INSTABILITIES OF COOLED ANTIPROTON BEAM IN RECYCLER *

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Abstract

The more beam is cooled, the less stable it is. In the 3.3 km Recycler Ring, stacked 8 GeV antiprotons are cooled both with stochastic (transversely) and electron (3D) cooling. Since the machine is staying near the coupling resonance, coupled optical functions should be used for stability analysis. To stabilize beam against the resistive wall instability, a digital damper is used. Digital dampers can be described as linear operators with explicit time dependence, and that makes a principle difference with analogous dampers. Theoretical description of the digital dampers is presented. Electron cooling makes possible a two-beam instability of the cooled beam with the electron beam. Special features of this instability are described, and the remedy is discussed.

INTRODUCTION

Analysis of transverse coherent instabilities in the Recycler forced us to resolve three theoretical problems, all of them being rather general. Here these problems with our solutions are described.

The first one relates to transverse coherent instabilities near the coupling resonance, $\{\nu_x\} = \{\nu_y\}$. As many machines, the Recycler operates in its vicinity. As a result, single-particle motion is coupled, and the conventional optical formalism can be not valid. This problem was considered by many authors, most extensively in Refs. [1], [2], [3]. Here, we suggest a solution, which is general and simple at the same time [4]. The leading idea is to use canonical coordinates and momenta associated with the optical eigenmodes. In this basis, beam motion gets to be uncoupled, and formally similar to conventional $x-y$ uncoupled case. To solve the problem in this way, the wakes (impedances) have to be properly projected on the optical eigenvectors. As a result, the coupled problem is effectively reduced to an uncoupled one, making the two problems identical - for any strength of coupling, any sort of bunching, any wake function, any space charge, etc.

The second problem addressed here relates to digital dampers. Due to periodical digitizing with time $\tau \equiv 1/f_s$, digital dampers are described as linear operators with explicit time dependence. Thus, a single frequency f at the entrance is transformed to an array of frequencies at the exit $f + nf_s$, $n = 0, \pm1, \pm2, \ldots$. As a result, coasting beam eigenmodes are linear superpositions of these combined frequencies, which is significant for high-frequency perturbations, $f \geq f_s/4$. Implementation of a digital damper in the Recycler [5] allowed to increase longitudinal phase space density several times, see more in Ref. [6].

The last problem discussed here relates to a possibility of coherent transverse two-beam instability between the cooled circulating antiproton beam and a cooling single-pass electron beam. This instability is discussed since early 90's [8], [9], but its understanding was not sufficient. Some observations rose a question about a possibility of this instability at the Recycler [10].

Since velocities of the two beams are identical, their interaction is local. Another important feature of this interaction is its skew character due to the magnetic field in the cooler. As a result, without $x-y$ coupling of antiproton optical modes, electron feedback to antiproton oscillations is insensible for antiprotons. In more details this issue is discussed in [11].

TRANSVERSE COUPLING: SUBSTITUTION RULES

For arbitrary coupling, the beam optics can be described in terms of 4D eigenvectors. Hereafter, a parametrization suggested in [12] is used, where the 4 eigenvectors $\mathbf{V}_1,\ \mathbf{V}_{-1} \equiv \mathbf{V}_1^*,\ \mathbf{V}_2,\ \mathbf{V}_{-2} \equiv \mathbf{V}_2^*$ of a revolution matrix $\mathbf{R}$ are presented as follows:

$$\mathbf{V}_1 = \left(\sqrt{\beta_{1x}}\,, -\frac{i(1-u)+\alpha_{1x}}{\sqrt{\beta_{1x}}}\,, \sqrt{\beta_{1y}}e^{i\nu_1}\,, -\frac{iu+\alpha_{1y}}{\sqrt{\beta_{1y}}}e^{i\nu_1} \right)$$
$$\mathbf{V}_2 = \left(\sqrt{\beta_{2x}}e^{i\nu_2}\,, -\frac{iu+\alpha_{2x}}{\sqrt{\beta_{2x}}}e^{i\nu_2}\,, \sqrt{\beta_{2y}}\,, -\frac{i(1-u)+\alpha_{2y}}{\sqrt{\beta_{2y}}} \right) \quad (1)$$

with $\mathbf{R}\cdot\mathbf{V}_m = \exp(-i\mu_m)\mathbf{V}_m$. Components of the 4D vectors are transverse coordinates and angles, $(x, \theta_x, y, \theta_y)$; in case of non-zero longitudinal magnetic field, the angles are modified according to a conventional rule for the canonical momenta. Eigenvector parameters β_{1x}, β_{2y}, etc. are determined by the machine optics. The symplecticity requires then a specific orthogonality

$$\mathbf{V}_m^+ \cdot \mathbf{U} \cdot \mathbf{V}_n = -2i\delta_{mn}\mathrm{sgn}(m)\,; \quad (2)$$

where a superscript $^+$ means Hermite-conjugation, δ_{mn} is the Kronecker symbol, $\mathrm{sgn}(m)$ is the sign function, and $\mathbf{U}$ is the symplectic unit matrix. This formalism is a development of Ripken-Mais presentation [13], and is closely related to the Edwards-Teng parametrization [14], [12]. Any vector $\mathbf{X}$ in the 4D phase space can be expanded over the eigenvectors:

$$\mathbf{X} = \sum_n C_n \mathbf{V}_n\ ;\ \ C_n = \frac{i}{2}\mathbf{V}_n^+ \cdot \mathbf{U} \cdot \mathbf{X}\,.$$

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An elementary act of two-particle interaction can be considered in terms of the eigenmodes. Following [15], the elementary kick for angles of the trailing particle $(\Delta\theta_x, \Delta\theta_y)$ is expressed as

$$\Delta\theta_x = -e^2 x W_x/(p_0 v_0);\ \ \Delta\theta_y = -e^2 y W_y/(p_0 v_0).$$

Here e is the particle charge, p_0 and v_0 are a longitudinal velocity and momentum in the laboratory frame, x and y are the offsets, and $W_{x,y}$ are the wake functions. In terms of the 4D vector $\mathbf{X}$ this can be expressed as a perturbation $\Delta\mathbf{X} = \mathbf{W}\cdot\mathbf{X}$ with the wake matrix elements $\mathbf{W}_{2,1} = -e^2 W_x/(p_0 v_0)$, $\mathbf{W}_{4,3} = -e^2 W_y/(p_0 v_0)$, and zeros for all other matrix elements. In terms of the complex amplitudes C_n, this kick is expressed as

$$\Delta C_n = \frac{i}{2}\mathbf{V}_n^+ \cdot \mathbf{U}\cdot\Delta\mathbf{X} \equiv \frac{i}{2}\sum_m \mathbf{G}_{nm} C_m\ . \qquad (3)$$

The kick matrix $\mathbf{G}$ is not diagonal generally; so, when the mode m is originally excited, the wake drives other modes $n \neq m$ as well. However, when the wake is small enough, it can be treated as a small perturbation of the coherent eigenmode amplitudes. In this case, in the first order of the perturbation theory, only diagonal elements of the perturbation count, similar to the Quantum Mechanics (see in more details [11]). The wake mixing can be considered sufficiently small when the tune separation of the two transverse modes is much larger than the wake-driven coherent tune shift:

$$|\nu_1 - \nu_2| \gg \Delta\nu_{\rm coh}\ . \qquad (4)$$

In reality, this condition is typically satisfied. If it is not, non-diagonal elements of the kick matrix $\mathbf{G}$ have to be taken into account as well, leading to some modification of results. Below, the condition (4) is assumed satisfied, so the perturbation formalism is valid. Thus, only diagonal matrix elements $\mathbf{G}_{nn} \equiv G_n$ in Eq. (3) count; they are calculated as follows:

$$G_n = -\frac{e^2}{p_0 v_0}(W_x\beta_{nx} + W_y\beta_{ny})\ ;\ (n = 1, 2). \qquad (5)$$

This result already shows how the wake is projected on the eigenmodes. However, one more step may be useful for understanding. The complex amplitudes C_n can be presented with explicitly written real and imaginary parts as

$$C_n = \frac{q_n}{2} + i\frac{p_n}{2}\ ;\ (n = 1, 2). \qquad (6)$$

It is straightforward to show that a linear phase space transformation from the original variables $(x, \theta_x, y, \theta_y)$ to the new variables (q_1, p_1, q_2, p_2) is canonical, since they are related to each other by a symplectic matrix, composed from real and imaginary parts of the eigenvectors $\mathbf{V}$ (see Ref. [12]). Thus, q_1, q_2 are new canonical coordinates, and p_1, p_2 are the corresponding canonical momenta. It follows then, that a single excited mode gets the wake-driven kick with

$$\begin{aligned} \Delta q_n &= 0\ ; \\ \Delta p_n &= G_n q_n = -\frac{e^2}{p_0 v_0}(W_x\beta_{nx} + W_y\beta_{ny}) q_n\ . \end{aligned} \qquad (7)$$

Equations (7) show how canonical momentum is perturbed by a small localized wake. Having that, the Vlasov equation with all its results in the phase space (q_1, p_1) are exactly identical to the uncoupled case (x, θ_x), with the following substitution rules for the tune $\nu_x = \mu_x/(2\pi)$, wake times beta-function $W_x\beta_x$, and, thus, impedance times beta-function $Z_x\beta_x$:

$$\begin{aligned} \nu_x &\rightarrow \nu_1\ ; \\ W_x\beta_x &\rightarrow W_x\beta_{1x} + W_y\beta_{1y}\ ; \\ Z_x\beta_x &\rightarrow Z_x\beta_{1x} + Z_y\beta_{1y}\ . \end{aligned} \qquad (8)$$

Note that these rules work both for coasting and bunched beam, and do not depend on a shape of the longitudinal potential well. Any solution of the Vlasov equation for an uncoupled beam can be immediately re-written to the coupled case with these simple rules. After that, the result looks formally similar, while its practical consequences are generally different because of two reasons. First, the incoherent betatron spectrum is changed by the coupling, $\nu_x \rightarrow \nu_1$; thus, the Landau damping is changed. This point is missed in Refs. [2], [3], where denominators of dispersion integrals are based on the uncoupled incoherent tunes. And second, an amplitude of the coherent shift $\propto Z_x\beta_{1x} + Z_y\beta_{1y}$ is a function of coupling as well. The wake substitution rule (8) is valid both for conventional driving (or dipole) wake, and for the detuning (quadrupole) wake (about the two wakes see e. g. Ref. [16]).

The substitution rules (8) show disagreement both with results of Ref. [1], and Refs. [2], [3].

A head-tail growth rate α was derived in Ref. [1] for a coupled optics within a two-particle model, and the two rates were found to be identical. In a simplified form, cited in Ref. [17], the rate looks as

$$\alpha \propto \nu'_x W_x + \nu'_y W_y\ ,$$

where ν'_x, ν'_y are the chromaticities far from the coupling resonance. Applied for the same problem, the substitution rules (8) lead to

$$\alpha_n \propto \nu'_n(\beta_{nx} W_x + \beta_{ny} W_y)$$

Clearly, the two results are significantly different. They may become identical only if accidentally $\nu'_x/\nu'_y = \beta_{nx}/\beta_{ny}$ both for $n = 1$ and $n = 2$. Generally, this condition cannot be correct: the left-hand side is determined by sextupoles, while the right hand side is given by coupled linear optics.

There are two significant disagreements between the rules (8) and Refs. [2], [3]. According to these papers, a localized skew-quad entangles coasting beam modes with different longitudinal numbers. We cannot agree with that. Indeed, localization of skew quads, as well as normal quads, still preserves the longitudinal wave number, since the growth time is much longer than the revolution time. When the Vlasov equation is averaged over fast variables, resulting equations on the slow growing coherent amplitudes become homogeneous over the ring, so the longitudinal Fourier harmonics are true eigenmodes of the coasting beam. The second disagreement between (8) and Refs. [2], [3] is that denominators in dispersive integrals (e. g. Eq. (7) of Ref. [2]) are uncoupled, which excludes correct calculation of Landau damping from those equations.

DIGITAL DAMPER

When the space charge tune shift is high compared with the coherent tune shift, $\Delta\omega_{\rm sc} \gg |\Delta\omega_Z|$, the stability threshold is almost independent on the impedance [6]. The stability condition can be approximately presented as

$$|\eta n - \xi|\delta p/p \,\geq\, \Delta\nu_{\rm sc}/x_{\rm th}\,, \qquad (9)$$

where $\eta,\ n,\ \xi$ are a slippage factor, harmonic number, and chromaticity; $x_{\rm th}$ is a numerical factor, $x_{\rm th} \simeq 3-5$ depending (logarithmically) on the space charge over the impedance tune shifts ratio, and reflecting the particle distribution over the momentum; $\Delta\nu_{\rm sc} = \Delta\omega_{\rm sc}/\omega_0$. By the same reason, the stability condition is not sensitive to the bunching factor, when the impedance is space-charge dominated, $\Delta\omega_{sc} \gg |\Delta\omega_Z|$. The stability condition can also be presented in terms of a threshold frequency $f_{th} \equiv n_{th}/T_0$:

$$f \geq f_{th} \equiv \frac{|\xi_{th}| - |\xi|}{|\eta| T_0}\,, \qquad (10)$$

where the threshold chromaticity ξ_{th} is

$$\xi_{th} = \frac{N r_0}{4\pi x_{\rm th}\gamma\varepsilon_\perp}\frac{mc^2 T_0}{\varepsilon_\parallel}\,, \qquad (11)$$

with $\varepsilon_\parallel = c\delta p T_0$ as the longitudinal r. m. s. emittance, When the chromaticity module is higher than the threshold, $f_{th} < 0$,the beam is stable for any frequency (mode number). If the chromaticity cannot be elevated as high, the beam is going to be unstable at harmonics below the threshold frequency.

When the synchrotron frequencies are small compared with the coherent tune shift, they can be neglected in the stability analysis. For the Recycler, the synchrotron periods are at the range of 1 second, while the instability growth time is typically at least an order of magnitude shorter. In this case, the tail of the bunch can act back on the head through the multi-turn resistive wake. However, for the space charge dominated impedance, the stability threshold is barely dependent on the wake value.

To suppress unstable modes, $f \geq f_{th} \simeq 10-100$MHz, transverse digital damper is implemented [5]. That choice is determined by required one-turn delay, which would make analog damper too expensive. An analog-digital converter (ADC) is a specific part of the digital damper, making its interaction with the beam different from a case of analog dampers.

The output signal of the analog-digital converter (ADC) goes with a sample frequency $f_s \equiv \omega_s/(2\pi) \equiv \tau_s^{-1}$, at the time of writing this statement $f_s = 53$ MHz, being exactly 588 harmonic of the revolution frequency (to filter out all the revolution harmonics). Presently, the input signal is digitized at $N_a = 4$ times higher frequency, and then an average of these N_a numbers goes as the output.

The ADC transforms any input frequency into a sequence of all the alias frequencies, shifted from the input one by multiples of the sample frequency. This equidistant sequence of frequencies includes a single one inside an interval $0 < \omega < \omega_s$, which can be taken as a parameter of the entire set of the cross-talking frequencies. This continuous parameter ω is referred below as a *marking* frequency. Incoming frequencies $\omega_p \equiv \omega + p\omega_s\ ,\ p = 0,\ \pm 1,\ \pm 2,$...are transformed by the ADC into outgoing frequencies $\omega_q \equiv \omega + q\omega_s\ ,\ q = 0,\ \pm 1,\ \pm 2,$.... Let $\widehat{T}$ be the linear operator of the ADC; then, it is straightforward to show that

$$\hat{T}\exp\left(-i\omega_p t\right) = \sum_{q=-\infty}^{\infty} T_{pq}\exp\left(-i\omega_q t\right)\ ; \qquad (12)$$

$$T_{pq} = \frac{2}{N_a}\exp\left[i\omega_p\tau_s\left(1-\frac{1}{2N_a}\right)\right]\frac{\sin^2\left(\omega_p\tau_s/2\right)}{\tau_s\omega_q\sin\left(\frac{\omega_p\tau_s}{2N_a}\right)}\,. \qquad (13)$$

Below, it is assumed that the phase factor in the ADC is compensated by a preceding delay line, providing all the matrix elements real:

$$T_{pq} = \frac{2}{N_a}\frac{\sin^2\left(\omega_p\tau_s/2\right)}{\tau_s\omega_q\sin\left(\frac{\omega_p\tau_s}{2N_a}\right)}\,. \qquad (14)$$

With the ADC, the frequency ω (representing actually the wave length of the beam perturbation) is no longer a good parameter for the beam modes, each consisting of all the composite harmonics. High enough harmonics are strongly damped by Landau damping (and possibly a low-pass filter); thus, they can be neglected and the infinite set of the composite amplitudes being cut.

Let A_p be an amplitude of the harmonic $\omega_p = \omega + p\omega_s$. Were the digital damper the only way for the beam to interact with itself, the time evolution of this harmonic would be described as

$$\frac{dA_p}{dt} = -\Lambda_0\sum_{q=-\infty}^{\infty} T_{qp}A_q \qquad (15)$$

with Λ_0 as a low-frequency rate, determined by the preamplifier. Influence of the low-pass filter is omitted here for simplicity, but can be easily included. A solution of this set of linear equations is expressed in terms of eigenvectors, whose eigenvalues are the damping rates of the beam modes. Impedance and Landau damping just add their terms to the matrix diagonal elements:

$$\frac{dA_p}{dt} = -\Lambda_0 \sum_{q=-\infty}^{\infty} T_{qp} A_q \; - \left(\Lambda_L\right)_p A_p - i \left(\Delta\omega_Z\right)_p A_p \quad (16)$$

Note that the matrix $\widehat{T}$ (14) is strongly degenerated: for any finite dimension it is reduced, all its eigenvalues but one are exact zeroes. With impedance, half of these zeroes are getting unstable; they could be stabilized by the Landau damping. For Gaussian distribution, the Landau damping rate $\left(\Lambda_L\right)_n$ is calculated as

$$\Lambda_L = \sqrt{\frac{\pi}{2}} \Delta\omega_{sc} x_n \exp(-x_n^2/2)\;, x_n \equiv \Delta\omega_{sc}/\Delta\omega_b(n), \quad (17)$$

where the chromatic frequency spread $\Delta\omega_b(n)/\omega_0 = |\eta n - \xi| \delta p/p$. If the distribution is not Gaussian, the correction is obvious. Note that the dimensionless energy spread x_n does not change, if the beam is adiabatically bunched: it depends on the longitudinal phase space density. In other words, growth of the space charge tune shift with the beam bunching is compensated by an equal growth of the momentum spread, so that the dimensionless spread x_n does not change. As a consequence, the Landau damping grows linearly with the bunching factor.

The described analysis predicts several times increase of the phase space density due to the digital damper. Currently, longitudinal phase space density is typically about twice higher than its stability threshold value without the damper.

This section is essentially based on Ref. [6]. A different way to present this problem was suggested later in Ref. [7]. Results of the two papers are close.

TWO-BEAM INSTABILITY AT ELECTRON COOLING

Electron cooling is a powerful tool to increase phase space density of hadron beams. It is successfully used at the Recycler [10], as well as at many other storage rings; the Recycler's beam kinetic energy is at least an order of magnitude higher than anywhere else. Circulating antiprotons are cooled because of their thermal collisions with electrons of a co-moving single-pass electron beam. The same-velocity beams share a small portion of the ring circumference (20 m from 3.3 km). While individual antiproton-electron scattering leads to cooling, a coherent interaction of the two beams may lead to a two-beam instability. Although this instability was never directly seen in the Recycler, it still can be suspected to have place at high frequencies or for quadrupole modes. A reason for this suspicion is that there is a lifetime degradation, and sometimes emittance growth, with increase of the antiproton density happened either with cooling or with longitudinal squeeze. Another possible explanation to these phenomena is an excitation of a single-particle resonances by a space charge of the cooled or squeezed antiproton bunch. A remedy depends on the reason, so it was important to understand if the antiproton-electron instability is responsible for the mentioned phenomena.

Two features of the beam-beam interactions are of principle importance. First, since the beams are moving with the same velocities, their interaction is local. Second, since there is a solenoidal magnetic field in the cooler, electron transverse motion is essentially a drift. Namely, a transverse offset of the ion beam causes a dipole electric field, forcing electrons to drift in the orthogonal transverse direction. This drift gives its own electric field, acting back on the ions. Being linear and local, this electron response can be described as a perturbation of the ion's revolution matrix. At first order, this non-symplectic perturbation matrix is proportional to a product of the electron and ion currents.

In the leading order, equations of motion for antiproton ("ion") and electron complex offsets $\xi_{i,e} = x_{i,e} + i y_{i,e}$ are reduced to the following set:

$$\begin{aligned} \xi_i'' - k_{ie}^2 \xi_e &= 0\,; \\ \xi_e' - i k_{ed} \xi_i &= 0\,. \end{aligned} \quad (18)$$

with k_{ie} and k_{ed} as wave numbers, describing the beams interaction. Here, modification of the beam-beam interaction by the antiproton Larmor rotation is neglected. Also, the beam-beam phase advances $\psi_{ie} = k_{ie} l$, $\psi_{ed} = k_{ed} l$ over the cooler length l are assumed to be small: $\psi_{ie}, \psi_{ed} \ll 1$. Solution of Eqs. (18) leads to the cooler's matrix for antiproton beam, perturbed by its interaction with the electrons; the perturbation is scaled by the interaction parameter

$$\alpha = \psi_{ie}^2 \psi_{ed}$$

proportional to both antiproton and electron currents. The beam-beam interaction can be described by means of the perturbed revolution matrix $\mathbf{R}$, its bare value $\mathbf{R}^{(0)}$ and the perturbation $\mathbf{P}$:

$$\mathbf{R} \equiv \mathbf{R}^{(0)} + \mathbf{P} \cdot \mathbf{R}^{(0)} \equiv (\mathbf{I} + \mathbf{P}) \cdot \mathbf{R}^{(0)}, \quad (19)$$

Complex shifts of the phase advances $\delta\mu_n \equiv \mu_n - \mu_n^{(0)}$ then follow by means of the perturbation theory:

$$\delta\mu_n = -\frac{1}{2} \mathbf{V}_\mathbf{n}^{(\mathbf{0})+} \cdot \mathbf{U} \cdot \mathbf{P} \cdot \mathbf{V}_\mathbf{n}^{(\mathbf{0})}\;, \quad (20)$$

where $\mathbf{V_n}$, $n = 1, 2$, are the optical eigenvectors (1). This yields growth rates $\Lambda_n = \mathrm{Im}(\delta\mu_\mathrm{n})/\mathrm{T}_0$, with T_0 as the revolution time. In the leading order, the perturbation 4×4 matrix $\mathbf{P}$, calculated by means of Eqs. 18, has skew structure; in terms of 2×2 blocks it has only anti-diagonal elements of equal values and opposite signs. The skew way of the two-beam interaction leads to a conclusion that this two-beam instability, if reveals itself at all, has to be highly sensitive to $x - y$ coupling of the unperturbed antiproton eigenmodes. Indeed, since the electron response goes in an orthogonal direction to the original antiproton offset, a work of the resulting force acting back on the antiproton beam is not zero only if the antiproton mode is not plane [11]. Thus, in a leading order, the growth rate has to be proportional to an antiproton coupling parameter, responsible to a degree of circularity of the antiproton optical mode. The described sequence of calculations leads to the following result for the growth rate:

$$\Lambda_{\rm c} = \pm \frac{\alpha \kappa_{xy}}{2T_0} \ ; \ \ \kappa_{xy} \equiv \frac{\sqrt{\beta_{1x}\beta_{1y}}}{l} \sin\nu_1 \qquad (21)$$

where κ_{xy} describes the degree of circularity of the antiproton optical mode. By a general property of the Twiss parameters $\sqrt{\beta_{1x}\beta_{1y}}\sin\nu_1 = \sqrt{\beta_{2x}\beta_{2y}}\sin\nu_2$ (see Ref [12]). In a special case, when the antiproton $x-y$ coupling is driven only by the solenoidal field in the cooler, the growth rate reduces to:

$$\Lambda_{\rm c} = \pm \frac{\alpha\ \beta_{\rm o}}{4T_0 l} \frac{1}{\sqrt{1+(\mu_{\rm x}-\mu_{\rm y})^2/\psi_{iL}^2}} \ , \qquad (22)$$

For the Recycler, assuming $300 \cdot 10^{10}$ antiprotons, evenly distributed over 50% of the circumference, and cooled with 0.5 A electron beam of 3 mm radius inside of a 20 m long cooler with 100 G field, taking the coupling parameter $\kappa_{xy} = 0.3$ (until recently, the working point was located at the coupling resonance), the rate is calculated as $\Lambda_{\rm c}^{-1} = 1$ s. For more bunching, this value grows as the local density; thus, it could easily be 10-20 times higher while compressed bunches are prepared for extraction.

Up to this point, a dipole beam-beam instability was discussed. However, the quadrupole instability can be excited as well, or even more likely [11]. Due to the same reason, the quadrupole instability has to be as much sensitive to the antiproton $x-y$ coupling, as the dipole.

Until not long ago, the Recycler stayed just at the coupling resonance. At that working point, an emittance growth and a lifetime degradation were observed, both associated with the antiproton peak current. Recent experimental studies indicate that while these phenomena depend on the tune position along the coupling resonance line, they are insensitive to the tune separation [10]. Based on these observations and results of the outlined model, the coherent antiproton-electron interaction can be excluded as a main reason for these antiproton intensity phenomena in the Recycler. Thus, excitation of single-particle resonances by a space charge of the cooled or squeezed antiproton bunch is singled out as an only remaining conjecture. In general, checking the influence of the horizontal-vertical coupling resonance appears to be a critical test to verify whether the coherent ion-electron instability causes lifetime degradation in a ring with electron cooling. Avoiding coupling resonances should be considered as a remedy against the instability, since horizontal-vertical coupling is its necessary (but not sufficient) condition.

SUMMARY

Three separated theoretical problems are presented here; they were encountered in stability analysis of the antiproton beam in the Recycler.

First, a method to treat $x-y$ coupling for analysis of beam transverse coherent oscillations is described. The method effectively reduces a coupled problem to an uncoupled one, making the two problems identical. Another problem outlined here relates to use of a digital damper for stabilization of beam coherent motion. Digital dampers are described by explicit time-dependent linear operators, so they do not preserve a frequency of the signal. Modification of the stability analysis with the digital damper is described. A third problem relates to two-beam coherent instability in electron cooling. The most important conclusion is extreme sensitivity of this instability to $x-y$ coupling of the antiproton optical modes. More precisely, it is sensitive to a degree of circularity of these modes. Although the problems were driven by specific conditions at Recycler, their solutions are general.

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