DESIGN OF A MAGNETIC LINEAR ACCELERATOR (MAGLAC) AS DRIVER FOR IMPACT FUSION (IF)∗

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IEEE Transactions on Nuclear Science, Vol. NS-26, No. 3, June 1979

ABSTRACT

Inertial confinement fusion programs currently underway seek to ignite microexplosions by applying various drivers to targets containing deuterium and tritium (D-T). It is generally accepted that to ignite a microexplosion requires over 1 megajoule (MJ) of energy. Accelerator and projectile elements are described. Within 10 ns (10¹⁴ W) into a target of volume 10⁻⁶ m³, projectiles to a velocity exceeding 10⁵ m/s are presented, without need to convert kinetic energy to momentum. The choice of material for the projectile and its construction may limit u or B or both. (b) Dissipative losses. In general, eddy currents will be set up in the projectile. These can limit the velocity that can be achieved, or overheat the projectile. (c) Transient effects. During acceleration, the applied fields may have nonzero frequency components in the rest frame of the projectile. (d) In order to get enough mass in a short projectile, magnetically inert material may have to be carried. (e) For obtainable vacua in the accelerator, the projectile may have to be protected by a heat shield. (f) In a thermonuclear explosion, we have to insure that the burn is not damped by contamination of the plasma by heavy ions from the projectile.

In the remainder of this section, we consider items (a) - (c). We consider three basic types of projectile. First is a ferromagnet with magnetic moment saturated. The other two are a superconducting solid and a superconducting solenoid wound around a magnetically inert core.

First we consider acceleration limits for the ferromagnet. The saturation B field from magnetization currents is Bsat = 2 T for a ferromagnet (iron). This gives a moment

\[ I = m^2 A B_{sat} / c_0, \]

where I is the projectile length and A its crosssectional area. If we assume superconducting current elements in the accelerator, then we can assume that the limits of B, the applied field, are -B < B < Bsat, where Bsat is the critical field for the superconductor. Then at most, \( B < 2 B_{sat} / I \). This gives the maximum acceleration for a projectile of mass m

\[ a_{max} = 2A B_{sat} B / \mu m \]

where \( A = 2 \pi \rho / c_0 \) is the density of the projectile. This implies an accelerator length of \( 2 \times 10^5 \) m using \( v_f \) for iron, Bsat = 2 T while \( B_m = 10^4 \). For a take 10⁻⁵ N. This yields \( a_{max} = 3 \times 10^6 \) m/s² if m = 10⁻⁵ kg.

Other problems also exist, but appear to be less fundamental in nature. These are:

(a) Intrinsic limits to acceleration. Centripetal force is \( F = B L \), where \( B \) is the (intrinsic or induced) dipole moment of the projectile. The choice of material for the projectile may not be able to respond to the field gradient and acceleration will not occur for all frequency components of the external field. Second, high frequency fields may destroy the magnetic moment. For example, electrons can be excited across the energy gap in a superconductor, or a ferromagnet can be depolarized.

Concept and design of a magnetic linear accelerator to accelerate a 0.1-1.0 gm superconducting solenoid projectile to a velocity exceeding 10⁵ m/s are presented. Transverse and rotational motions are stable. Accelerator and projectile elements are described. We find no obstacle to development of this concept.
For a bulk Type II superconductor, a fraction $\varepsilon$ of the external flux $B_c$ is excluded giving an effective moment $M = \varepsilon B_c$ per unit area $A$. From equation (2), we see that we can apparently achieve an acceleration which is similar to that in a ferromagnet. Using $\varepsilon = 10^{-4}$, we estimate that a magnetization current density of order $J = 10^9 A/m^2$. This means that a bulk superconductor can not sustain the acceleration required, assuming a size comparable to the ferromagnetic projectile above.

The last possibility is the use of a superconducting solenoid with a permanent dipole moment from a permanently circulating current. The discussion of this alternative is similar to that of the bulk superconductor discussed above. We only need to point out that:

(1) Thin filaments of Nb$_3$Sn seem able to sustain a current density over $j = 10^9 A/m^2$. This current density in a solenoid can give the required moment.

(2) Since flux need not be excluded inside the superconductor, we need not worry about the large normal volume in a type II superconductor near $B = B_c$.

We now turn to the question of dissipative losses. For a ferromagnetic projectile this may be a serious problem. If the accelerator maintained a constant magnetic field at the projectile, dissipative losses would not occur in the projectile. Unfortunately this is not practical. It is straightforward to show that:

$$v_{\text{limit}} = \frac{8p}{\mu_0} \frac{m v_{\text{sat}}^2}{J_c} = v_{\text{sat}}$$

For the proposed accelerator, $V_{\text{accl}}$ must be about $10^3 N$. It is then found that

$$v > 10^{-3} \text{ m/s} (dB/dt = \xi \nu dB/dz)$$

For an iron projectile $\xi = 10^{-7}$, so that $\xi < 10^{-2}$. This means the field must vary $< 11$, which may be difficult.

We can also estimate ohmic heating. If we let the limiting velocity $v_L$ exceed the desired final velocity of $v_L = 10^{-5} m/s$, then the ohmic heating rate is about $P_{\text{ohm}} = F_{\text{accl}} v_L$. This means that even if $v_L < 10^{-4} v_L$ (a much stronger condition than 8), the projectile temperature will rise by about $10^6 \text{ K}$. Therefore the ohmic heating condition is more severe than the limiting velocity condition and requires

$$a > \xi^2$$

To prevent evaporation of the projectile, it may be possible to get effective $\xi$'s of much larger than $10^{-2}$ for iron by either elimination of the projectile or by using ferrite. However there is a penalty in terms of reduced acceleration and simplicity of construction.

Ohmic effects are greatly reduced for a superconducting solenoid projectile; the superconductor traps flux reducing $dB/dt$. However, this flux trapping has a finite relaxation time, so if the magnetic field at the projectile has high frequency components, the flux may penetrate and cause losses in the normal fraction of the superconductor and in the inert filler.

(c) Transient Effects: On a more fundamental level, changing magnetic field in the projectile results in high frequency fields which are unavailable for acceleration. We can make some estimates of these frequencies. For ferromagnetic projectiles, the transient problem occurs because the magnetization currents are not built up instantaneously so that a constant $v$ can not be maintained in rapidly changing external field. We estimate the critical frequency for this. The magnetization comes from a superposition of spin waves. If $J$ is the spin interaction energy, $S$ the average spin, and the lattice spacing, it is found that for a spin wave $\omega = 2 \delta^2 J / (k a^2)$. The phase velocity of the spin wave is then

$$v_p = \frac{\omega}{k} = 2 \delta \frac{J^2}{S} / a^2$$

For a best case estimate, let $k = k_{\text{max}} = a^{-1}$. $J$ is about 20 ke Joules, where $k_B$ is Boltzmann's constant. $S$ is taken as unity and $a = 10^{-4} \text{ m}$. We find $v = 10^9 \text{ m/s}$. The penetration depth for a spin wave is a magnetic domain size $d = 10^{-6} \text{ m}$. The relaxation time is then

$$\tau = \frac{d}{v_p} = 10^{-9} \text{ s}$$

The highest frequencies the projectile sees will be of this order if $dB/dt$ is large.

For a type II superconductor the relevant frequencies are those at which the skin depth for the magnetization current is smaller than superconducting skin depth. We can estimate this to occur at $\omega > 10^6 / c^2 \text{ Hz}$. Such high frequencies are no problem. We also note that the flux lines in a type II superconductor have a natural resonant frequency in the range of $10^6 / c^2 \text{ Hz}$. At higher frequencies we expect the flux lines to be "frozen" and not be excited. So there might be critical ac frequencies which have to be avoided.

In summary, it appears that the solenoidal projectile offers the best possibility to attain the required velocity. However, the difficulties of the bulk field magnet with transient and ohmic losses may be surmountable so this option should continue to be explored.

II THE MAGNETIC LINEAR ACCELERATOR

To approach the problems of actual accelerator design it's useful to review magnetic levitation. Suppose we want to keep a dipole $u$, on the axis of a circular current loop. Let the loop have radius $a$ and carry current $I$. Let $z$ be the vertical coordinate with $z = 0$ in the plane of the loop. We use a scalar potential

$$\phi = \frac{\mu_0}{c^2} \frac{z}{r^2}$$

If the dipole is on the $z$ axis with $u$ vertical, it feels a force

$$F_z = -\frac{\partial \phi}{\partial z} = u \frac{\partial^2 \phi}{\partial z^2} = \frac{\mu_0 u I a^2 z}{c^2 + z^2}$$

The first requirement for levitation is to balance gravity. If the dipole mass is $m$,

$$F_z + mg = 0$$

The second requirement for levitation is stability: if the dipole wanders away from the equilibrium point, there must be a force to push it back. Consider first vertical stability. There are two regions of vertical stability:

- $a/2 < z < 0$, and $z > a/2$. The force itself has opposite sign in the two regions; they are qualitatively different. For example, a superconductor levitated by Meissner effect ("flux exclusion") would be vertically stable for $z > a/2$; an iron object levitated by induced ferromagnetism would be vertically stable at $-a/2 < z < 0$. But radial stability is also required. In any region not enclosing currents, $z$ must satisfy Laplace's equation.

In cylindrical coordinates ($r^2 = x^2 + y^2$)

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

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was as follows: The loop at position \( z \) is turned on when the solenoid position, \( z \), and velocity, \( v \), satisfy

\[
\frac{\partial^2 \phi}{\partial t^2} = \frac{\partial^2 \phi}{\partial z^2} + \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -\frac{1}{\beta} \left( \frac{\partial^2 \phi}{\partial z^2} \right)
\]

Then, if \( u \) is directed along \( z \),

\[
\frac{\partial^2 \phi}{\partial t^2} = \frac{\partial^2 \phi}{\partial z^2} = \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \phi}{\partial y^2} = -\frac{1}{\beta} \left( \frac{\partial^2 \phi}{\partial z^2} \right)
\]

The negative sign means radial and vertical stability are mutually exclusive. This is a special case of Earnshaw's theorem. Thus magnetic levitation can be stable either radially or vertically, never both at once. The usual choice is to select radial stability and get vertical stability by feedback from a sensor.

**IV ACCELERATOR STRUCTURE**

By the principle of equivalence, a levitation scheme is accelerated only if there is a corresponding mutual inductance between the loops. To avoid having to switch large currents, we drive each loop from a capacitor \( C \), through a diode and a switch. When the switch is turned on, the LC circuit executes \( l/2 \) period of an oscillation before being quenched by the diode.

In a loop turned on at \( t = t_0 \), the current is

\[
I = 0, \quad t < t_0 \quad \text{and} \quad t > t_0 + \pi \frac{v}{C} L
\]

\[
I = I_{\max} \sin \left( \frac{t-t_0}{\pi \frac{v}{C} L} \right)
\]

Here \( C \) is the capacitance, \( L \) is the self inductance of the loop, and the maximum current, \( I_{\max} = \sqrt{V_0 C L} / L \), depends on the initial voltage, \( V_0 \). Before presenting results of simulation of this model, we discuss some qualitative features. The dipole tends to line up as to be sucked into the region of highest field. The opposite case, using Meissner effect, is not considered here.

Then the radial motion will be stable, if and only if the dipole is farther than \(-a/2\) behind the peak current. Then \( z \) stability (longitudinal) must come from feedback, i.e. the switching on of the current loops must be synchronized with the dipole motion. We assume that an arbitrary trigger function of position and velocity is possible. As a first order proof-of-principle, a crude model has been simulated by numerically integrating the \( z \) equation of motion of a dipole through a section of a hypothetical accelerator. The simulation parameters are given in Table I. The trigger scheme used was as follows: The loop at position \( z_0 \) is turned on when the solenoid position, \( z \), and velocity, \( v \), satisfy

\[
z + v \cdot \frac{L}{C} = z_0
\]

This trigger, which was picked arbitrarily, is such that the extrapolated time when the solenoid will cross the plane of the loop, will be the end of the current cycle for that loop. Acceleration functions \( A_0 \) are shown in Figure 1. The focussing function, \( k \), of the accelerator is shown in Figure 2. For \( d \lesssim 1.0 \text{ cm} \), the accelerator is always negative, thereby providing continuous radial focussing.

**V ENGINEERING CONSIDERATIONS**

We have not yet studied in detail engineering aspects of the accelerator elements. The superconducting coils might pose formidable engineering tasks. Nb3Sn has been made to small wire dimensions. Construction of coils should not be too difficult. The vacuum chamber is made of tubular ceramic material to prevent eddy current heating and should be strong enough to withstand repeated coil forces. Cryogenic conditions are maintained by compressing helium gas at 4K to flow through the outer region of the coil. Energy storage capacitors are also circular and are to be charged up from generators outside the accelerator structure. The entire assembly is insulated and kept at low temperature. The cross section is expected to be less than 30 cm, and is segmented for ease of maintenance. (See Figure 3.)

<table>
<thead>
<tr>
<th>Table I. Parameters Used In Simulation</th>
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<tbody>
<tr>
<td>( a ), radius of loop</td>
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<tr>
<td>( d ), separation</td>
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<tr>
<td>( L ), inductance of loop</td>
</tr>
<tr>
<td>( C ), capacitance per loop</td>
</tr>
<tr>
<td>( V_0 ), applied voltage</td>
</tr>
<tr>
<td>( \tau ), ( V_0 / CL )</td>
</tr>
<tr>
<td>( I_{\max} ), peak current</td>
</tr>
<tr>
<td>( v ), initial velocity</td>
</tr>
<tr>
<td>( m ), mass of dipole</td>
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<tr>
<td>( u ), moment of dipole</td>
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</table>

**Figure 1--Simulated Acceleration functions, \( A_0 \)**

**Figure 2--Radial focussing function, \( k \)**

Fabrication of the solenoid projectile is not difficult. A diamagnetic filler weighing \(-0.1 \text{ g} \) can be made from SiO2. Superconducting films can be vacuum deposited on the surface to form the solenoid. The shape of the projectile is dictated by vacuum considerations. The heat gained by the projectile is

\[
\Delta Q = \rho g v^2 A L
\]

where \( \rho g \) is the residue gas density, \( v \) and \( A \) the projectile velocity and cross section, and \( L \) the length of the
accelerator. If vacuum is worse than $10^{-6}$ Torr, $\Delta Q$ is less than 0.5 J. To maintain superconductivity, the vacuum needs to be improved by at least 3 orders of magnitude well within current state-of-the-art, or a prefabricated heat shield has to be placed in front of the projectile. (See Figure 4.)

To specify parameters required for the projectile, we assume it to be a cylinder of length $L$. The solenoid is filled with magnetically inert material which carries most of the mass ($m$). The average density of the projectile, $\rho$, to calculate the magnetic moment of the projectile, we assume the thickness of the superconducting wire is $\delta = (r_2 - r_1) << r_2, r_1$.

Before discussing the numbers, we consider some basic points. The main requirement is to deliver energy ($\sim 10^6$ J) to the target in a short time ($10^{-4}$ s). If $v_f$ is the final velocity of the projectile, then one has a restriction:

$$\epsilon \ll v_f t$$

The kinetic energy $K$ depends upon the magnetic moment per unit length ($\mu/J$), the applied field $B$, and the length of the accelerator $L$ through the following approximate equation.

$$K = \text{Force} \cdot L = B(\mu/J) \cdot L$$

For a solenoid with a number of turns per unit length, and current density $j$, one can obtain an equation determining $\epsilon$ in terms of the basic parameters of the projectile and the impact time and the accelerator length $L$.

Typical values of the projectile parameters are given in Table I. To obtain various entries for the table, we have used $j = 5 \times 10^{10}$ A/m$^2$, $\sigma = 5 \times 10^3$ Kg/m$^3$, $t = 2 \times 10^{-4}$ s, and $\delta = 2 \times 10^{-4}$ m. The final kinetic energy delivered is $\sim 10^8$ J, slightly higher than the needed value.

<table>
<thead>
<tr>
<th>Table I. Typical Projectile Parameters</th>
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<tbody>
<tr>
<td>Heat Shield</td>
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<tr>
<td>Diameter of Solenoid, $2r_2$</td>
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<tr>
<td>Thickness of Nb$_3$Sn wire, $\delta$</td>
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<tr>
<td>Field at the Projectile, $B_0$</td>
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<tr>
<td>Mass of the projectile, $m$</td>
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</table>

VI CONCLUSION

None of the considerations of this paper indicate any intrinsic problems which indicate that a magnetic acceleration to velocities of $10^5$ m/s is unfeasable. A superconducting solenoid projectile with a permanent dipole moment seems capable of reaching these velocities in an accelerator of $\sim 2$ km length.

We close by noting that a more detailed version of this report is available. [8]

We believe that the present work indicates that impact fusion can be expected to be a technologically viable method of achieving thermonuclear power generation. We are encouraged to pursue more detailed questions of design. Among these are:

(i) Mutual induction effects between the accelerating coil and projectile.

(ii) Projectile geometry for optimal fusion ignition.

(iii) Vacuum and dissipation constraints

(iv) Injection and feedback design for the longitudinal motion of the projectile.

In light of the favorable results reported here we suggest that a significant effort be made to do further design and conceptual work on impact fusion, including proof-of-principle experiments.

VII ACKNOWLEDGEMENT

The assistance of M. Sarafaraz and E. Salberta is acknowledged. One of us (K.W.C.) has been benefited from discussions with F. Winterberg, R. Muller, B. Richter, A. Trivelpiece and G. Stuart.

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[4] The magnetic levitation idea has been used extensively in magnetic monopole search experiments, e.g., J. Beams, et al., J. Appl. Phys. 17, 856 (1946).


