SIMULATION STUDY OF THE HORIZONTAL HEAD-TAIL INSTABILITY OBSERVED AT INJECTION OF THE CERN PROTON SYNCHROTRON

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Abstract
For many years, a horizontal head-tail instability has been observed at the CERN Proton Synchrotron during the long 1.2 s injection flat-bottom. This slow instability has been damped using linear coupling only, i.e. with neither octupoles nor feedbacks. Using the nominal machine and beam parameters for LHC, the sixth head-tail mode number is usually observed. Several other modes were also observed in the past by tuning the chromaticity, and these observations were found to be in good agreement with Sacherer’s formula. The purpose of this paper is to present the results of assessing the effect of chromaticity and linear coupling on this slow head-tail instability using the HEADTAIL simulation code, and to compare these simulations with both measurements performed over the last few years, and theoretical calculations.

INTRODUCTION
Linear coupling was proposed as a third method for beam stabilization (in addition to Landau damping & feedbacks) in Ref. [1]. Some aspects of the theoretical predictions were benchmarked against measurements in Ref. [2], and since then the CERN Proton Synchrotron (PS) beam for LHC was stabilized at injection (1.4 kinetic) energy by linear coupling only, i.e. with neither octupoles nor feedbacks. This method proved to be very robust and reproducible over the years. The purpose of the present paper is to try and deepen our understanding of the phenomenon using the HEADTAIL code, which simulates single-bunch phenomena [3].

The measurements performed in the CERN PS are first reviewed in Section 1. A comparison between Sacherer’s analytical formula to compute the (slow) head-tail instabilities [4] and the HEADTAIL code vs. chromaticity is then shown in Section 2, assuming a horizontal impedance ~ 16 times larger than the real one to reduce the simulation time. Note that the driving impedance considered in this paper is the resistive-wall impedance induced by the elliptical stainless steel vacuum pipe. Full-scale simulations were performed to investigate the effect of linear coupling on the head-tail instability, which are presented in Section 3. It is worth mentioning that space charge has been neglected in this analysis. Even though the (small amplitudes) space charge tune shift is quite high (~ 0.2), the effect of space charge is believed to be small. This aspect will be analyzed in detail in the future.

MEASUREMENTS IN THE CERN PS
When the current of the skew quadrupoles in the PS is set to have the minimum of linear coupling between the transverse planes, i.e. \( I_{\text{skew}} = 0.33 \, \text{A} \) (see Fig. 1(b)), with the working point \( Q_x = 6.22, \, Q_y = 6.25 \), a head-tail instability develops, usually with mode \(|m| = 6\) (see Fig. 2, exhibiting 6 nodes). Typical beam losses due to this instability are shown in Fig. 3(b). Note that the beam from the PSBooster (PSB) is injected into the PS in 2 batches, the first of 4 bunches and the second of 2, to...
overcome space charge effects in the PSB. The drawback of this filling scheme is that the bunches of the first batch (injected at 170 ms in Fig. 3) have to wait 1.2 s at 1.4 GeV kinetic energy, and during that time about 75% of the beam is lost due to the above instability, if no counter measures are taken. In the presence of coupling, the beam losses can be removed as observed in Fig. 3(a), for $I_{\text{skew}} = -0.4$ A, corresponding to a “closest-tune approach” of $\sim 0.05$.

The natural (relative) chromaticities were measured in the past to be $\xi_x = -0.9$ and $\xi_y = -1.3$ [5]. Different head-tail modes were also observed when the horizontal chromaticity was scanned, as seen in Fig. 4.

Figure 4: Measured $\Delta R$ signals from a radial beam-position monitor during 20 consecutive turns, in the PS with minimum coupling [5]: (a) $\xi_x = -0.5$, (b) $\xi_x = -0.7$, (c) $\xi_x = -1.1$, (d) $\xi_x = -1.2$, (e) $\xi_x = -1.3$. Time scale: 20 ns/div.

### EFFECT OF CHROMATICITY

As a first step, the HEADTAIL code has been benchmarked against Sacherer’s analytical formula for the (slow) head-tail instabilities using the PS and beam parameters reported in Table 1, but using for the length of the resistive-wall impedance 10000 m instead of the circumference (628 m) to reduce the computation time by $\sim 16$. The results compared to Sacherer’s predictions using a parabolic bunch are shown in Fig. 5. It is found that a good agreement is obtained as concerns both the instability rise-times and the mode numbers, except for $\xi_x = -0.1$, where the head-tail mode $m = 0$ is found to be unstable in the simulation (even for 0 chromaticity). This is explained by the fact that the threshold for the Transverse Mode–Coupling Instability was reached for the lower values of chromaticity, due to the artificially increased impedance. A relatively good agreement is also obtained for the real part of the tunes. The difference might be explained by the fact that only the dipolar impedance was considered in the analytical computation (with the correct Yokoya factor), whereas the detuning impedance was also included in the HEADTAIL simulations. Note that using a Gaussian bunch [6] the rise-times and mode numbers are very different for large chromaticities (see Fig. 6), which indicate that for high-order head-tail modes a parabolic bunch should be preferred when applying Sacherer’s formula, even if the transverse bunch profile is Gaussian.

Figure 5: Comparison of the instability rise-times (left) and real part of the tunes (right) with the associated mode number $|m|$ between Sacherer’s theory (using a parabolic bunch) and HEADTAIL simulations vs. chromaticity.

Figure 6: Instability rise-times (with the associated mode number $|m|$) vs. chromaticity predicted from Sacherer’s formula using a Gaussian bunch.

As in the measurements reported in Fig. 4, different head-tail modes are observed when the horizontal chromaticity is varied (see Fig. 7). Note that in the simulation, the beam goes through the centre of the beam position monitor, whereas in the measurements the (stable) beam had an offset. This explains why the unstable motion was measured on top of a Gaussian-like profile.

### Table 1: Basic beam and PS parameters relevant for this simulation study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
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<tr>
<td>Circumference</td>
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<td># of bunches</td>
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<td>Relativistic $\gamma$</td>
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<td># of protons / bunch</td>
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<td>Horiz. tune</td>
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</tr>
<tr>
<td>Vert. tune</td>
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</tr>
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<tr>
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<td>m</td>
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<td>Rms long. mom. spread</td>
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<tr>
<td>Cavity harmonic number</td>
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<tr>
<td>Mom. compaction factor</td>
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<td>mm</td>
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<tr>
<td>Beam pipe resistivity</td>
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<td>$\Omega$m</td>
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FRPMN074
Figure 7: Examples of head-tail modes observed in the HEADTAIL simulations, when superimposing every 10 turns of the 20000 simulated turns at a beam position monitor, for various horizontal chromaticities.

EFFECT OF LINEAR COUPLING

The effect of linear coupling on the beam stability has been studied for the particular case of (relative) chromaticities $\xi_x = -0.5$ and $\xi_y = -1$. These chromaticities have been chosen to clearly reveal the effect predicted in Ref. [1], i.e. the sharing of the transverse instability growth rates, which heavily depends on chromaticities. The real case of chromaticities (which need to be precisely re-measured) of $\xi_x = -0.9$ and $\xi_y = -1.3$ will be simulated in the future as it requires more CPU time due to the large vertical chromaticity.

The intensity vs. time is shown in Fig. 8, whereas the evolution of both horizontal and vertical centroid motions are depicted in Fig. 9 (upper plots). It is shown in Fig. 8 that without linear coupling about 60% of the beam is lost after 500000 turns, i.e. ~1.1 s as the revolution period is ~2.3 $\mu$s, and that for the highest normalized integrated coupling strengths ($K \geq 0.012$ m$^{-1}$, corresponding to a “closest-tune approach” $\geq 0.03$) no beam losses are observed. The simulated loss pattern is very similar to the measured one of Fig. 3(b). The evolution of both horizontal and vertical rms beam sizes are shown in Fig. 9 (lower plots), revealing that when the beam is stabilised no significant emittance blow-up is taking place.

Figure 8: Simulated bunch intensity vs. PS turns for various integrated normalized coupling strengths.

Figure 9: (a) Evolution of both horizontal (upper left) and vertical (upper right) centroid motions vs. PS turns, and (b) evolution of both horizontal (lower left) and vertical (lower right) rms beam sizes. The curves are plotted on top of each other starting from the case without coupling in blue ($K = 0$) and ending with the case with the highest coupling ($K = 0.016$ m$^{-1}$). For the highest coupling strengths all the curves are almost flat.

CONCLUSION

The HEADTAIL simulation code has been benchmarked against Sacherer’s formula in the case of the CERN PS low-energy horizontal resistive-wall instability (artificially increasing the impedance and therefore decreasing the simulation time by ~16) for various chromaticities. A good agreement was revealed when a parabolic bunch was used for the analytical computations, whereas a poor agreement was obtained for the higher-order head-tail modes using a Gaussian bunch.

Full-scale HEADTAIL simulations during ~1.1 s also revealed the possibility to stabilise the beam by linear coupling (only) when an asymmetry between the two transverse planes is introduced through chromaticities, as predicted in Ref. [1]. The simulated case used $\xi_x = -0.5$ and $\xi_y = -1$, whereas the measured chromaticities [5] are $\xi_x = -0.9$ and $\xi_y = -1.3$. The next steps will consist to re-measure precisely the chromaticities and simulate the real case, introducing also space charge.

REFERENCES