POLE FACE SHAPE DESIGN IN HIGH-ENERGY ACCELERATORS

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Abstract

An analytic method for designing pole face shape is presented, which gets the desired value of the field index in a given region, with a prescribed error.

The method has three stages, corresponding to the successive computation of :

- the ideal pole face shape and ideal equipotential set (previously developed by the author) ;
- the equipotential set, taking into account fringing field effects, by means of conformal mapping ;
- the necessary corrections at the edges of the ideal pole face by means of conformal mapping, for obtaining the desired field distribution.

As an example, the pole face shape for a 300-GeV synchrotron with n=5040 is computed.

The method applies to every magnetic device having a radially constant field index.

Introduction

In a previous work, the author developed a method for designing the pole face shape for high-energy particle accelerators.[1] This method is based on the concept of an ideal pole face, which has an infinite radial extension, is made out of a magnetic material of infinite permeability and which lacks remanence and saturation.[3] The ideal pole face generates a magnetic field distribution in the median plane given by

$$B_z\ (r,\theta,0) = B_0\ \left(\frac{r}{r_0}\right)^{-n} \qquad (1)$$

where (r,θ,z) are the cylindrical coordinates,

$B_z(r,\theta,0)$ - the magnetic induction in the median plane,
r_o - the radius of the central orbit,
B_o - the magnetic induction at r_o,
n - the field index.

This expression determines unequivocally the magnetic field distribution in the whole volume of the air gap, particularly the main parameters of this distribution : the magnetic scalar potential, the magnetic induction, the equipotential set, and the potential network.[2,3]

According to the previous method, the correction of the ideal pole face at the edges, in order to obtain the genuine pole face, is made by a step-by-step procedure, computing in every case the potential network and trying to making it coincide with the ideal one.

According to the method presented here, the desired field index is computed by means of an equipotential line.[4] First, two ideal equipotential lines are computed : the 10% equipotential - used as a reference for determining the field index - and the 100% - the ideal pole face. Second, limits are set at the radial extension of the ideal pole face, and the new reference equipotential, taking into account fringing field effects, is computed by means of conformal mapping. Here, an idea proposed by Rogowski is followed, which consists in using an equipotential line in the field of a simple boundary shape to represent an actual boundary of a complicated shape.[5] In this way, the limited ideal pole face is represented, in a good approximation, by the 80% equipotential line of another equipotential set, the 100% line of which is composed of straight segments. Third, this 100% equipotential is corrected for edge effects, and the resulting 80% equipotential line represents the genuine corrected pole face.

The new method is applied to an actual case, being used for the computation of a pole face shape, giving the desired field index with a relative error less than $5\cdot10^{-4}$.

Theoretical considerations

Ideal equipotential set

Starting from eq.(1), the ideal equipotential set equations were derived [3], where n is a constant, taking into account the cylindrical symmetry :

$$Z=Z_0 \sum_{v=0}^{\infty} A_{2v+1}\, b^{2v+1} \left(\frac{Z_0}{r_0}\right)^{2v} \xi_{r_0}^{2v+1} \left(\frac{r}{r_0}\right)^{2v(n-1)+n} \tag{2}$$

where $2z_0$ is the total height of the gap at radius r_0,

b - equipotential order,

$$\xi_{r_0} = \sum_{v=0}^{\infty} \frac{(-1)^v}{(2v+1)!} \prod_{\ell=0}^{v-1} (n+2\ell)^2 \left(\frac{Z_0}{r_0}\right)^{2v} \tag{3}$$

and the coefficients A_{2v+1} are given by

$$\begin{aligned} A_1 &= 1 , \\ A_3 &= \frac{n^2}{3!} , \\ A_5 &= 3\left(\frac{n^2}{3!}\right)^2 - \frac{n^2}{5!}(n+2)^2 , \ldots \end{aligned} \tag{4}$$

By definition, the ideal pole face is the equipotential of the order 100%(b=1). Inserting this condition into eq.(2), we obtain :

$$z=z_0 \sum_{v=0}^{\infty} A_{2v+1}\, b^{2v+1} \left(\frac{Z_0}{r_0}\right)^{2v} \xi_{r_0}^{2v+1} \left(\frac{r}{r_0}\right)^{2v(n-1)+n} \tag{5}$$

Field index computation by means of an equipotential line

Starting from eq.(1), one obtains

$$n = -\left[r/B_z(r,\theta,0)\right]\left[\partial B_z(r,\theta,0)/\partial r\right] . \tag{6}$$

The field index may be computed by means of an equipotential line, with the formula [4] :

$$n' = \frac{r_0}{h}\,\frac{z_2 - z_1}{z_2 + z_1} \tag{7}$$

where z_1 and z_2 are the ordinates of two points on a reference equipotential line, and 2h is the difference between their abscissae.

This method for computing the field index introduces two errors : an error of method and one of determination. The error of method comes from the use of the approximate eq.(7) instead of the exact one (6). It has the form :

$$\left(\frac{\Delta n}{n}\right)_m = 2\,\frac{\displaystyle\sum_{u=0}^{\infty}\sum_{v=0}^{\infty} \frac{n(v-u)-(v+u)}{(2u+1)!} A_{2v+1}\, b^{2v+1} \xi_{r_0}^{2v+1} \left(\frac{Z_0}{r_0}\right)^{2v} \prod_{j=0}^{2u-1}\left[2v(n-1)+n-j\right]\left(\frac{h}{r_0}\right)^{2u}}{\displaystyle n\sum_{u=0}^{\infty}\sum_{v=0}^{\infty} \frac{1}{(2u)!} A_{2v+1}\, b^{2v+1} \xi_{r_0}^{2v+1} \left(\frac{Z_0}{r_0}\right)^{2v} \prod_{j=0}^{2u-1}\left[2v(n-1)+n-j\right]\left(\frac{h}{r_0}\right)^{2u}} . \tag{8}$$

The error of determination is :

$$\left(\frac{\Delta n}{n}\right)_d = \frac{\Delta r_0}{r_0} + \frac{\Delta h}{h} + \left(\frac{r_0}{nh} - \frac{nh}{r_0}\right)\frac{\Delta z}{z} \tag{9}$$

The eq.(8) may be used for correcting the computed field index values. From eqs.(7) and (8) we obtain the correct field index value

$$n = \frac{r_0}{h}\,\frac{z_2-z_1}{z_2+z_1} - 2\,\frac{\displaystyle\sum_{u=0}^{\infty}\sum_{v=0}^{\infty} \frac{n(v-u)-(v+u)}{(2u+1)!} A_{2v+1}\, b^{2v+1} \xi_{r_0}^{2v+1} \left(\frac{Z_0}{r_0}\right)^{2v} \prod_{j=0}^{2u-1}\left[2v(n-1)+n-j\right]\left(\frac{h}{r_0}\right)^{2u}}{\displaystyle\sum_{u=0}^{\infty}\sum_{v=0}^{\infty} \frac{1}{(2u)!} A_{2v+1}\, b^{2v+1} \xi_{r_0}^{2v+1} \left(\frac{Z_0}{r_0}\right)^{2v} \prod_{j=0}^{2u-1}\left[2v(n-1)+n-j\right]\left(\frac{h}{r_0}\right)^{2u}} . \tag{10}$$

Edge effect determination and correction

The determination and correction for fringing field effects at the two edges of a pole piece, will be treated sepparately for each edge. After that, the two corrected pole piece edges are joined in the central region by the ideal pole face. We consider an edge of an air gap, between two pole pieces, each of them bounded by two semi-infinite planes, the inner planes - pole faces - having an angle 2α between them, and the outer ones being vertical.

Following the idea of Rogowski, these pole pieces will be used as a simple boundary to generate a field distribution fairly identical with the ideal one. The equipotential line of the v^{th} order, obtained by means of the Schwarz - Christoffel differential equation, is given by :

$$x = -\cot g\,\alpha + \frac{g}{\sin\alpha}\,\frac{\displaystyle\sum_{\ell=0}^{\infty} \frac{\prod_{i=0}^{i=\ell}\left(\frac{3}{2}-\frac{\alpha}{\pi}-i\right)}{\ell!\,(\ell+\alpha)}\, e^{u\left(\ell+\frac{\alpha}{\pi}\right)} \cos v\left(\ell+\frac{\alpha}{\pi}\right)}{\displaystyle\sum_{\ell=0}^{\infty}(-1)^{\ell}\,\frac{\prod_{i=0}^{i=\ell}\left(\frac{3}{2}-\frac{\alpha}{\pi}-i\right)}{\ell!\,(\ell+\alpha)}}, \tag{11}$$

$$z = \frac{g}{\sin\alpha}\,\frac{\displaystyle\sum_{\ell=0}^{\infty} \frac{\prod_{i=0}^{i=\ell}\left(\frac{3}{2}-\frac{\alpha}{\pi}-i\right)}{\ell!\,(\ell+\alpha)}\, e^{u\left(\ell+\frac{\alpha}{\pi}\right)} \sin v\left(\ell+\frac{\alpha}{\pi}\right)}{\displaystyle\sum_{\ell=0}^{\infty}(-1)^{\ell}\,\frac{\prod_{i=0}^{i=\ell}\left(\frac{3}{2}-\frac{\alpha}{\pi}-i\right)}{\ell!\,(\ell+\alpha)}}, \tag{12}$$

where x is the abscissa counted from the outer vertical surface of the pole piece and g is the vertical distance between the vertex and the median plane. Eqs.(11) and (12) are written for the edge with greater aperture.

In order to correct for the edge effect, we consider an infinite thin extension of the outer vertical surface of the pole pieces, protruding to the median plane. In this case, using again the Schwarz - Christoffel differential equation, we obtain the equation of the same equipotential line :

$$x = -\cot g\,\alpha + \frac{g}{\sin\alpha}\,\frac{\displaystyle\sum_{\ell=0}^{\infty} \frac{\prod_{i=0}^{i=\ell}\left(\frac{3}{2}-\frac{\alpha}{\pi}-i\right)}{\ell!\left(\ell+\frac{\alpha}{\pi}\right)}\, e^{u\left(\ell+\frac{\alpha}{\pi}\right)} \left[1+(a-1)\,\frac{\frac{1}{2}-\frac{\alpha}{\pi}-\ell}{\frac{1}{2}-\frac{\alpha}{\pi}}\right] \cos v\left(\ell+\frac{\alpha}{\pi}\right)}{\displaystyle\sum_{\ell=0}^{\infty}(-1)^{\ell}\,\frac{\prod_{i=0}^{i=\ell}\left(\frac{3}{2}-\frac{\alpha}{\pi}-i\right)}{\ell!\left(\ell+\frac{\alpha}{\pi}\right)}\left[1+(a-1)\,\frac{\frac{1}{2}-\frac{\alpha}{\pi}-\ell}{\frac{1}{2}-\frac{\alpha}{\pi}}\right]}, \tag{13}$$

$$z = \frac{g}{\sin\alpha}\,\frac{\displaystyle\sum_{\ell=0}^{\infty} \frac{\prod_{i=0}^{i=\ell}\left(\frac{3}{2}-\frac{\alpha}{\pi}-i\right)}{\ell!\left(\ell+\frac{\alpha}{\pi}\right)}\, e^{u\left(\ell+\frac{\alpha}{\pi}\right)} \left[1+(a-1)\,\frac{\frac{1}{2}-\frac{\alpha}{\pi}-\ell}{\frac{1}{2}-\frac{\alpha}{\pi}}\right] \sin v\left(\ell+\frac{\alpha}{\pi}\right)}{\displaystyle\sum_{\ell=0}^{\infty}(-1)\,\frac{\prod_{i=0}^{i=\ell}\left(\frac{3}{2}-\frac{\alpha}{\pi}-i\right)}{\ell!\left(\ell+\frac{\alpha}{\pi}\right)}\left[1+(a-1)\,\frac{\frac{1}{2}-\frac{\alpha}{\pi}-\ell}{\frac{1}{2}-\frac{\alpha}{\pi}}\right]} \tag{14}$$

where the coefficient a is choosen according to the length of the vertical extension.

Practical application

Let us consider, as a example, a strong focusing 3oo-GeV synchrotron with the following basic parameters[6] :

$$\frac{n}{\rho} = 6\ \text{m}^{-1}$$

$$\rho = 840\ \text{m}$$

$$2z_o = 70\ \text{mm}$$

inside vacuum chamber dimensions = 55 x 9o mm.

Ideal equipotential lines

Starting from eq.(5) the ideal pole face equation is, within an error $\Delta z < 5.10^{-6}$ mm in the region 839,9oo $< r <$ 84o,1oo mm

$$z = 34.74331700\left(\frac{r}{840000}\right)^{5040} + 2.51631542\cdot 10^{-1}\left(\frac{r}{840000}\right)^{15118} + 4.92021786\cdot 10^{-3}\left(\frac{r}{840000}\right)^{25196} + 1{,}27254936.10^{-4}\left(\frac{r}{840000}\right)^{35274} + 3.76306312\cdot 10^{-6}\left(\frac{r}{840000}\right)^{45352} \text{mm} \tag{15}$$

The reference ideal equipotential line, obtained for b=o.1 from eq.(2) is, within an error $\Delta z < 5.10^{-7}$mm and in the same region,

$$Z=3.47433170\left(\frac{r}{840000}\right)^{5040}+2.51631542\cdot 10^{-4}\left(\frac{r}{840000}\right)^{15118}+4.92021786\cdot 10^{-8}\left(\frac{r}{840000}\right)^{25196} \text{mm} \quad (16)$$

The field index distribution, given in table 1 is obtained by means of eq.(1o), where the values for z_1 and z_2 are computed from eq.(16) and h = 5 mm. In order to obtain a relative error $\left(\frac{\Delta n}{n}\right)_d \leq 5.10^{-4}$, we must consider $\Delta r \leq 5.10^{-5}$mm, as shown by eq.(9).

The values listed in table 1 show that the ideal pole face and equipotential lines satisfy the condition for an accurate field index distribution.

Table 1

x	n	$\frac{\Delta n}{n}\cdot 10^4$	x	n	$\frac{\Delta n}{n}\cdot 10^4$	x	n	$\frac{\Delta n}{n}\cdot 10^4$
-70	5039.50	-1	-20	5038.55	-2.9	+30	5040.32	+0.6
-65	5040.42	+0.8	-15	5040.44	+0.9	+35	5038.27	-3.4
-60	5040.40	+0.8	-10	5040.25	+0.5	+40	5039.52	-1
-55	5039.35	-1.3	-5	5040.70	+1.4	+45	5040.04	-0.1
-50	5040.50	+1	0	5039.10	-1.4	+50	5039.52	-1
-45	5037.25	-5.5	+5	5040.22	+0.4	+55	5040	0
-40	5038.86	-2.3	+10	5041.44	+2.9	+60	5040.90	+1.8
-35	5042.04	-4	+15	5040.37	+1.3	+65	5040.90	+1.8
-30	5040.53	+1	+20	5039.26	-1.5	+70	5039.65	-0.7
-25	5037.60	-4.8	+25	5040	0			

Edge effect determination and correction

Choosing a value for α = o.27577861 rad, we obtain from eqs.(11) and (12),

$$x=-3.53370114\,g+0.23851107\,g\sum_{\ell=0}^{\infty}\frac{\prod_{i=0}^{i=\ell}\left(\frac{3}{2}-\frac{\alpha}{\pi}-i\right)}{\ell!\left(\ell+\frac{\alpha}{\pi}\right)}e^{u\left(\ell+\frac{\alpha}{\pi}\right)}\cos v\left(\ell+\frac{\alpha}{\pi}\right), \quad (17)$$

$$z=0.23851107\,g\sum_{\ell=0}^{\infty}\frac{\prod_{i=0}^{i=\ell}\left(\frac{3}{2}-\frac{\alpha}{\pi}-i\right)}{\ell!\left(\ell+\frac{\alpha}{\pi}\right)}e^{u\left(\ell+\frac{\alpha}{\pi}\right)}\sin v\left(\ell+\frac{\alpha}{\pi}\right), \quad (18)$$

representing the equipotential lines modified by the edge effect.

We need a value for g so as the corresponding 8o% equipotential line, computed with eqs.(17) and (18), be coincident in the largest possible region with the ideal pole face. For g = 61.831837 mm, we find that the two curves are fairly coincident in the region 84o,ooo < r < 84o,o55 mm.

The reference equipotential line, computed from eqs.(17) and (18), - in this case for 8% -, is shifted with respect to the reference ideal equipotential of 1o%, computed from eq.(16). In order to make them coincide in the largest possible region, a coefficient a=1.4 is choosen. The corrected equipotential set is obtained from eqs.(13) and (14)

$$x=-3.53370114\,g+0.16265117\,g\sum_{\ell=0}^{\infty}\frac{\prod_{i=0}^{i=\ell}\left(\frac{3}{2}-\frac{\alpha}{\pi}-i\right)}{\ell!\left(\ell+\frac{\alpha}{\pi}\right)}\left[1+0.4\,\frac{\frac{1}{2}-\frac{\alpha}{\pi}-\ell}{\frac{1}{2}-\frac{\alpha}{\pi}}\right]e^{u\left(\ell+\frac{\alpha}{\pi}\right)}\cos v\left(\ell+\frac{\alpha}{\pi}\right), \quad (19)$$

$$z=0.16265117\,g\sum_{\ell=0}^{\infty}\frac{\prod_{i=0}^{i=\ell}\left(\frac{3}{2}-\frac{\alpha}{\pi}-i\right)}{\ell!\left(\ell+\frac{\alpha}{\pi}\right)}\left[1+0.4\,\frac{\frac{1}{2}-\frac{\alpha}{\pi}-\ell}{\frac{1}{2}-\frac{\alpha}{\pi}}\right]e^{u\left(\ell+\frac{\alpha}{\pi}\right)}\sin v\left(\ell+\frac{\alpha}{\pi}\right). \quad (2o)$$

The corrected reference 8% equipotential obtained from eqs.(19) and (2o) coincide with the reference ideal 1o% equipotential in the region

840,000 $< r <$ 840,050, the differences being smaller than 10^{-3}mm. With the same eqs.(19) and (20), the 80% equipotential line is computed; it represents the corrected shape of the pole face.

The same method works also for the smaller aperture edge. Joining the results, we obtain the coordinates of the corrected pole face, as listed in table 2.

We note that in the region 839,970 $< r <$ 840,020, the corrected pole face coincides with the ideal pole face.

The above computations were checked by means of an electrolytic tank. Within the errors of the experimental set-up, which were of $\pm$ 0.1 ... 0.2 mm, a good agreement was found between the experimental equipotential lines, including the edge effect, and the ideal and corrected equipotential lines.

Table 2

$r-r_0$	z	$r-r_0$	z	$r-r_0$	z
-70	26	-42.3492	26.6134	+21.4277	39.8597
-66.4842	24.9427	-37.1588	27.7015	+21.2901	41.6846
-63.5528	23.9309	-31.7018	28.8776	+37.4413	43.6108
-60.1041	23.7916	-20	30.9929	+45.8451	45.6399
-56.1888	24.1424	-10	32.9340	+54.3985	47.7602
-51.8901	24.7895	0	35	+62.5121	49.7861
-47.2656	25.6320	+10	37.1998	+75	58
		+20	39.5428		

A final check, of the high accuracy required, may be done only by computing the field index by means of the potential network.[1]

References

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