STUDY AND DESIGN OF NAL MAIN RING ACCELERATOR CAVITY

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Introduction

This cavity is to be used for accelerating the protons in the Main Ring of the National Accelerator Laboratory 200 GeV Synchrotron.

The main objectives of this study and design program were:

1. Develop a peak voltage of at least 240 kV across the gaps in each cavity which would interact with the protons when the cavity is driven by an RF source of 20 kV.

2. Develop a cavity which was tunable over a frequency range of 32.512 MHz to 53.105 MHz.

3. Dampen the higher order cavity modes having resonant frequencies up to the 20th harmonic of the fundamental frequency. This will minimize interaction between these modes and harmonic components of the beam.

4. Obtain a minimum Q of 5000 from the prototype cavity.

Cavity Study

A cavity similar to that described by Kerns et al., was used as the starting point. Cold tests were performed on a 1/5 scale model. Figure 1 is a sketch of this cavity. This cavity was later modified to ease the problem of vacuum integrity.

In order to better understand the operation of the cavity, it was approximated by a combination of transmission lines and lumped circuit elements. The TEM modes of the cavity can be fairly accurately represented in this manner.

The equivalent circuit that was chosen is shown in Figure 4. The solutions were found with ABCD type matrices. The ABCD matrix can be used to express the circuit transfer characteristics as in (1):

\[
\begin{bmatrix}
V_1 \\
I_1 \\
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D \\
\end{bmatrix}
\begin{bmatrix}
V_n \\
I_n \\
\end{bmatrix}
\]

(1)

Each circuit element can be represented by such a matrix. The final ABCD matrix is obtained by multiplying...
the individual matrices. The resonances are determined by finding the poles of $Z_1$ (where $Z_1 = V/I_1$), with $I_m = 0$. Therefore from Equation (1), $A/C = 0$. The resonances occur when $A = \infty$ or $C = 0$.

Figure 4. Equivalent Circuit of Cavity

The voltages and currents at different parts of the equivalent circuit can be found from the $A$ and $C$ values at the position of interest.

Values of the circuit elements that gave good agreement with experimental results are listed below.

<table>
<thead>
<tr>
<th>Gap Capacitance</th>
<th>25 pF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Inner Coaxial Line</td>
<td>0.59 m</td>
</tr>
<tr>
<td>Length of Outer Coaxial Line</td>
<td>0.62 m</td>
</tr>
<tr>
<td>Corner Capacitance</td>
<td>8 pF</td>
</tr>
<tr>
<td>Corner Inductance</td>
<td>0.018 uH</td>
</tr>
<tr>
<td>Coupling Inductance</td>
<td>0.04 uH</td>
</tr>
</tbody>
</table>

A comparison between the measured resonant frequencies and calculated values is tabulated below. The measured frequencies were scaled to the full scale model.

<table>
<thead>
<tr>
<th>Measured Frequency (MHz)</th>
<th>Computed (MHz)</th>
<th>% Error</th>
<th>Mode Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>34</td>
<td>0</td>
<td>Even</td>
</tr>
<tr>
<td>48</td>
<td>48</td>
<td>0</td>
<td>Odd</td>
</tr>
<tr>
<td>92</td>
<td>89</td>
<td>-3.3</td>
<td>Even</td>
</tr>
<tr>
<td>112</td>
<td>118</td>
<td>+5.0</td>
<td>Odd</td>
</tr>
<tr>
<td>186</td>
<td>183</td>
<td>+10.2</td>
<td>Even</td>
</tr>
<tr>
<td>220</td>
<td>217</td>
<td>-1.5</td>
<td>Odd</td>
</tr>
<tr>
<td>262</td>
<td>268</td>
<td>+2.3</td>
<td>Even</td>
</tr>
<tr>
<td>290</td>
<td>318</td>
<td>+13.1</td>
<td>Odd</td>
</tr>
</tbody>
</table>

The results above indicate that the equivalent circuit used was quite accurate, especially at the lower frequencies.

Voltage Gain Design

The cavity was to be excited by a tetrode with an RF voltage of 20 kV peak. The required RF gap voltage across the gaps is 240 kV peak, thus requiring a "gain" of 12 to 1. Power from the tetrode was to be coupled to the cavity by extending the center conductor of the coaxial transmission line from the tetrode directly to the intermediate cylinder of the cavity. This is shown in Figure 5. This coupling arrangement capacitively loads the cavity with about 150 pF, which unbalances the gap voltages by about 20%. This would deteriorate the mode loading to be described later and could also cause uneven heating of the cavity. For these reasons, a compensating capacitor was used on the opposite side of the cavity midpoint. This is also shown in Figure 5. This capacitor was in the form of an open circuited transmission line that was shorter than a quarter wavelength. The length of the inner cylinder could be varied to obtain a good balance for the gap voltages.

Figure 5. Sketch of Prototype Cavity

The equivalent circuit was also used to find the proper contact position for the coupling line to the intermediate cylinder. There is a voltage drop in the coupling line since the distance from the interaction region of the tetrode to the coupling point is an appreciable part of a quarter wavelength. This voltage drop was minimized by making the inner conductor diameter of this coupling line as large as possible considering voltage holdoff requirements. The length of this line was kept as short as possible. The voltage ratio between the gap and the coupling point can be increased by moving the coupling point to a low voltage point. The voltage at the coupling point is approximately given by $V = Z_0 \tan \theta$, where $Z_0$ is the characteristic impedance of the line and $\theta$ is the electrical length of the line between the coupling point and the center of the cavity. Due to the large dimensions of the coupling line, compensating condenser, and mode loading appendages, the coupling point could not be moved sufficiently close to the midpoint to obtain the required gain. However, the voltage at the coupling point could be decreased by reducing $Z_0$. $Z_0$ could be reduced by increasing the diameter of the inner cylinder of the outer coax line of the cavity. The equivalent circuit program was used to find the proper diameter which would yield sufficient voltage "gain". When the scale model using these modifications was cold tested, the voltage gain was measured to be 12.7 to 1. This was close to the value predicted from the equivalent circuit model.

Higher Order Mode Suppression

There are two techniques to prevent interaction between harmonics of the proton beam and higher order cavity modes: (1) prevent overlap of resonances with harmonic frequencies, and (2) dampen the modes. At the higher frequencies, the mode density is so great that complete prevention of overlapping would be almost impossible. However, at the lower frequencies where the mode density is smaller, this technique can be used. The design of this cavity incorporated both techniques.
Figures 2 and 3 showed the fields of the odd and even modes respectively. The center plane of the cavity is a good position for loading both types of modes. The odd modes should couple strongly to a waveguide as shown in Figure 6. The even modes shown in Figure 3 would best be coupled to coaxial lines. This is shown in Figure 7. Since coaxial lines do not have a cutoff frequency, even modes at all frequencies will be coupled to the coaxial line.

The waveguide used had a cutoff at 345 MHz to keep the waveguide to a reasonable size. The even modes shown in Figure 3 were best coupled to coaxial lines, This is shown in Figure 7. Since coaxial lines do not have a cutoff frequency, even modes at all frequencies will be coupled to the coaxial line.

The coaxial TEM modes are circularly symmetrical. Thus, one waveguide load and one coaxial load should be sufficient to load all these modes. However, some of the higher order modes do not have circular symmetry and it is possible to have a mode so oriented as to have negligible coupling to a single waveguide or coaxial line. For this reason, two waveguides and two coaxial lines were used for the mode loading. This loading scheme should permit damping of all modes up to at least the 20th harmonic. The fundamental mode was negligibly affected.

A mode search was made for frequencies up to the 20th harmonic. Those modes which fell close to a beam harmonic or had a substantial E field in the gap were further investigated. The Qs and impedances of these modes were measured. These values were then introduced into a beam interaction computer program (described below), to find the effects of these higher order modes on the gap voltage.

**Beam Cavity Interaction Computer Program**

The effective voltage which accelerates the beam in each cavity is a consequence of all the harmonic voltages produced by the axial electric fields in the interaction gap. The approach was to calculate the effective harmonic voltage component due to each TM mode of any significance (up to the 20th harmonic) to determine which (if any) modes were present that would produce a significant effective voltage as compared to the fundamental mode.

The general expression for the harmonic effective voltage due to a single mode is given by:

\[
V_{\text{eff}} = \frac{M^2 I_N (R/Q)}{1 + (R/Q) Q L G_b} \]

where
- \(V_{\text{eff}}\) = effective accelerating voltage
- \(V\) = gap voltage
- \(M\) = gap coupling coefficient
- \(I_N\) = harmonic beam current
- \(R/Q\) = intrinsic gap impedance
- \(G_b\) = beam loading conductance
- \(Q_L\) = loaded Q of mode without beam present
- \(f_N\) = harmonic frequency
- \(f_0\) = resonant frequency of mode

In order to be conservative in calculating the effective accelerating voltages, several assumptions were made. Each mode was assumed to coincide with the nearest harmonic frequency, thus eliminating the second term in the square root of the denominator. In addition, a 1 nanosecond rectangular pulse was assumed instead of a gaussian shape. This would increase the amplitude of the harmonic beam current. The resulting equation simplifies to:

\[
V_{\text{eff}} = \frac{M^2 I_N (R/Q) Q L}{1 + (R/Q) Q L G_b} \]

The basic equation for the square of the axial coupling coefficient between the cavity gap and the beam is:

\[
M^2 = \frac{1}{\pi b^2} \int_0^{2\pi} d\theta \int_0^b r dr \left( \int_{-\infty}^{\infty} E_z(r, \theta, z) \frac{1}{V} e^{j\beta_e z} \right)^2
\]

where
- \(\beta_e = \omega_{ce} = 2\pi f \)
- \(E_z\) = axial component of electric field in the gap
- \(V\) = gap voltage
- \(b\) = beam radius
- \(u_o\) = beam velocity
At the highly relativistic beam energies considered, the velocity remains nearly constant through the range and thus $M^2$ is nearly independent of beam energy. The small signal beam loading conductance $G_b$ was derived from ballistic theory neglecting the space charge field (a good assumption for a high impedance beam). Including the relativistic corrections for the mass and the velocity, the result is given by: 

$$G_b = \frac{I_0}{V_0 R (R + 1)} \beta_o M \frac{\partial M}{\partial \rho}$$

where

$I_0 = \text{dc beam current}$
$V_0 = \text{dc beam energy, eV}$
$R = \frac{m}{m_0}$
$m_0 = \text{rest mass of particle (in this case a proton)}$
$m = \text{total mass of particle in lab frame of reference}$

Computations were performed for beam energies covering the range from 8 to 200 GeV. The beam size was assumed to vary inversely with the total mass, which approximates the synchrotron oscillation effects. $G_b$ was found to be small for most modes due to the high impedance of the beam.

The second term in the denominator of expression (3) was small except for modes with very high values of $Q$, and relatively high values of beam loading. In general, the beam loading conductance may be either positive or negative; however, since the values were very small there is no danger of instability due to a negative loading term.

In order to be able to treat modes with an even number of half-wave variations of longitudinal electric field along the axis of symmetry, the gap voltage is defined for all modes as the product of the maximum value of longitudinal electric field and the gap length. For modes with electric field on the axis, the peak value of field on the axis is assumed. For modes with no electric field on the axis, the peak value of field (as a function of $\theta$ and $Z$) at a specified radius off the axis is used. A perturbation measurement is made at this position to provide data for calculating the $R/Q$.

Using the approximations, which would provide conservative values, the maximum ratio of any higher order mode voltage to the fundamental voltage was less than 0.2%.

Prototype Test Results

A prototype cavity was built and tested. The cavity was constructed of stainless steel for strength and then copper-plated to reduce the RF losses. A photo of the cavity is shown in Figure 8. Some of the pertinent results are:

1. A peak voltage of 270 kV was developed across the gap.
2. With the single tuner in place, the maximum ratio of the voltage for a higher order mode to that of the fundamental was less than 0.2%.
3. A $Q$ of 5700 was obtained; the minimum objective $Q$ was 5000.

The cavity tuning has not been fully evaluated yet since only one tuner was available.

![Photo of Prototype Cavity](image)

References