Absolute Bunch Length Measurements at the ALS by Incoherent Synchrotron Radiation Fluctuation Analysis

Fernando Sannibale

Lawrence Berkeley National Laboratory

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Credits

Based on the method described in Zolotorev, Stupakov, SLAC-PUB 7132 (1996)

The team for the ALS experiment:
Fernando Sannibale, Max Zolotorev (LBNL),
Daniele Filippetto (INFN-LNF),
Gennady Stupakov (SLAC).

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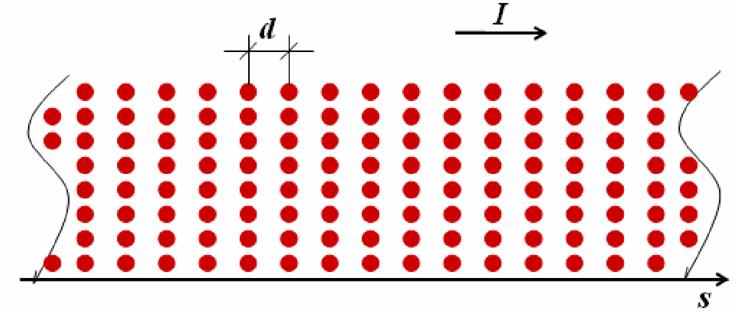
Incoherent Radiation from Charged Particles

Moving charged particles can radiate photons by synchrotron radiation, Cerenkov radiation, transition radiation, etc. For all such processes, the incoherent component of the radiation is due to the random distribution of the particles along the beam.

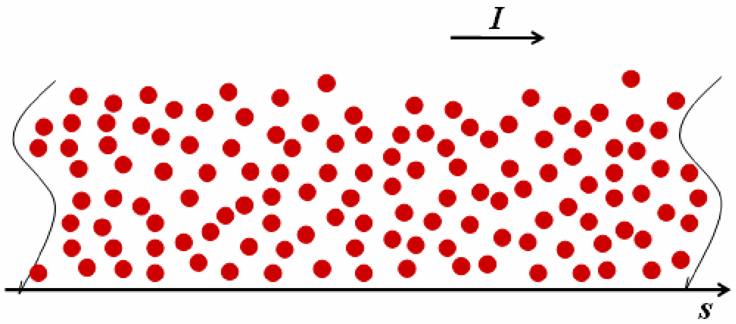
Example: "Ideal" coasting beam moving on a circular trajectory with the particles equally separated by a longitudinal distance d :



No synchrotron radiation emission for frequencies with $\lambda > d$.

The interference between the radiation emitted by the evenly distributed electrons produces a vanishing net electric field.

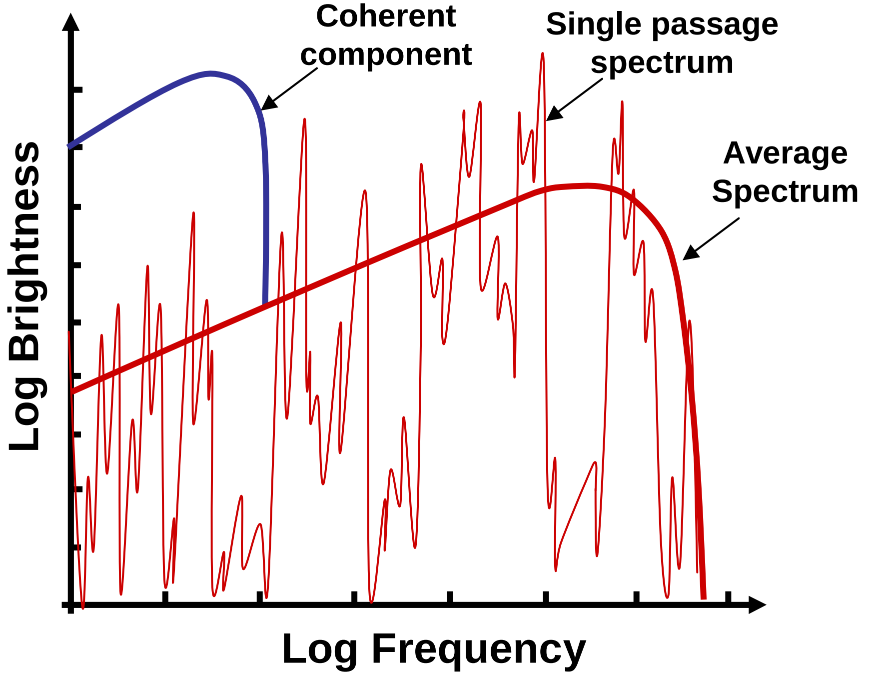
In a more realistic coasting beam, the particles are randomly distributed causing a small modulation of the beam current. The interference is not fully destructive anymore and the beam radiates also at longer wavelengths.



Radiation Fluctuations

If the particle turn by turn position along the beam changes (longitudinal dispersion, path length dependence on transverse position), the current modulation changes and the radiated energy and its spectrum fluctuate turn by turn.

By averaging over multiple passages, the measured spectrum converges to the characteristic incoherent spectrum of the radiation process under observation. (synchrotron radiation in the example).



In the case of bunched beams, a strong coherent component at those wavelengths comparable or longer than the bunch length shows up But the higher frequency part of the spectrum remains unmodified.

More Quantitatively...

The electric field associated with the radiation emitted by the beam at the time *t* is:

$$E(t)=\sum_{k=1}^{N}e(t-t_k)$$

where *e* is the electric field of the electromagnetic pulse radiated by a single particle and t_k is the randomly distributed arrival time of the particle (Poisson process).

In the frequency domain:

$$\hat{E}(\omega)=\int_{-\infty}^{\infty}E(t)\,e^{i\omega t}\,dt=\hat{e}(\omega)\sum_{k=1}^{N}e^{i\omega t_k}$$

And for the radiated power per passage:

$$\boxed{P(\omega)\propto\left|\hat{E}(\omega)\right|^2=\left|\hat{e}(\omega)\right|^2\sum_{k,l=1}^{N}e^{i\omega(t_k-t_l)}}$$

The previous quantity fluctuates passage to passage, and the average radiated power from a beam with normalized distribution *f (t)* is:

$$\left\langle P(\omega)\right\rangle\propto\left|\hat{e}(\omega)\right|^2\sum_{k,l=1}^{N}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dt_k\,dt_l\,f(t_k)f(t_l)e^{i\omega(t_k-t_l)}=\left|\hat{e}(\omega)\right|^2\left[\underbrace{N}_{\text{Incoherent term}}+\underbrace{N(N-1)\left|\hat{f}(\omega)\right|^2}_{\text{Coherent term}}\right]$$

More Quantitatively...

The energy *W* radiated per passage by incoherent radiation can be obtained by integrating *P* over ω neglecting the coherent contribution. It can be shown that the relative variance for *W* is given by:

$$\delta^2 = \frac{\sigma_W^2}{\langle W \rangle^2} = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\hat{e}(\omega)|^2 |\hat{e}(\omega')|^2 \left|\hat{f}(\omega-\omega')\right|^2 d\omega d\omega'}{\left(\int_{-\infty}^{\infty} |\hat{e}(\omega)|^2 d\omega\right)^2}$$

The shape of $e(\omega)$ is defined by the radiation mechanism properties or by the frequency acceptance of the system used for the measure of δ.

If we use a bandpass filter with gaussian transmission curve with rms bandwidth σ_ω and the bunch is gaussian with rms length in time units σ_τ, we can integrate the above expression and obtain:

$$\boxed{\delta^2 = 1\Big/\sqrt{1+4\sigma_\tau^2\sigma_\omega^2}}$$

Possibility of absolute bunch length measurements !

A Simple Physical Interpretation

For $\sigma_\tau >> 1/2\sigma_\omega$

$$\delta^2 \cong \frac{1}{2\sigma_\tau \sigma_\omega}$$

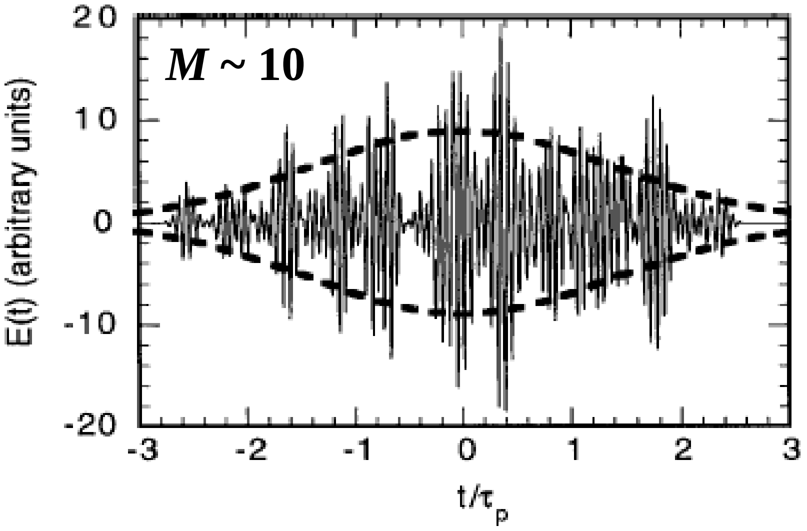
When the bandwidth σ_ω, is fixed, the uncertainty principle defines the coherence length σ_{tc}. For the gaussian case:

$$\sigma_{t_C}\sigma_\omega = \frac{1}{2}$$

The electric field of photons radiated within the coherence length σ_{tc} and within the bandwidth σ_ω adds coherently. σ_{tc} defines a radiation "mode".

$$\boxed{\delta^2 \cong \frac{\sigma_{t_C}}{\sigma_\tau} = \frac{1}{M}}$$

The previous equation shows that in a bunch of length σ_τ, there are $M = \sigma_\tau / \sigma_{tc}$ independent modes radiating simultaneously within the bandwidth σ_ω.



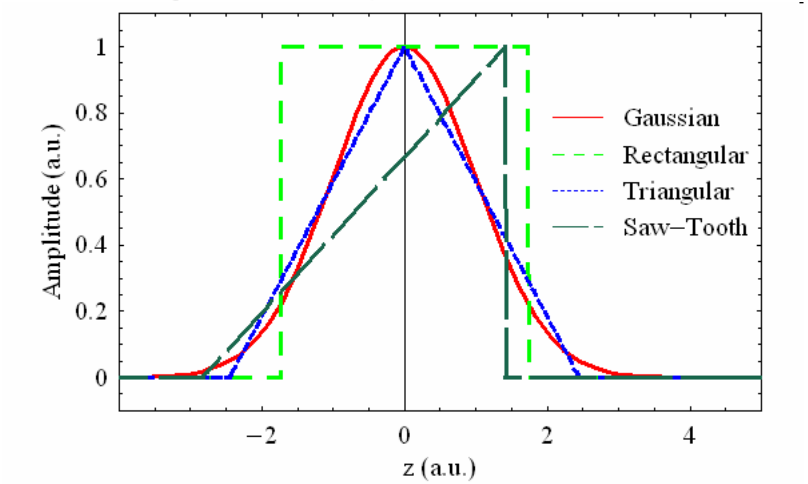
Each mode shows 100% intensity fluctuation, and the variance of the combined intensity scales as $1/M$ (M combined Poisson processes).

Dependence on Longitudinal Distribution

The previous expressions have been obtained for gaussian beams. In the general case:

$$\sigma_\tau = F_{Filter}\frac{F_{Dist.}}{\delta^2}$$

The filter form factor can be measured but the bunch longitudinal distribution is generally unknown.



Distribution	Form Factor ($F_{Dist.}$)	Error Assuming Gaussian
Gaussian	$\frac{1}{2\sqrt{\pi}} \cong 0.2821$	**0.0 %**
Rectangular	$\frac{1}{2\sqrt{3}} \cong 0.2887$	**-2.3 %**
Triangular	$\sqrt{\frac{2}{27}} \cong 0.2722$	**+3.6 %**
Saw-Tooth	$\frac{4}{3\sqrt{18}} \cong 0.3143$	**-10.2 %**

The table shows that by using the expression for gaussian beams for different distributions, the consequent error is at the few % level for most cases, as long as the distributions are represented by their rms length and do not include microstructures with characteristic length << σ_τ.

Transverse Beam Size Effects

In the previous derivation, a beam with no transverse size was assumed.

Analogously to the longitudinal case, the finite transverse size introduces additional independently radiating transverse modes (M_x, M_y). The resulting intensity fluctuation variance becomes:

$$\delta^2 \approx \frac{1}{M} \times \frac{1}{M_x} \times \frac{1}{M_y}$$

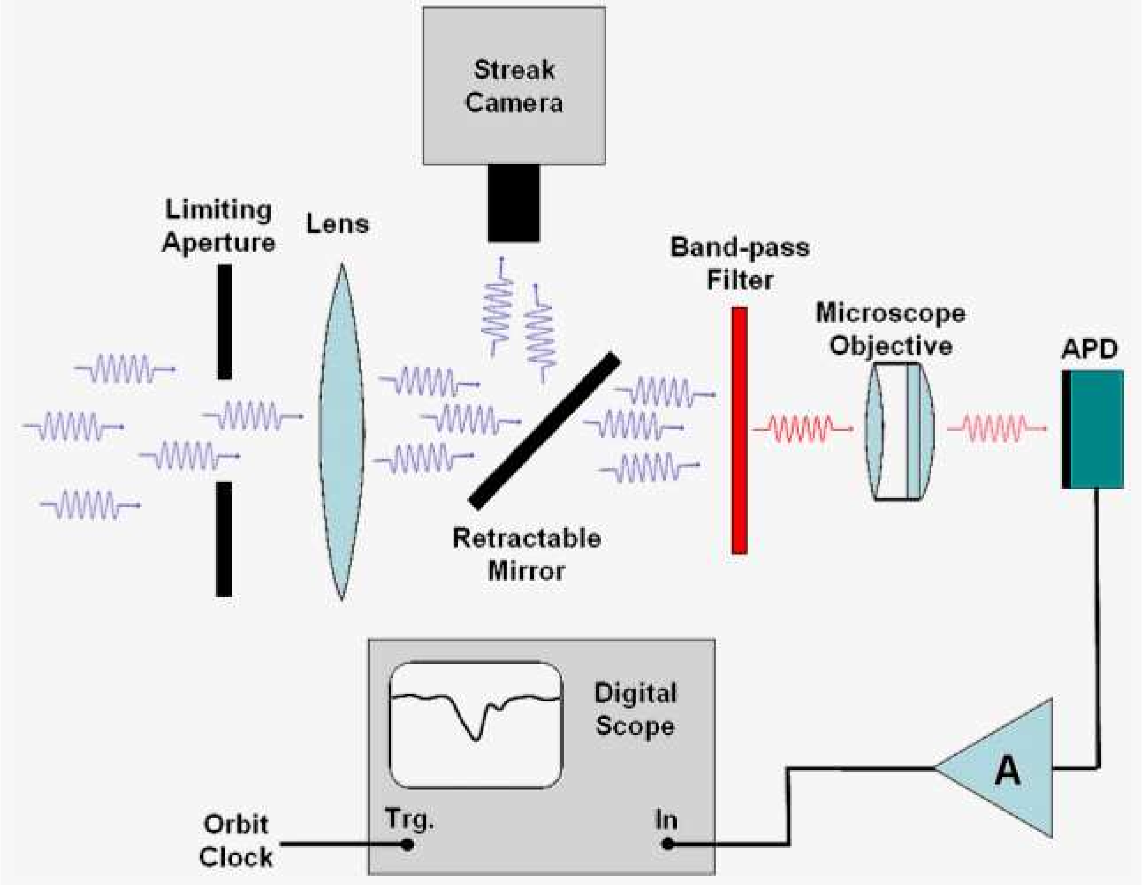
For example, for the full gaussian case one obtains (with σ_x and σ_y the rms transverse beam sizes):

$$\delta^2 = \left(1+\sigma_\tau^2/\sigma_{tc}^2\right)^{-\frac{1}{2}}\left(1+\sigma_x^2/\sigma_{xc}^2\right)^{-\frac{1}{2}}\left(1+\sigma_y^2/\sigma_{yc}^2\right)^{-\frac{1}{2}}$$

The transverse coherence lengths σ_{xc} and σ_{yc} are defined by the radiation mechanism and include diffraction effects introduced by any limiting apertures.

σ_{xc} and σ_{yc} can be analytically calculated in the simpler cases or numerically evaluated (SRW, ...)

The Tests at BL7.2 of the ALS



BL7.2 collects the synchrotron radiation from a dipole magnet.

The limiting apertures were defined by the beamline acceptance 5.5/2.8 mrad (H/V)

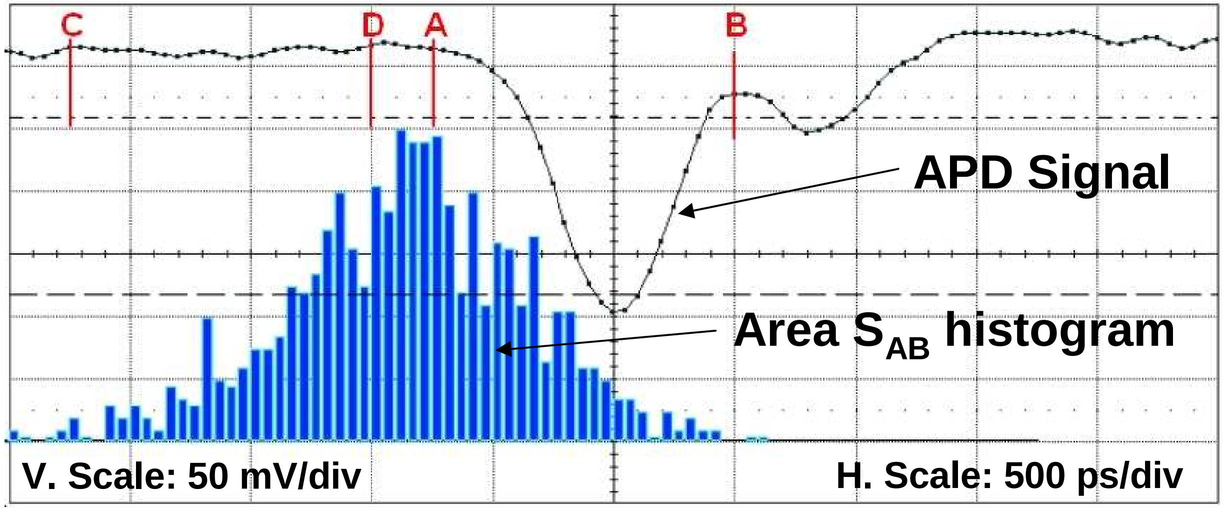
BP filter: gaussian filter 632.8 nm, 1nm FWHM

The signal from the avalanche photo-diode (APD) was amplified and sent to a digital scope for data acquisition and analysis. (LeCroy Wavepro 7300 A)

The setup allowed for comparison with streak camera measurements.

Data Acquisition & Analysis

The scope was set to measure the areas of the signal between the points A and B (S_{AB}) and between C and D (S_{CD}), and their statistical moments.



S_{AB} is proportional to the pulse energy convoluted with some electronic noise. S_{CD} is a measure of such a noise.

$$\Rightarrow \delta_M^2 = \frac{\sigma_{S_{AB}}^2 - \sigma_{S_{CD}}^2}{\left(\langle S_{AB}\rangle - \langle S_{CD}\rangle\right)^2}$$

A complete 5 ksample measurement required ~ 1 minute

Quantum Noise Effects

The number of photons impinging on the APD is finite. Additionally, APDs exploit stochastic processes for the photon-to-electron conversion and for the amplification.

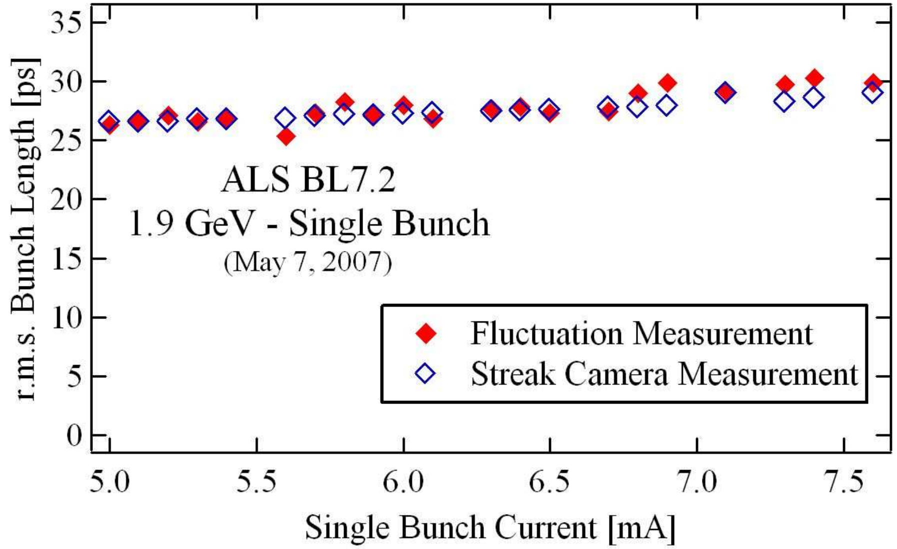
All this effects generate extra-fluctuations (shot noise) that need to be accounted.

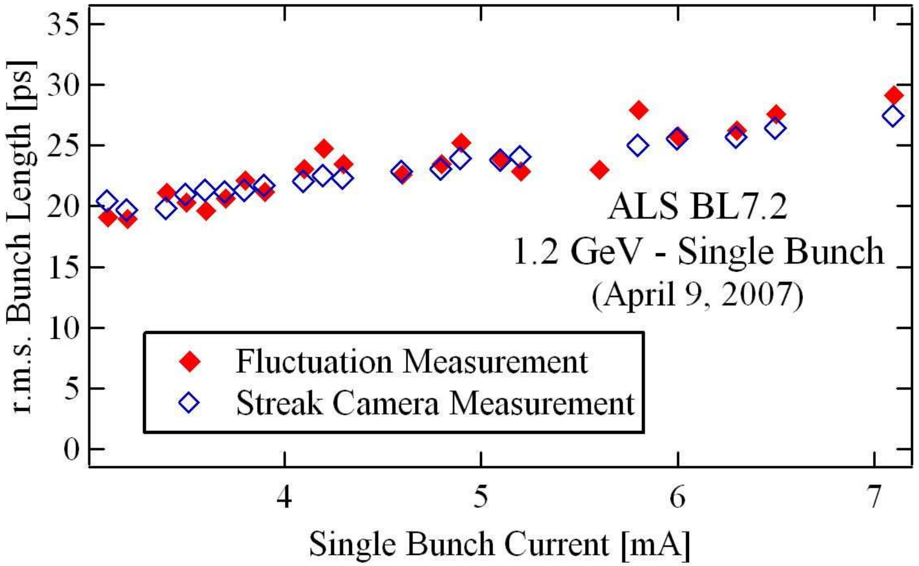
$$\sigma_\tau^2 = \frac{1}{4\sigma_\omega^2}\left[\left(\delta_M^2 - \kappa^2\right)^{-2}\left(1+\frac{\sigma_x^2}{\sigma_{xc}^2}\right)^{-1}\left(1+\frac{\sigma_y^2}{\sigma_{yc}^2}\right)^{-1} - 1\right]$$

The term κ represents the total photon shot noise and accounts for all the terms above mentioned.

κ needs to be measured once forever, and this can be easily done by performing 2 or more measurements of δ_M^2 for the same bunch length for different number of photons impinging on the APD (using neutral density filters for instance).

A Sample of the ALS Results





Remarkably good agreement with streak-camera data.

No parameter has been adjusted to fit the data.

It can be shown that the statistical contribution to the error is given by (~ 2% with 5 ksamples):

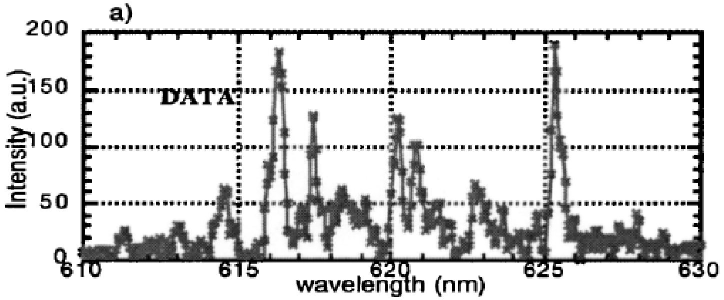
$$\frac{\Delta\sigma_\tau}{\sigma_\tau}=\sqrt{\frac{2}{N_{Samples}}}$$

The typical rms difference between the streak camera and the fluctuation data was ~ 4 %. The extra error is probably associated with the shot noise term that in our measurements was comparable to δ^2.

Frequency Domain Applications

Frequency domain versions of the method have been already successfully used.

They require a more complex scheme with a photon spectrometer, but potentially allow for single shot measurements.



Catravas. ATF measurement

P. Catravas et al., PRL 82, Number 26, June 99

First measurement using the spectrometer technique with undulator radiation at ATF.

V. Sajaev, EPAC 2000, p. 1806

V. Sajaev, BIW 04 p. 74

Measurements using the spectrometer technique with undulator radiation at LEUTL, Argonne.
Phase retrieval techniques.

M. Yabashi et al., PRL 97, 084802 (2006)

First measurement using X-rays.
Important for SASE applications!

Conclusions

We have demonstrated an absolute bunch length measurement technique based on the analysis of the fluctuations in the incoherent part of the radiation emitted by a particle beam.

The scheme is non-destructive, shows a remarkable simplicity and can be applied in both circular and linear accelerators including cases where the very short length of the bunches makes difficult the use of other techniques.

Future Upgrades

By splitting the signal in two branches with filters with the same bandwidth but with different central frequencies, it is possible to discriminate between the transverse and longitudinal fluctuation contributions by exploiting the fact that the longitudinal term depends only on the bandwidth while the transverse ones depend only on the central wavelength. Such a capability allows removing the dependence on the transverse plane and can be useful when the transverse beam size changes during operation.

We want also test the system by coupling the light from the source into an optical fiber. This will allow having the measurement setup separated from the source area for an easy accessibility and tuning of the system.