IBS EFFECTS IN A WIGGLER-DOMINATED LIGHT SOURCE
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Abstract
Intra-beam scattering (IBS) is often thought of as a fundamental limitation to achieving lower emittance and hence higher brightness in modern storage ring light sources. However, as we show in this paper analytically and by simulations using ZAP and SAD codes, this limitation may no longer be relevant in a wiggler-dominated 3rd generation light source. Instead, lowering the emittance by increasing the amount of wiggler radiation does not result in significant IBS-induced emittance blow-up, as higher beam density (and IBS rates) is compensated by faster radiation damping. We show that under some practical assumptions the relative ratio of the emittance including the IBS effect to the emittance at zero current is emittance-independent.

INTRODUCTION
Small angle Coulomb scattering within a beam leads to the excitation of betatron and synchrotron oscillations of particles which usually increases beam emittances in all phase planes. This effect, often called intra-beam scattering (IBS) or multiple Coulomb scattering, is proportional to the beam 3D phase-space density and depends strongly on beam energy, becoming more severe for high intensity, low energy machines.

When IBS is included, the steady-state beam properties with radiation damping are defined by

\[ \varepsilon_x = \frac{\varepsilon_{x0}}{1 - \tau_x / T_{x}}, \quad (1a) \]
\[ \varepsilon_y = \frac{\varepsilon_{y0}}{1 - \tau_y / T_{y}}, \quad \sigma_p^2 = \frac{\sigma_{p0}^2}{1 - \tau_p / T_p}, \quad (1b, c) \]

where \( \varepsilon_{x,y} \) stand for emittances, \( \sigma_p \) is the relative energy spread, subscript 0 indicates the beam properties in the absence of IBS, \( \tau_{x,y,p} \) stand for synchrotron radiation damping times, and \( T_{x,y,p} \) are the IBS growth times discussed below. These equations indicate that the IBS effect becomes important when IBS rates are significant in comparison with the radiation damping rates. Because the IBS growth times \( T_{x,y,p} \) depend on beam current as well as beam emittances, energy spread, and bunch length, the above equations are coupled, and solving them requires some iterative procedure. Sometimes a fourth equation is added that expresses the current-dependent relation of the bunch length to the energy spread to account for the potential well distortion. If the vertical emittance is dominated by weak coupling (which is what we assume in this paper) the effect simplifies to 2D, and the second equation, Eq. (1b), is replaced by

\[ \varepsilon_y = \kappa \varepsilon_x, \]

where \( \kappa \) stands for the coupling coefficient.

The basic theoretical framework of IBS effect was established long time ago by Piwinski [1] and Bjorken and Mtingwa (BM) [2] using two different approaches. These theories express IBS rise times \( T_{x,y,p} \) as complicated integrals of beam parameters, such as energy and phase space density, as well as lattice properties. The BM theory has been generalized by Kubo and Oide [3] to include arbitrary vertical-horizontal and vertical-longitudinal coupling. The resulting growth rates are local quantities, and have to then be averaged around the lattice. Many accelerator physics codes include some variations of the BM approach. In addition, since these general procedures are fairly computer intensive, there are exist a number of more approximate formulations of IBS effect that simplify the treatment for certain parameter regimes. For example, Bane [4] has recently shown the equivalence of Piwinski and BM treatments in the regime applicable for high-energy machines. In this regime Bane has found that BM results reduce to fairly compact expressions for IBS rise-times, which we will use later in this paper.

The IBS approaches mentioned above result in the growth times proportional to the so-called Coulomb log factor, equal to \( \ln(b_{\text{max}}/b_{\text{min}}) \), where \( b_{\text{max, min}} \) are impact parameters, which are not well defined. Often, \( b_{\text{max}} \) is taken equal to \( \sigma_y \). To fix \( b_{\text{min}} \) a so-called “tail cut” procedure was suggested by Raubenheimer [5]. He pointed out that, since the IBS results in non-Gaussian beam distributions, tail particles could be overemphasized; therefore one must chose \( b_{\text{min}} \) to eliminate interactions having collision rates smaller than synchrotron radiation damping rates. We should mention that a more recent approach to the IBS problem by Nash et al [6] does not introduce the Coulomb log factor and related ambiguities.

In a wiggler-dominated light-source most of the synchrotron radiation power is emitted from the wigglers as opposed to the bending magnets, therefore the beam emittance is essentially defined by the amount of wiggler radiation. Consequently, both the (emittance-dependent) IBS growth times and the synchrotron radiation damping times in Eqs. (1) depend strongly on the amount of wiggler radiation. The IBS effect is therefore expected to have interesting scaling with the beam emittance, which is reported in this paper. To better illustrate our results we first present IBS calculations using two different beam dynamics codes for wiggler-dominated light source NSLS-II, and then generalize these results with the help of analytical considerations using Bane’s formalism.

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ZAP AND SAD RESULTS FOR NSLS-II

NSLS-II is a high brightness 3rd generation 3 GeV storage ring light source under design at BNL [7]. NSLS-II is planning to achieve sub-nm emittance via the use of damping wigglers (as well as other IDs) installed in zero-dispersion straight sections.

Rather than assuming some fixed ID make-up for the initial NSLS-II studies we have calculated IBS effects as a function of radiation losses in the machine, having $\varepsilon_{x,0}$ vary from the ~2 nm bare lattice value down to about 0.4 nm. The zero current vertical emittance $\varepsilon_{y,0}$ was fixed at varying from ~0.5% for bare lattice to about 2% for the lowest $\varepsilon_{x,0} = 0.4$ nm.

Most of NSLS-II IBS calculations have been performed with the code ZAP [8], which implements the 2D procedure of the BM theory, i.e. the vertical emittance is assumed dominated by coupling. We used 500 mA for the total ring current, and assumed it uniformly distributed into 80% of 500 MHz RF buckets. As we changed the amount of radiation losses, the RF voltage was adjusted to keep the RF energy acceptance at 3% (see Fig. 1). Electron beam parameters in the absence of IBS, used as input to ZAP (such as horizontal emittance, energy spread, zero-current bunch length, and radiation damping times) were calculated analytically by scaling bare lattice values by the amount of radiation losses.

Energy spread and bunch length are shown in Fig. 2, and zero-current horizontal emittance is shown in Fig. 3 (blue circles). Note that while the energy loss changes by a factor of 5, the bunch length remains almost constant (due to our choice of constant RF acceptance of 3%).

The results of ZAP calculations of IBS effect are shown in Fig. 3 (magenta triangles). Note that IBS induced relative emittance blow-up doesn’t exceed 20%, and remains fairly independent of the amount of radiation losses (and hence the emittance). This implies that for smaller emittances, increased IBS rates due to denser bunches are offset by the increase in radiation damping. This is quite contrary to a more typical situation in a storage ring light source not dominated by IDs, where decreasing the emittance by adjusting the lattice (and hence keeping the synchrotron radiation damping rates fixed) can result in an increase of IBS-induced emittance blow-up.

We have also performed similar calculations using the code SAD from KEK [9]. This code has been extensively bench-marked against the IBS measurements at the ultra-low emittance ATF storage ring. Apart from a more general treatment of the IBS effect (described in detail in [3]) SAD calculations are different from ZAP in another important aspect. Specifically, being a lattice code, SAD allowed us to self-consistently calculate the beam phase-space dimensions as a function of radiation losses. To do this we added a simple model of a damping wiggler (a periodic sequence of sector magnets) into every straight section of NSLS-II lattice. We then varied the wiggler strength (by adjusting the bending angles of each magnetic element of each wiggler). Simultaneously we adjusted the RF voltage to keep 3% RF acceptance as shown in Fig. 1, as well as adjusted the coupling to keep the vertical emittance constant as described above. SAD then calculated beam emittances, energy spread, and bunch length with and without the IBS effect.

Our SAD results came out very similar to what is presented in Fig. 3, specifically the IBS-induced relative emittance blow-up remained fairly independent of the amount of radiation losses (and hence the emittance). SAD, however predicted this emittance blow-up of only about 7%. We confirmed that this difference between the
two codes was mostly due to different ways of calculating the Coulomb log. The calculation of Coulomb log used in 
ZAP sets $b_{\text{max}}$ equal to $\sigma_\gamma$ and it doesn’t include the tail cut procedure. SAD on the other hand, does include the tail cut, generally resulting in smaller Coulomb log values. For example these values for NSLS-II bare lattice case were about 17 for ZAP and about 10 for SAD.

It should be noted that while SAD implements a full 3D treatment of IBS we have not taken advantage of this yet. This will be explored in the future to study the effects of finite vertical dispersion for NSLS-II (zero vertical dispersion is assumed in this paper). It has been recently demonstrated [10] that introducing periodic vertical dispersion to control the vertical beam-size (as opposed to betatron coupling) could increase the Touschek lifetime in a 3rd generation light source with small-gap IDs.

NALYTICAL ESTIMATES

In Bane’s formalism [4] the IBS growth rate for the horizontal emittance is expressed as

$$T_\gamma^{-1} = \frac{\gamma^\prime c N (\log)}{16 \gamma^2 \epsilon^{3/4} \sigma_\alpha \sigma_\beta} \left[ H_x \sigma_\beta g \left( \frac{\epsilon \beta_x}{\sqrt{\epsilon \beta_y}} (\beta_x \beta_y)^{1/4} \right) \right],$$

(2)

where $\langle \cdots \rangle$ denote the lattice average, $r_0$ is the classical electron radius, $N$ is the number of particles per bunch, log is the Coulomb log factor, $H_x$ is the dispersion invariant, $\sigma_{\alpha}^{-2} = \sigma_p^{-2} + H_x / \epsilon_x \beta_{x,y}$ are the beta functions, and $g[\alpha]$ is approximated by $g[\alpha] = \alpha^{0.021-0.044 \ln \alpha}$.

Let’s study the scaling of Eq. (2) as a function of $\epsilon_x$ (or, equivalently, the inverse of the energy loss per turn, $E_{\text{loss/turn}}$) in the regime of interest. For couplings 0.5% $< \kappa < 1$ and $\beta_x = \beta_y$, $g[\kappa^{1/2}]$ changes by less then 30%, so for rough estimates its variation in Eq. (2) could be ignored. Similarly, we ignore the variation of $\sigma_\gamma$ (see Fig. 2). Taking $\sigma_{\alpha}^{-2} = H_x / \epsilon_x$, and assuming that the energy spread scales as $\sigma_{\epsilon} \sim E_{\text{loss/turn}}^{1/4} \sim \epsilon_x^{1/4}$ (approximate, see Fig. 2) we obtain $T_\gamma \sim \epsilon_x^{-1} / E_{\text{loss/turn}}$. Since the radiation damping time scales the same way, $\tau_\gamma \sim 1 / E_{\text{loss/turn}}$, we observe that the denominator in Eq. (1a) is independent of the radiation loss. Thus we have derived the scaling confirming our earlier observations in Sec. 2 above, that the IBS-induced relative emittance blow-up, $\epsilon_x / \epsilon_0$, is emittance-independent. Plugging NSLS-II values into Eq. (2) (no approximations, $\log = 17$) we obtain $T_\gamma / T_x = 0.17$ for bare lattice and $\tau_\gamma / T_x \approx 0.16$ for 1.4 MeV/turn loss.

CONCLUSION

To summarize, our NSLS-II calculations to date indicate that under pessimistic assumptions IBS-induced relative emittance blow-up for NSLS-II should not exceed 20% at nominal bunch intensity and therefore it should not present a problem. We emphasize that these estimates are based on several conservative assumptions. First of all, calculations are performed at zero current bunch length, while in reality the bunch will be longer due to potential well distortion. At even higher single bunch currents, used in special operating modes, the microwave instability will result in an even stronger increase in bunch length as well as energy spread, reducing the IBS effect further. On top of that, we believe that due to the Coulomb log calculation procedure ZAP overestimates the IBS effect, and that SAD results, predicting weaker IBS-induced beam blow-up, are more realistic. Finally, NSLS-II is planning to include a 3rd harmonic RF system (to increase the Touschek lifetime) that will reduce the peak current and will therefore reduce the IBS effect even further.

We note that this paper presents the results of IBS calculations for the NSLS-II CDR lattice only. However, our conclusion on the small magnitude of the IBS effect at nominal NSLS-II operating parameters held true for all previous lattices considered in the NSLS-II design. Specifically, these included DBA lattices with stronger (shorter bending radius) dipoles, as well as the TBA lattice considered for NSLS-II CD0. Therefore the IBS effect has been shown to be largely irrelevant to the NSLS-II machine design.

More fundamentally, we have found the magnitude of the IBS-induced emittance blow-up in a wigglerroriented light source to be fairly independent of the emittance. This implies that for smaller (horizontal) emittances, increased IBS rates due to denser bunches are offset by the increase in radiation damping. We believe that this result is fairly general, as long as our assumptions of fixed vertical emittance, coupling not being extremely small, and slowly varying bunch length apply. These assumptions are usually true for modern light sources, which obtain diffraction limited vertical beam sizes (for most wavelengths of interest) with $\kappa$ about a fraction of a % or higher; further reduction in the vertical emittance doesn’t directly benefit the users.

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REFERENCES

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