam-beam interaction, which is supposed to lead to "cooling" of the proton beam at the cost of some "heating" of the electron beam. Because of the large energy and since the electron beam is already focused by the lattice quadrupoles and RF cavities, we ignore space charge effects on the trajectory of the electrons. It is easily verified that at larger energies

$$\delta \ll \frac{1}{\sqrt{\beta}} ,$$

where $\delta_{11}$ and $\delta_{12}$ are respectively the longitudinal and transverse relative momentum spreads. This is true for both beams. The coupled equations for the beam emittances are

$$\frac{d\epsilon_p}{d\tau} = \tau_0 \epsilon_p - 2K \epsilon_p$$

(4)

$$\frac{d\epsilon_e}{d\tau} = 2K \epsilon_e$$

(5)

where $\tau_0$ and $\tau_1$ are the lattice beta-values at the interaction region length and $m_p$ and $m_e$ are the masses at rest. The relaxation times are given by

$$\tau_0 = \frac{6\epsilon_p 4 \beta \gamma}{\epsilon_p m_p c^2} \frac{1}{1 - \frac{\epsilon_p}{\epsilon_p^*}}$$

(6)

and

$$\tau_1 = \frac{6\epsilon_e 4 \beta \gamma}{\epsilon_e m_e c^2} \frac{1}{1 - \frac{\epsilon_e}{\epsilon_e^*}}$$

(7)

Here $\beta$ and $\gamma$ are the usual relativistic factors which are common to both beams, $\epsilon_p$ is the ratio $i/c_p$ of the interaction region length to the proton ring circumference, $\epsilon_p$ is defined in a similar way and accordingly, $\tau_0$ and $\tau_1$ are the beam currents within a bunch, $L$ is the Coulomb logarithm ($-15$) which we assume to be the same for both beams, finally $\delta_{11}$ and $\delta_{12}$ are the two beams transverse radii.

Ignoring intrabeam scattering we can solve these equations to obtain the equilibrium conditions. By the following we shall assume that the motion of the bunches in the two rings is synchronized in such a way that $\eta_0 = \eta_1$; this occurs when one bunch interacts always with the same one in the other beam. Also the bunches in the two beams are assumed to have the same length.

From the definition of emittance we have

$$a^2 = c\beta^*$$

and $\delta^2 = c/\beta^*$

(8)

which we can use in the right-hand side of (6) and (7). Then (4) and (5) can be replaced by

$$\frac{d\epsilon_p}{d\tau} = 2K \epsilon_p$$

(9)

$$\frac{d\epsilon_e}{d\tau} = -2K \epsilon_e$$

(10)

with

$$K_0 = \frac{6\epsilon_p 4 \beta \gamma}{\epsilon_p m_p c^2} \frac{1}{1 - \frac{\epsilon_p}{\epsilon_p^*}}$$

(11)

$$K_e = \frac{6\epsilon_e 4 \beta \gamma}{\epsilon_e m_e c^2} \frac{1}{1 - \frac{\epsilon_e}{\epsilon_e^*}}$$

(12)

In absence of intrabeam scattering an equilibrium emittance of the proton beam would be reached

$$\epsilon_p = \frac{6\epsilon_p 4 \beta \gamma}{\epsilon_p m_p c^2} \frac{1}{1 - \frac{\epsilon_p}{\epsilon_p^*}}$$

(13)

where $\epsilon_p$ is given by Eq. (3) and

$$\tau_0 = \frac{6\epsilon_p 4 \beta \gamma}{\epsilon_p m_p c^2} \frac{1}{1 - \frac{\epsilon_p}{\epsilon_p^*}}$$

(14)

Crude this formula indicates that the proton damping time will be longer than the synchrotron damping time since $m_p > m_0$ and in a realistic situation $\eta_0 < \eta_0$. However it is conceivable that $\tau_0 >> \tau_1$ for low energy electron storage rings and thus $\tau_0$ may be reduced. In practice $\tau_0$ will depend on the lattice of the storage rings, the effects of intrabeam scattering the amount of synchrotron radiation in the electron storage ring and the maximum electron current. Typically for $I_p \approx 5$ amperes, $F_p \approx 125 MeV$ and a proton storage ring such as the SPS and Fermilab (but with a very good vacuum in each case) the damping times will be of order 100-1000 seconds. This time is short enough to be useful in realistic situations since $\tau_0 < \tau_1$ lifetime due to beam blowup.

Intrabeam scattering in the electron storage ring causes an increase in this cooling time. The effect of the intrabeam scattering depends very much on the dispersion of the lattice. In the following we shall consider only the average contribution, namely we shall spread smoothly the dispersion around the machine. Also we shall assume the same amount of dispersion in both planes. We consider the case of energies of both beams is well above the transition energies of the respective machines.

If the dispersion is considerably high and because $\eta_0 < \eta_0$, the diffusion due to intrabeam scattering in the transverse plane can be decoupled from the longitudinal emittance of the beam. Then all the other effects are ignored one would have

$$\frac{d\epsilon_p}{d\tau} = \frac{Q}{e}$$

(15)

and

$$Q = \frac{6\epsilon_p 18\gamma^2}{m_e c^2} \frac{1}{1 - \frac{\epsilon_p}{\epsilon_p^*}}$$

(16)
where $\bar{g}$ is the average value of the beta-function around the lattice and $\gamma_T$ is the ratio of the transition energy to the rest energy.

The coupled equations for the emittances now become

\[
\frac{dc}{dt} = D_e - 2K_e \frac{e}{p} \left( \frac{e}{p} + \frac{e}{p} \right) + \frac{e^{3/2}}{p} \frac{e^{3/2}}{p} + \frac{Q_e}{p}
\]

and

\[
\frac{dc}{dt} = D_e - 2K_e \frac{e}{p} \left( \frac{e}{p} + \frac{e}{p} \right) + \frac{e^{3/2}}{p} \frac{e^{3/2}}{p} + \frac{Q_e}{p}
\]

The equilibrium solution is obtained by setting $dc/dt = dc_e/dt = 0$.

Intrabeam scattering effects can be reduced by one of two techniques: 1) reduce the dispersion in the machine to very low values and 2) arrange the electron beam emittances such that

\[\sigma_x^e = \sigma_y^e = \sigma_y^e\]

i.e., an isotropic distribution of velocities in the rest frame of the beam particles. Our calculation including intrabeam scattering indicates that the cooling time $t_0$ probably does not increase appreciably in a practical situation.

We now turn to a preliminary design of a small storage ring that could be used to study relativistic electron cooling at Fermilab or CERN.

The ring, Fig. 3, could be situated in the present tunnel at Fermilab at one of the 15 m straight sections. The proton and electron beams would coincide in one of the straight sides of the electron ring, where the electrons would cool the protons. The electrons would then radiate their transverse energy away in the four 90° bending magnets and in wiggler magnets in the straight side opposite the cooling region.

Fig. 3. Lattice of electron storage ring or cooling protons.

In designing the lattice we tried to satisfy a number of requirements, not all of which are easily compatible. We list them briefly, not necessarily in the order of importance.

1. The proton and electron beams should have roughly equal transverse dimensions and angles so that the cooling proceed efficiently. That implies comparable emittances, about $10^{-8}$ mrad, and beta-values, in the range 50-100 m.

2. So that longitudinal collisions between electrons and protons do not excite transverse electron oscillations, the dispersion function should be zero or small in the cooling region.

3. Electron damping times should be as short as possible. This leads to strong fields in the dipoles and to the addition of wiggler magnets.

4. The ring should be stable over the expected electron energy range.

The emittance requirement also leads to zero or low dispersion in the straight sections, because large dispersion in the bending magnets or wiggler magnets would give large emittance due to quantum excitation.

We first worked on a lattice without wiggler magnets. This ring has reflection symmetry about both axes, and two superperiods, while the ring with wiggler magnets has only left-right reflection symmetry and one superperiod (see Fig. 1). Zero dispersion in the straight sections uniquely determines the strength of the QF1 quadrupoles. Next, one may specify the beta-function values at point A and adjust QF and QD to produce a match at the center of QF1. One does not always find solutions, nor do those found always seem feasible with regard to the betatron tunes, emittance, or chromaticity.

The lattice elements of the ring design selected are shown in Fig. 3 and Table I, and the operating and orbit parameters in Tables II and III, respectively. Apart from the long damping times, the ring might cool if vertical emittance were introduced by use of solenoids or skew quadrupoles.

Table I. Lattice Elements

<table>
<thead>
<tr>
<th>Type Name</th>
<th>Length (m)</th>
<th>Wiggler Off</th>
<th>Wiggler On</th>
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<tr>
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Table II. Operating Parameters

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<td>Degrees</td>
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<tr>
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<td>m</td>
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<td>Number Periods in WH</td>
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<tr>
<td>Number Periods in WW</td>
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<tr>
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Table III. Emittance Requirement

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<th>Type Name</th>
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Betatron Tunes
Beta-Functions in Cooling Region
Dispersion
Beta-Functions at Center of Vertical Wigglers
Transition Energy
Damping Times
Emittances ε < κ²/β
Energy Spread

<table>
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<th>Wiggler Off</th>
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</tr>
<tr>
<td>ε_f</td>
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</tr>
</tbody>
</table>

Table III. Orbit Parameters

To reduce damping times we investigated use of wiggler magnets in the ring. We found that if the orbit bends through large angles, that it is nearly impossible to make the complete ring stable. Therefore, we propose short period wigglers, as indicated in Fig. 3. Different combinations of horizontal and vertical wigglers were tried, introduction of further quadrupoles among them and use of gradients. We concluded that no further quadrupoles are needed or gradients in the wigglers. The combination shown in Figs. 3, 4, and Tables I, II, and III appeared to maximize stability among the horizontal and the vertical plane. The coupling can be made by skewed quadrupoles or a solenoid.

To counteract the blowup of the proton or antiproton beam by root gas scattering and beam-beam interaction an average current of about 1 Amp has to be stored in the electron cooling ring. As the beam becomes more stable with increasing energy one directly injects at 125 MeV. This can be done with one of the three injectors: a linac, race track microtron, or a combination of linac and race track microtron.

In conclusion the addition of a damping mechanism to high energy proton or antiproton beam is useful to increase stability and luminosity in these machines. Damping times of 100-1000 seconds seem feasible with our present knowledge of the required parameters of the electron storage ring. We have designed a small storage ring that can be used to carry out an experiment of an electron cooling in order to demonstrate the principle and explore the further possibilities of this technique.

References

6. A. Garren, Synch Program Description.