SIMULATION OF ULTRA-SHORT PULSES IN A STORAGE RING *

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Abstract

Simulation study was performed with the tracking code Elegant [M. Borland, APS Report LS-287] to show beam quality evolution for a short, intense electron bunch after being injected to the SPEAR3 storage ring. The electron bunch with an intensity of 1 mA (0.78 nC) and a length of nearly 1 ps (FWHM) is found to degrade rapidly due to coherent synchrotron radiation (CSR) which causes large uneven longitudinal phase space distortion. The bunch length remains short and the longitudinal line density remains smooth for about 10 turns. For such a beam to circulate in the ring, a total of 10 MV rf power is needed to compensate for the energy loss.

INTRODUCTION

As a future upgrade option, we were considering injecting intense (1 nC/bunch), ultra-short (0.1–1 ps FWHM) electron bunches to the SPEAR3 ring and circulating for up to 50-100 turns. The short X-ray pulses and the Tera-Hertz coherent synchrotron radiation (CSR) generated by such beam can be valuable to our user community. This study was a preliminary attempt to investigate the possibility and identify the difficulties, especially to find out how many turns such a beam can survive and remain usable. Since beam dynamics under CSR is a subject that needs more theoretic study, our main approach for the study was multi-particle tracking. We used the code Elegant [1, 2] which has incorporated CSR effects to the beam motion. Simple theoretic considerations were also applied to gain some physics insights.

The main difficulties in our considerations were the additional one-turn energy loss due to CSR and the longitudinal phase space distortion due to its spatial distribution within the bunch. For simplicity, we consider a gaussian bunch. Using the form factor $g(\lambda) = e^{-\left(\frac{2\pi\sigma_z}{\lambda}\right)^2}$ and considering the vacuum pipe shielding effect [3, 4, 5], one can derive the one-turn CSR energy loss

$$U_{\rm csr} = 3.250 N r_0 E_0 \frac{\rho^{1/3}}{\sigma_z^{4/3}} F\Big(\frac{2\pi\sigma_z}{\lambda_0}\Big), \qquad (1)$$

where N is the number of electrons in the bunch, $E_0 = 0.511$ MeV is the electron rest energy, $r_0 = 2.818 \times 10^{-15}$ m is the classical electron radius, ρ is the bending radius, σ_z is the bunch length and $\lambda_0 = 2b\left(\frac{b}{\rho}\right)^{1/2}$ is the shielding cutoff wavelength. The function $F(x) = \int_x^\infty u^{1/3} e^{-u^2} du$ is plotted in Figure 1. The CSR energy loss of an electron depends on its longitudinal position in the bunch. Not considering shielding effect, the longitudinal distribution of CSR energy loss for a gaussian bunch is described by [6]

$$U_{\rm csr}(s) = 3.476 N r_0 E_0 \frac{\rho^{1/3}}{\sigma_z^{4/3}} G\Big(\frac{s}{\sigma_z}\Big), \qquad (2)$$

The function $G(x) = -\int_{-\infty}^{x} \frac{u e^{-u^2/2}}{(x-u)^{1/3}} du$ is plotted in the right plot of Figure 1, from which we see that electrons in the center of the bunch lose more energy to CSR than those in both ends. The negative values of $G(x)$ at $x > 1.1322$ indicates that the bunch head actually gains energy from the CSR wake. This unevenly distributed CSR wake will distort the longitudinal phase space distribution of the electron bunch.

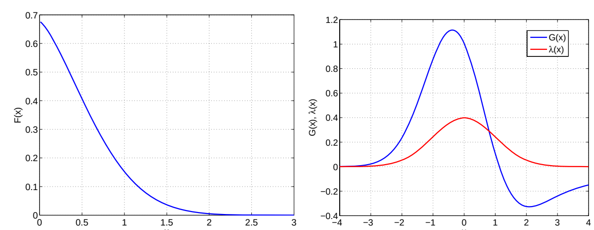

Figure 1: Left: the function $F(x)$ as shown in Eq. (1). Right: The function $G(x)$ as shown in Eq. (2). Also plotted is the gaussian line density function $\lambda(x) = e^{-x^2/2}/\sqrt{2\pi}$.

In the above discussion a circular orbit is assumed for the electrons. In reality, however, beam trajectory is curved only inside bending magnets. It is pointed out in Ref. [6] that the results from the circular motion model is applicable to bending magnet cases when

$$\rho/\gamma^3 \ll l_b \ll \rho\phi_m^3/24, \qquad (3)$$

where l_b is the bunch length and $\phi_m \ll 1$ is the bending angle of the magnet.

For the SPEAR3 ring, some relevant parameters are: bending radius $\rho = 8.14$ m, circumference $2\pi R = 234.1$ m, beam energy $E = 3.0$ GeV, vacuum pipe height $2b = 0.034$ m, from which we derive that $\lambda_0 = 1.55$ mm. The bending angle for a regular dipole magnet is 0.1848 rad. The slippage length defined as $\rho\phi_m^3/24$ is 2.1 mm for SPEAR3, much larger than the assumed bunch length ($\sigma_z = 0.1$ mm) in this study. So it is valid to apply Eqs. (1,2) in the analysis. For an electron bunch of beam current 1.0 mA(or 0.78 nC) at rms bunch length $\sigma_z = 0.1$ mm, we find $U_{\rm csr} = 4.60$ MeV. If we don't consider the shielding effect, we would get $U_{\rm csr} = 6.67$ MeV. For such a bunch length, the shielding effect reduces CSR energy loss by roughly 30%.

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SIMULATION WITH THE CODE ELEGANT

The tracking code Elegant simulates CSR effects for both dipoles and drift spaces [2]. It takes into account the transient effects of CSR when the beam enters and leaves the dipoles. But effects due to the finite transverse dimensions of the bunch and the shielding effects are not considered.

For an intense beam with CSR to circulate in a ring for multiple turns and remain short, the lattice must be near isochronous. Otherwise the bunch center will overtake (assuming positive phase slippage factor) the head quickly because of the energy difference induced by CSR. The longitudinal bunch shape is then distorted and the bunch get elongated as we have observed in simulation. In our study we chose a lattice with a zeroth order factor $\alpha_0 = 5.95 \times 10^{-7}$, which was 2000 times smaller than the nominal. The first order factor was reduced to $\alpha_1 = -3 \times 10^{-6}$ by adjusting the sextupoles. The second order factor was $\alpha_2 = 0.053$. The quasi-isochronous lattice has an equilibrium horizontal emittance of $\epsilon_x = 43$ nm while $\epsilon_x = 17$ nm for the nominal SPEAR3 lattice.

In our simulation, the number of slices for each dipole was chosen to be $N_k = 20$. The number of bins in longitudinal density function calculation was $N_{\rm bin} = 600$. The drift space was cut into slices of 5 cm long to calculate the CSR effects. As simulation will show, a longitudinal bunch tail develops and extends far away from the bunch centroid. Since in CSR wake calculation all particles are binned by their z-coordinate, as the longitudinal span of the beam increases, binning becomes less efficient and the line density calculation as well as the CSR wake calculation become spurious. To overcome this issue, we used the Elegant element CLEAN to remove all particles strayed away from the bunch centroid beyond a limit, typically 2 ps.

The number of macro-particles we used ranges from 100k to 1 million. Comparison was made between the two extreme cases and little difference was seen in the bunch profile. The initial phase space distribution was assumed gaussian for all three dimensions. The initial bunch length was $\sigma_z = 0.1$ mm. The initial momentum spread was set to either $\sigma_\delta = 0.001$ or $\sigma_\delta = 0.01$ for comparisons. Both the initial horizontal and vertical emittances were set to 43 nm even though there was no particular reason to do so for the vertical plane. Since in general CSR tends to be stronger for bunches with more longitudinal structure and the gaussian distribution is the smoothest distribution, the cases considered here are the "best" situation (i.e., least CSR effects). We mainly considered the intense beam case with a single bunch current $I_b = 1$ mA . We have also considered a weaker intensity case with $I_b = 0.1$ mA for comparison. Since CSR causes much additional energy loss, we change the rf gap voltage to have a total of 18 MV for the $I_b = 1$ mA case. We adjusted the synchronous phase to compensate the average total one-turn energy loss.

TRACKING RESULTS

We first examined the CSR-induced energy loss issue. We tracked a bunch with $I_b = 0.1$ mA and various bunch length for one turn. The average energy loss was obtained and the incoherent synchrotron radiation energy loss was subtracted from it to get $U_{\rm csr}$. The CSR energy loss by tracking was then compared to calculation results obtained with Eq.(2) in Figure 2. The tracking result gave $U_{\rm csr} = 0.91$ MeV and the calculation showed $U_{\rm csr} = 0.67$ MeV for $\sigma_z = 0.1$ mm. The difference may come from the transient effects considered by Elegant. At $\sigma_z = 0.1$ mm and $I_b = 1.0$ mA, the one turn CSR energy loss was $U_{\rm csr} = 9.1$ MeV by tracking, which means the CSR energy loss is linearly scalable with single bunch current, at least for the first turn in tracking.

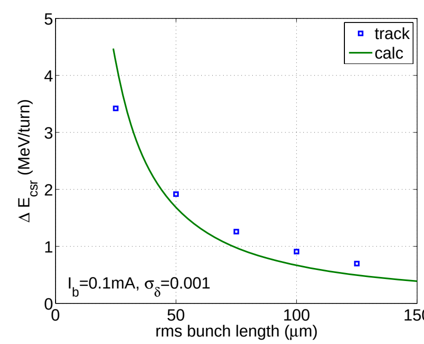

Figure 2: The CSR energy loss $U_{\rm csr}$ by Elegant tracking or by calculation for the $I_b = 0.1$ mA case. Note the usual synchrotron radiation energy loss is not included.

The longitudinal bunch profiles after 5, 10, 15 turns of two cases with different initial momentum spread are shown below (Fig. 3). Both cases assumed $I_b = 1$ mA and $\sigma_z = 0.1$ mm and were tracked with 400k macro-particles. But the initial momentum spread were $\sigma_p = 0.001$ and $\sigma_p = 0.01$, respectively. The longitudinal line density is also shown in each plot. Fig. 4 shows the percentage of particles that remain within 2 ps from the center, rms momentum spread and bunch lengths in 20 turns. The momentum spread keeps growing each turn for both cases due to CSR. When particles have large momentum deviation (above 3%) , they shift backward due to higher order momentum compaction factors and the bunch develops a tail. The case with smaller initial momentum spread (0.001) is more distorted. The other case develops the tail earlier because it has larger initial momentum deviation. But it keeps a smooth longitudinal density function. In both cases more and more particles are lost to the tail and the bunch length grows. The bunch remain usable for only about 10–15 turns in terms of a smooth longitudinal profile and keeping the majority of particles within the centroid.

We also simulated a case with weaker current in which $I_b = 0.1$ mA, $\sigma_z = 0.1$ mm and initial $\sigma_p = 0.001$. In this case particles start losing to the tail after 50 turns. The

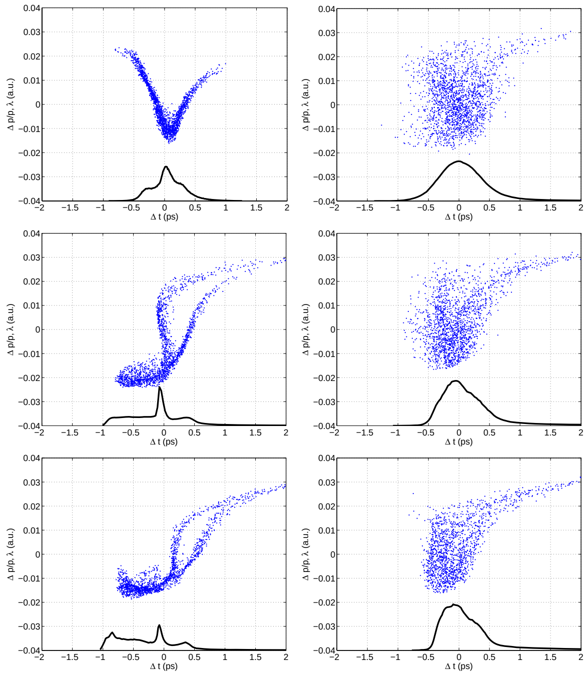

Figure 3: Longitudinal phase space profiles after the 5th, 10th and 15th turn for cases with initial rms momentum spread 0.001 (left) and 0.01 (right). The black curves are the longitudinal line density in arbitrary unit.

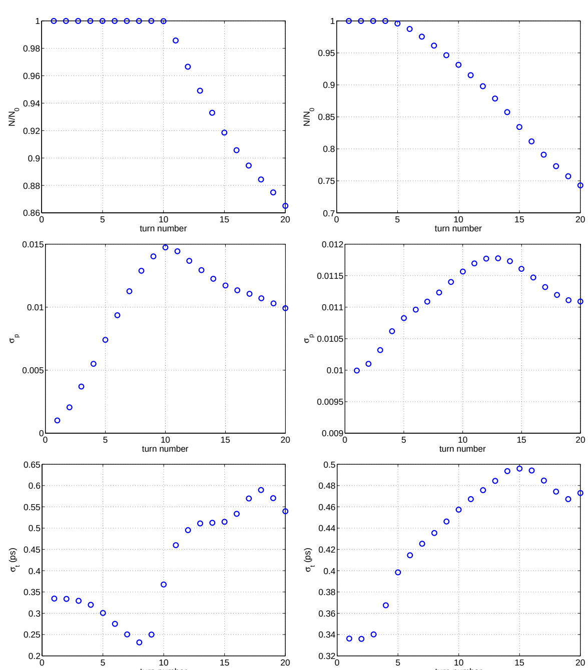

Figure 4: Comparisons of remaining particles, momentum spread and bunch length for the two cases (left column with initial $\sigma_p = 0.001$, right column $\sigma_p = 0.01$).

evolution of momentum spread and bunch length is shown in Fig. 5.

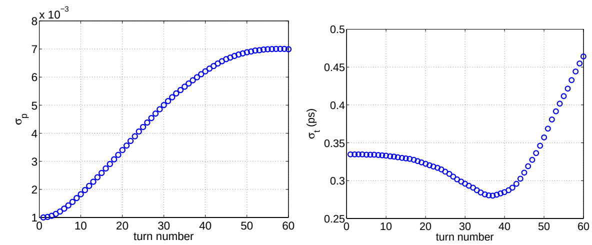

Figure 5: The evolution of momentum spread and bunch length for the case with $I_b = 0.1$ mA.

CONCLUSION

The challenges of keeping intense, ultra-short bunches in a storage ring include extra rf power needed to compensate the CSR energy loss and the severely distorted longitudinal phase space profile. The latter comes from the uneven longitudinal distribution of CSR and the subsequent filamentation due to phase slippage effect. A strictly isochronous lattice is desirable. A larger initial momentum spread seems to ease up the distortion. However, the ever-growing momentum spread still poses a strong limitation on how many turns the bunches remain usable.

For the SPEAR3 case, our simulation shows that bunches with current 1 mA and initial bunch length 0.1 mm remain usable for about 10–15 turns before a large fraction of particles are lost to the tail due to large momentum deviation. This is less than 50–100 turns as expected by our users. They may have to accept a weaker beam or longer bunch length to get 50 turns of usable beam.

ACKNOWLEDGMENTS

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