END EFFECTS IN SUPERCONDUCTING BEAM-TRANSPORT MAGNETS*

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Introduction

The magnets considered in this report provide a uniform or "dipole" field, a quadrupole field, or a higher-order multipole field over the volume of a right circular cylinder. The field is transverse to the axis of the cylinder, and the particle beam passes through the cylinder roughly parallel to the axis. The field is generated by a symmetrical array of longitudinal electrical conductors lying in an annulus surrounding the cylinder. The conductors may be surrounded by an iron annulus either fitting closely or spaced some distance from the conductors. Such configurations are particularly applicable to superconducting magnets.

Ideally, the field within the cylinder would consist only of a single two-dimensional multipole component. At the ends of the cylinder the field would drop abruptly to zero, but Nature demands a more gradual falling-off in this fringing-field region. The conductors on one side of each magnetic pole must somehow be connected to those on the other side at the ends, and the manner in which these connections are made controls the character of the fringing field.

The Field-Integral Criterion

For many purposes an acceptable criterion for judging the quality of a magnet is the degree to which the field integrals, along paths parallel to the z-axis

$$\int_{-\infty}^{\infty} B_x \, dz \, , \qquad \int_{-\infty}^{\infty} B_y \, dz$$

approach the desired pure multipole distribution.

Magnets satisfying this integral criterion can be classified and ranked roughly as follows: (1) Best of all is a magnet having a pure two-dimensional field in the middle and short end regions in which the field-integral condition is satisfied. (2) Next best is one in which the field in the middle is slightly distorted to compensate for the aberrations of the ends. (3) Least desirable is one where the field on all transverse planes departs severely from the desired pure multipole field, but one which nevertheless satisfies the integral condition. All magnets having a small length diameter ratio fall into this category.

This categorizing is rough, because there is a continuous spectrum of magnets differing only in degree.

The Integral Theorem**

For a magnet of finite length having no iron, the field integrals,

$$\int_{-\infty}^{\infty} B_x \, dz \, , \qquad \int_{-\infty}^{\infty} B_y \, dz$$

are related to the current integrals

$$\int_{-\infty}^{\infty} I_z \, dz$$

in the same way that the field components B_x, B_y are related to the currents I_z in an infinitely long two-dimensional magnet. This is also true if the conductors are surrounded by regions of uniform magnetic permeability, the surfaces of which are generated by lines parallel to the z-axis.

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** Glen Lambertson--unpublished.

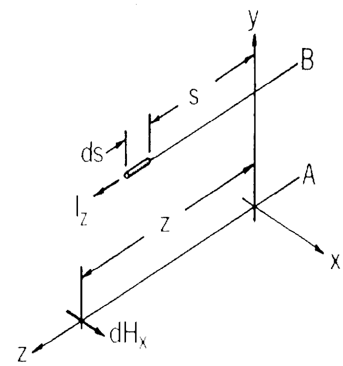

Fig. 1. Nomenclature for current-integral theorem.

For a permeability >> 1, only the inner iron surface is important. The law holds if the iron cross section changes abruptly, or the iron ends suddenly, in a region where the conductors are parallel to the axis, or in a region where the field is very small. This theorem is of great convenience, for it permits the field integrals to be calculated for real magnets having ends, by the methods used to calculate two-dimensional fields, rather than calculating the actual field and integrating it. A proof, of sorts, follows.

Lines A and B (Fig. 1) are parallel and situated in a Cartesian coordinate system as shown. From Ampere's law it follows that a current element having a z-component $I_z(s)$ and a length ds produces a magnetic field $dH_x(Z)$ having a finite integral of magnitude proportional to $I_z(s)$ ds :

$$\int_{-\infty}^{\infty} [dH_x(z)] \, dz = \text{const} \times I_z(s) \, ds \, .$$

The integration of H_x is performed along line A, and that of I_z along line B. Upon integrating over all conductor elements along line B we obtain

$$\int_{-\infty}^{\infty} H_x(z) \, dz = \text{const} \times \int_{-\infty}^{\infty} I_z(s) \, ds \, .$$

An x-component of current on line B produces only y- and z-components of field at line A--these have no contribution to the integral of H_x. A y-component of current on line B produces an x-component of field at line A, but the integral of H_x is zero because of symmetry. Therefore the above equations remain valid whether or not there are x- or y-components of current on line B. The axes can be shifted and rotated to include the effect of all conductors on all field-integration paths, so by superposition the theorem can be applied to the whole magnet.

When iron having surfaces generated by lines parallel to A and B is present, image currents are generated in the iron. We normally think of image currents in the two-dimensional sense; nevertheless, we can define three-dimensional image currents into existence. The preceding reasoning for the field integrals associated with the current elements can also be applied to the images of those current elements. Symmetry arguments can be brought to bear for iron that ends abruptly on a plane perpendicular to the axis in regions where the conductors are parallel to the axis, and the general law can be shown to be still valid. However, when the iron changes cross section in a region where the conductors are not parallel to the axis, all bets are off.

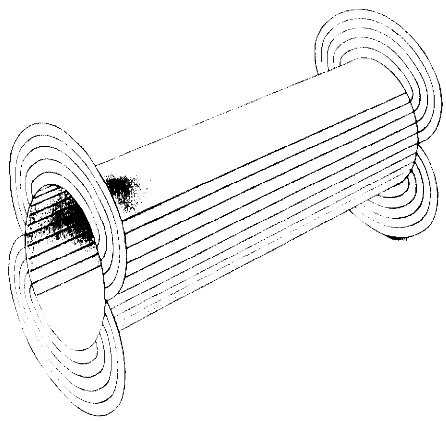

Fig. 2. Dipole with plane ends. Only one layer is illustrated.

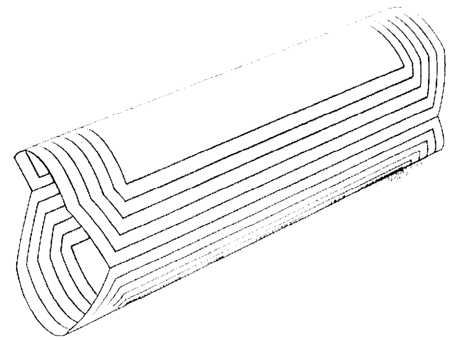

Fig. 5. Layer-type dipole with Lambertson ends. Only one layer is illustrated.

Configurations Satisfying the Integral Criterion

Several configurations have been devised that have no end aberrations in the integral sense. In one the longitudinal part of every conductor has the same length (Fig. 2). The end connections lie in a transverse plane and do not contribute to the field integrals A further advantage is that the effective magnetic length is the same as the physical length of the winding--in all other configurations the physical length is longer. Disadvantages are that the ends are bulky in their transverse direction, and that the large longitudinal force on the ends must be transmitted to some auxiliary structure.

A configuration (devised by Glen Lambertson and reported in these Transactions by Avery et al.) that is particularly applicable to very short magnets is shown in Figs. 3 and 4. The turns, in the development, are rectangular. The length of the longitudinal part of the conductor is a function of θ, and the conductor spacing, or current density, are tailored so that the following equation is satisfied:

$$J_z z = A \cos\theta .$$

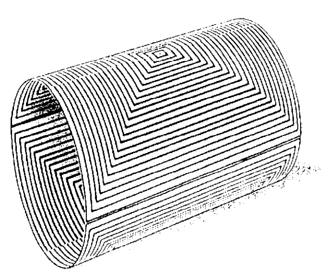

Fig. 3. Lambertson dipole.

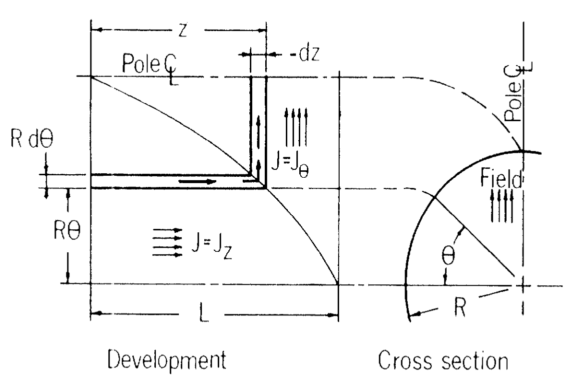

Fig. 4. Nomenclature for Lambertson dipole.

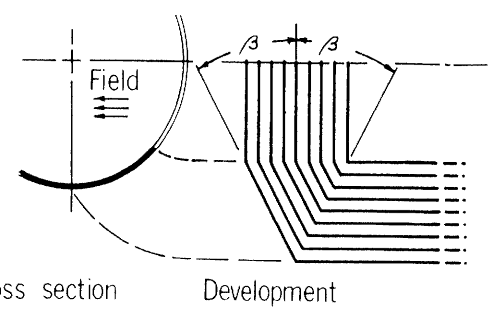

Fig. 6. Geometry of one layer of a magnet having Lambertson ends.

Another configuration satisfying the integral criterion (Figs. 5 and 6) is particularly applicable to magnets consisting of concentric cylindrical shells of conductors (devised by Glen Lambertson). The effective lengths, in the current-integral sense, of all conductors in a layer are the same. The two angles β must be identical. For $\beta \geq 30°$, the conductor spacing in the angled region is equal to or greater than that in the middle. The maximum field at the conductor is typically only 5% higher than in the aperture. The longitudinal forces on the ends must be transmitted to an auxiliary structure for high-field magnets.

Two-Dimensional Configurations

Two uniform-current-density configurations are in current fashion at LRL: the layer type (Figs. 7 and 8) and the block type (Fig. 9). Both can, in principle, be designed to eliminate, in the two-dimensional sense, as many orders of multipole components, higher than the fundamental, as there are degrees of freedom in the dimensions. A magnet of symmetrical construction can have only harmonics of order n = mj, where n is the

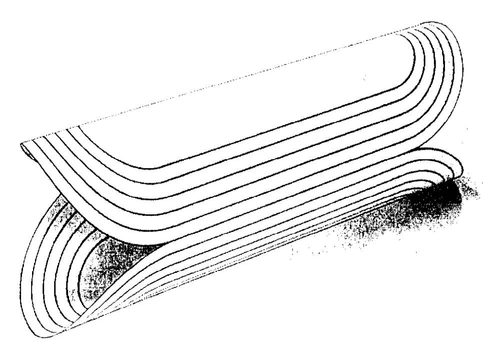

Fig. 7. Layer-type dipole with semicircular ends. Only one layer is illustrated.

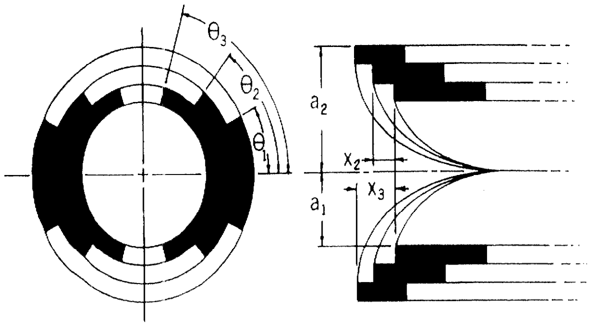

Fig. 8. Cross sections of layer-type dipole with semicircular ends.

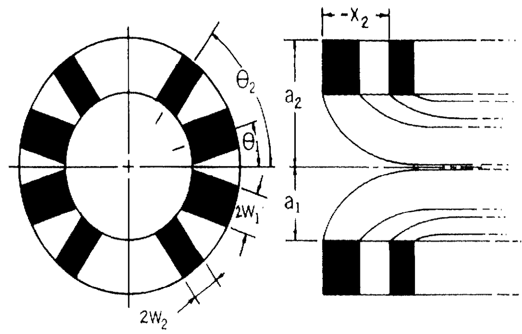

Fig. 9. Cross sections of block-type dipole with semicircular ends.

multipole order corresponding to n pairs of poles, m is the number of pairs of magnet poles, and $j = 1, 3, 5, 7, \cdots$. A quadrupole magnet ($m = 2$) can only have harmonics of $n = 2, 6, 10, 14, \cdots$. Harmonics $n = 6$ and 10 can be eliminated from a magnet having two current blocks, or two layers of equal thickness. (If the layer interface radii are free variables, additional harmonics can be eliminated.)

For layer-type magnets an iterative procedure (Newton's method, applied to a function of several variables) is used to solve the simultaneous nonlinear equations. C_n represents the magnitude of the rotating field vector of order n at some reference radius, corresponding to an initial set of layer angles, or a set resulting from a previous iteration; θ_1, and the partial derivatives, are evaluated for those same angles. New values, C'_n, can be expressed in terms of the old values, the derivatives, and increments of the angles as in the following example for a two-layer dipole:

$$C'_1 = C_1 + (\partial C_1/\partial\theta_1)d\theta_1 + (\partial C_1/\partial\theta_2)d\theta_2$$
$$C'_3 = C_3 + (\partial C_3/\partial\theta_1)d\theta_1 + (\partial C_3/\partial\theta_2)d\theta_2$$
$$C'_5 = C_5 + (\partial C_5/\partial\theta_1)d\theta_1 + (\partial C_5/\partial\theta_2)d\theta_2 \quad \text{etc.}$$

In the second and third equations, C'_3 and C'_5 are set equal to zero, and the pair are solved simultaneously to give increments $d\theta_1$, $d\theta_2$ which are added to the old values for the next iteration.

The procedure used for the block construction is slightly different. The block centerline angles are set at the angles determined by Beth (Ref. 1) for his stepwise-variable current density configurations:

$$\theta_i = \frac{\pi}{2}(2i - 1)/[m(2k+1)] \; ,$$

where m is the number of pairs of poles, i is the current-block index, and k is the number of blocks per half-pole. This automatically makes the harmonic of order $n = (2k+1)m$ equal to zero, and reduces by one the number of equations to be handled. Angles θ are replaced by the widths w_i as the independent variables. Formulas used for evaluating the terms in these equations are presented in the Appendix.

Simple computer programs have been written to solve these problems on LRL's Berkeley Remote Facility (BRF) system. This is an on-line time-share system of limited speed and capacity, utilizing a pidgin-Fortran. Input-output is by teletype console tied to a CDC 6600 computer. The programs work very well for the block construction but not as well for the layer construction, particularly for high-order multipole magnets, a large number of layers, and a very thick conductor region. Usually a solution can be obtained by approaching it in small steps from a known solution. If the iteration works at all, the unwanted harmonics are down by a factor of 10^{-10} from the fundamental after about five iterations. Various cosine approximations can be used to estimate the starting dimensions. The programs, and results for related series of magnets, are presented in Ref. 2. The complete theory is presented in Ref. 3. The more general problem of a continuously curved outer boundary has been treated by Morgan (Ref. 4).

Aberrations in the Ends of Layer- and Block-Type Magnets

In the designs currently favored at LRL, the conductor crossovers at the ends of the magnet are made so that no appreciable length of any conductor lies in a single plane perpendicular to the magnet axis. For such a shape the large longitudinal forces on the crossovers are transmitted to the straight section of the conductor where they appear as simple tension. They are adequately resisted by the tensile strength of the conductor, so no auxiliary structure is required to hold the crossovers in position. For simplicity of construction and analysis the conductors are semicircles in the development of a layer. The current integrals for such a design are presented in Refs. 5 and 6.

In principle, the end aberrations can be compensated by corrector windings at the ends. A dipole magnet ($m = 1$) requires correctors for $n = 3, 5, 7$, etc. A corrector for $n = 3$ generates multipole components of order $n = 3, 9, 15$, etc., so in general a separate multipole corrector is required for compensating each undesired aberration component. Even one corrector multipole would add appreciably to the length or diameter of a magnet. For superconducting magnets of the type we have been concerned with, adding to the diameter would increase the cost and decrease the utility of a magnet. Increasing the length by a certain factor is, for our purposes, the same as decreasing the field strength by the same factor, and therefore the use of correctors has not been considered practical. The end aberrations can be compensated by shifting the ends of the layers or blocks longitudinally.

Since lengthening of the whole magnet does not alter the end aberrations, there is one fewer degree of freedom than there are layers or blocks. A three-layer dipole can have multipole components of $n = 3, 5$, and 7 eliminated in the straight section, but only $n = 3$ and 5 eliminated from the ends. Equations of the form

$$C'_i = C_i + (\partial C_i/\partial x_2)dx_2 + (\partial C_1/\partial x_3)dx_3 + \cdots$$

can be written for each layer as in the two-dimensional case, where x_i represents an increment of axial shifting. The partial derivatives are simply the contributions of unit length of straight section to each multipole field component. The partials are independent of x, hence the equations are linear. Upon setting as many C' values equal to zero as there are layers to be moved and solving for the dx values, the required solution is found without iteration.

The shifting required for a related series of 36 magnets has been calculated (Ref 5.) The series includes both quadrupoles and dipoles, and coil radius ratios from 1.0 to 1.5. The presence or absence of iron, closely fitting around the coil, is considered. The following conclusions can be drawn:

For magnets having thin windings, the shifting required for a quadrupole is exactly half that of a dipole, that of a sextupole is one third, etc., and the amount of shifting is independent of the presence or absence of iron. This is approximately true for thick windings.

The total shifting required for a layer-type magnet is roughly proportional to the number of multipole components that are compensated. Compensation of only one aberration component of a dipole requires shifting of the layers by 1.6 to 2.4 coil-inside-radii at each end.

For a dipole of block-type construction, compensation of four components can be achieved with an added length of 1.1 to 1.5 coil-inside-radii at each end.

For the 8-in.-diameter layer-type dipole and quadrupole doublet being constructed at LRL it was not practical to use the above method of compensation because too much field-integral would be lost. For these magnets, the two-dimensional fields were distorted, by about 2%, to compensate for the aberrations of the ends.

Conclusions

Several ways of reducing the aberrations at the ends of magnets have been discussed. For superconducting magnets, in addition to field aberrations, one must consider the maximum magnetic field in the end region of the conductors, and many structural, mechanical, and fabrication problems. There is no single "best" solution to this combination of problems.

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APPENDIX

Appendix: Formulas for Two-Dimensional Magnets

The following formulas are for a 2m-pole array of uniform-current-density layers or single filaments. Units are gauss, cm, amperes.

J_L = lineal current density, A/cm

J_A = areal current density, A/cm^2

I = current in filament, A

R = arbitrary reference radius, cm

a = average radius of thin layers, or radius to filament, cm

a_1 = inside radius of thick layer, cm

a_2 = outside radius of thick layer, cm

b = inside radius of infinitely permeable iron, cm

r = local radius, cm

m = number of pairs of magnet poles

n = multipole field order, corresponding to n pairs of poles

α = angular extent of layer, or angular position of filament, measured from a line bisecting two adjacent poles

θ = angle to point in aperture measured from a line bisecting two adjacent poles

B = magnetic field vector having components B_x and B_y, G

C_n = nth order multipole component of field, G

$C_n = C_{n,1} + C_{n,2}$

$C_{n,1}$ = C_n for iron-free magnet

$C_{n,2}$ = contribution to C_n by the iron.

The field in the aperture is conveniently expressed in terms of the coefficients C_n. The field corresponding to a single C_n is

$$B_x = C_n(r/R)^{n-1} \sin n\theta \qquad B_y = C_n(r/R)^{n-1} \cos n\theta$$

Note that $|B|$ at $r = R$ is simply C_n.

Thin layer:

$$C_{n,1} = 0.8(m/n)J_L \sin n\alpha \, (R/a)^{n-1},$$

$$C_{n,2} = C_{n,1} \, (a/b)^{2n} .$$

Thick layer:

$$C_{n,1} = A \, (2-n)^{-1} \, (a_2^{\,2-n} - a_1^{\,2-n}) ,$$

$$C_{n,2} = A \, (2+n)^{-1} \, (a_2^{\,2+n} - a_1^{\,2+n}) \, b^{-2n} ,$$

where

$$A \equiv 0.8(m/n)J_A R^{n-1} \sin n\alpha .$$

For n = 2, replace

$$(2-n)^{-1} \, (a_2^{\,2-n} - a_1^{\,2-n}) \text{ by } \ln\,(a_2/a_1) .$$

Filament:

$$C_{n,1} = 0.8(m/a)I \cos n\alpha \, (R/a)^{n-1} ,$$

$$C_{n,2} = C_{n,1}(a/b)^{2n} .$$

Field at iron surface at pole, for all three cases, is

$$2(b/R)^{n-1}C_{n,2} .$$

The above formulas are valid only for $n = km$, where $k = 1, 3, 5, \cdots$. For all others, $C_n = 0$.

It is to be noted that $C_n = 0$ for a 2m-pole array of filaments located at $\alpha = \pi j/(2n)$, where j = integer, for $0 \le \alpha \le \pi/(2m)$, or for a pair of filaments situated symmetrically about a radial line at angle α, or for a layer segment so situated.

These formulas were obtained by Fourier analysis of the equations presented by Blewett (Ref. 7)