THE EFFECT OF IONS ON THE SYMMETRICAL THROBBING BEAM MODE

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Introduction

It has been demonstrated that the presence of ions produced by a beam of particles in an accelerator can cause unstable coherent oscillations in the same way (apart from a sign difference) that the presence of the vacuum chamber with finite conductivity can produce unstable oscillations. However, for the symmetric mode, where the radius of the beam oscillates, there is a great deal of difference between the effects with ions and that with finite wall conductivity.

For a round beam in a round chamber, a change in the radius of a uniform beam does not change the electric or magnetic field at the vacuum chamber wall so that the only effect that the chamber wall has upon the beam is due to the variation of the beam radius with azimuthal position around the accelerator. Therefore, unstable symmetric oscillations due to the chamber wall have very slow growth rates. In contrast, the motion of an ion inside of the beam is affected by the beam radius and can produce unstable symmetric oscillations of the beam by leaving behind a "memory" field. It is the purpose of this report to investigate the stability and growth rates for symmetric oscillations with ions present.

It is assumed that the beam has a bunch structure, such that the time between bunches is long compared to the oscillation period of electrons in the potential well of the beam. Then the electrons produced by the beam escape of the beam and do not affect the motion of the beam. It is also assumed that the movement of the positive ions during the passage of the beam is slow enough that the beam can be assumed to be continuous for the purpose of calculating the dynamics of the ions.

Dynamics of the Ions

The radial force \( F_r \) on an ion with mass number \( A \) inside of a beam of radius \( a \) and linear particle density \( \lambda \) is given by

\[
\frac{1}{A m_p} F_r = \frac{2 \lambda}{A^2} r^2 r,
\]

with \( m_p \) and \( r_p \) the mass and classical radius of the proton, and \( c \) the velocity of light.

We assume that the beam radius oscillates as

\[
a = a_0 + r_n e^{i(\omega t + n \theta)},
\]

with \( \omega = (\nu - n)(\omega_0) \), \( n \) integral, \( \omega \) the revolution frequency, and \( \nu \) in general complex.

To first order in \( \Delta r / a_0 \) we obtain the following equation for the transverse motion of the ions:

\[
y_1 + \left[ - \frac{2}{\omega_0} + \frac{2 \lambda}{A_0} \right] r_n e^{i(\omega t + n \theta)} y_1 = 0,
\]

with

\[
\mu_2 = \frac{2 \lambda \omega_0^2}{A_0^2}.
\]

The ions are produced with negligible transverse energy uniformly within the radius of the beam. With these initial conditions Eq. (3) is solved and the density of the ions produced at \( t_0 \) is obtained as a function of time. By integrating over \( t_0 \) from \( -\infty \) to \( t \) we obtain, to first order in \( \Delta r / a_0 \), the ion density

\[
\rho_i = \left( \frac{1}{n_0} \right) \left( \frac{1}{4 \pi} \right) \left\{ 1 - \left[ c_1 \left( \frac{\omega}{\omega_0} \right) \right. \right.

\left. - \left. \left. 1 \frac{\omega}{\omega_0} \right] \frac{\Delta r}{a_0} e^{i(\omega t + n \theta)} \right\},
\]

with \( \Gamma \) the ion production rate per beam particle,

\[
oc_1(z) = \frac{4}{4+z^2}, \text{ and } c_2(z) = \frac{2z}{4+z^2}.
\]

The function \( c_2 \) has a broad maximum of value \( \frac{1}{8} \) at \( z = 2 \).

Dynamics of the Beam

The total force on a beam particle is the sum of: the external force due to magnetic focusing elements, the space-charge force due to the beam particles, and the space-charge force due to the ions. The equation for the transverse motion of the protons valid to first order in \( \Delta r / a_0 \) is

\[
y_p + \left[ \nu_0^2 \Omega^2 - [2 \nu_0 c \Delta v c^2 + \alpha] \right.

\left. + \left[ 4 \nu_0 \Delta v c^2 + \alpha \right] o_1 \left( \frac{\omega_0}{\nu_0} \right) \right.

\left. - \left. \left. 1 \right] \frac{\Delta r}{a_0} e^{i(\omega t + n \theta)} \right\} y_p = 0,
\]
with \( v_0 \) the single particle betatron oscillation frequency

\[
\Delta v = \left( \frac{\lambda r_p R^2}{2 v_{00} B^2 \gamma^2 a_0^2} \right),
\]

\( R \) the average machine radius, \( B \) the bunching factor \((B < 1)\), \( \beta \) and \( \gamma \) the relativistic parameters, and

\[
\sigma = \frac{\eta(2\lambda \lambda r_p R^2)}{\gamma a_0}. \tag{9}
\]

We demand that the solution of Eq. (7) be self consistent, i.e. that the radius of the beam be given by Eq. (2). This determines the oscillation frequency of the beam radius.

If \( v_0 \) is the betatron oscillation frequency including the incoherent shift due to both the beam and the ions, then to zero order in \( \xi_{in}/a_0 \) we have

\[
v_{00}^2 \Omega^2 = v_{00}^2 \Omega^2 - \left[ 2v_{00} \Delta v \Omega^2 + \sigma \right]. \tag{10}
\]

To first order in \( \xi_{in}/a_0 \), we obtain

\[
\sigma - \frac{4v_{00} \Delta v}{\Omega} \Omega^2 + \frac{\sigma \Omega}{\mu_0} \left[ \frac{(v_0 - n)}{\Omega} \right] - i \sigma \frac{2}{4v_{00} \Omega} \left( \frac{2z}{4z^2} \right), \tag{11}
\]

The quantities \( 4v_{00} \Delta v \), \( \frac{\sigma \Omega}{\mu_0} \) and \( \frac{\sigma \Omega^2}{\mu_0} \) are usually compared to \( v_{00} \) so that we obtain

\[
\Omega \approx 2v_{00} \Omega + \Delta \Omega + \frac{\sigma}{4v_{00} \Omega} \left( \frac{2z}{4z^2} \right), \tag{12}
\]

with

\[
z = (2v_0 - r) \left( \frac{\Omega}{\mu_0} \right). \tag{13}
\]

Thus one has the imaginary part \( v \) is negative when the imaginary part of \( v \) is negative. Thus the oscillations of the beam radius are unstable for \( n < 2v_0 \) with a growth rate

\[
\frac{1}{\tau} = \frac{\sigma}{2v_{00} \Omega} \left( \frac{2z}{4z^2} \right). \tag{14}
\]

In order for unstable oscillations to be damped it is necessary for the spread in the frequency \( 2v_0 \Omega \) to be larger than the coherent frequency shift for the mode under consideration. If the ion density is sufficiently low compared to the beam density the coherent frequency shift is, from (12), just \( \Delta \Omega \), and we require a betatron \( v \)-spread for damping given by

\[
\delta v_{00} > \frac{\Delta v}{2} = \left( \frac{\lambda r_p R^2}{2 v_{00} B^2 \gamma^2 a_0^2} \right). \tag{15}
\]

**Numerical Results**

Values of parameters for the CPS Booster and the CPS are used as numerical examples.

For the Booster at injection energy \((50 \text{ MeV})\) and a pressure of \(10^{-7} \text{ mmHg}\) we have an ion production rate \( \eta = 143/s \). The modes \( n = 8 \), and \( n = 9 \) have the fastest growth rates given by

\[
\frac{1}{\tau} = \begin{array}{ll}
8.8/s & \text{for } n = 9 \\
8.2/s & \text{for } n = 8.
\end{array} \tag{16}
\]

Thus for the Booster we would expect unstable oscillations of the symmetric throbbing beam mode to have a growth time of about \( 0.1 \) s.

This result is rather insensitive to most of the values of the parameters. Quite considerable factors in \( \omega_{00} \) or in the fractional part of \( v_0 \) may shift the emphasis between the modes, but cannot much change the largest value of \( \Omega \). So the growth rate will be largely determined by \( \rho \) where the vacuum \((\Pi^0)\) and the beam size \((a_{0}^{-2})\) are important factors, while the mass number of the ions and the proton intensity only come in the square root.

In order for unstable oscillations in the Booster to be damped it is necessary that the spread in the betatron oscillation frequency \( \delta v_0 \) satisfy the following condition:

\[
\delta v_0 > \frac{\Delta v}{2} = 0.6. \tag{17}
\]

For the CPS at an energy of \(150 \text{ MeV}\) and a pressure of \(5 \times 10^{-6} \text{ mmHg}\), the ion production rate is \(5.2 \times 10^3/s\). The node \( n = 12 \) has the fastest growth rate given by

\[
\frac{1}{\tau} = 310/s \quad \text{for } n = 12. \tag{18}
\]

Thus one expects an unstable symmetric throbbing beam mode in the CPS to have a growth time of about \(3 \text{ ms}\). The betatron frequency spread needed to stabilize this mode is

\[
\delta v_0 > \frac{\Delta v}{2} = 0.07. \tag{19}
\]

**References**

4. A more detailed report on these sections is available upon request as an internal CERN report.