CHARACTERIZATION OF ORBITAL ANGULAR MOMENTUM MODES IN FEL RADIATION

E. Hemsing*, G. Andonian, J. Rosenzweig, PBPL-UCLA
M. Babzien, V. Yakimenko, BNL, Upton, Long Island, NY, A.Gover, Tel-Aviv University, Israel

Abstract

It is well known that Gaussian modes of paraxial light can possess orbital angular momentum (OAM) as a result of a exp($-il\phi$) azimuthal angular dependence[1]. Recent experiments on the VISA free-electron laser operating as self-amplified spontaneous amplifier (SASE) at λ=840 nm have produced transverse electromagnetic radiation intensity patterns that suggest the presence of one or more OAM modes. Though the intensity profiles from VISA are compelling, work remains to discern the phase structure since OAM modes are revealed by a characteristic helical phase dependence, with a pathological singularity on axis. Here we discuss current efforts to characterize the mode content of the VISA FEL through experiments, theoretical models, and simulations. Possible methods to generate OAM modes *in situ* are discussed.

INTRODUCTION

Free-electron lasers (FELs) have long been used as tunable sources of coherent electromagnetic (em) radiation. In an FEL the generation of light that possesses orbital angular momentum (OAM) may have particular applications for broad sections of the scientific community. The interaction between OAM modes and matter has been studied intensely in recent years, where it has been shown that the angular momentum in the em field can drive the particles to rotate or orbit the beam axis, allowing the possibility of light driven rotors, propellers, pumps and so on[3]. Since future generation FEL light sources will be used to probe matter with coherent x-rays, FEL-OAM modes on this scale would permit the transfer of torque on atomic length and time scales. Though most OAM modes in the longer wavelengths can be readily generated using optical elements in the beam path, the present interest regarding in situ generation of OAM in an FEL is motivated by the on-going development of high-brightness, ultra-short pulse length FELs that may render such extrinsic methods impractical due to size and damage constraints. It is therefore of interest to explore the possibility of generating OAM modes by intrinsic coupling to the source e-beam, eliminating the requirement for external mode conversion.

The possibility of producing OAM light in an FEL is motivated by the appearance of hollow and spiral transverse intensity patterns generated during SASE and the VISA (visible to infrared seeded or spontaneous amplifier) FEL experiment at Brookhaven. The apparent structure of the patterns suggest the possible interplay of single or multiple OAM modes, and experiments have been launched to fully characterize the mode content through planned intensity and phase topology measurements. We discuss several experimental tools that are planned. We also report on recent results from both simulations and predictive models that suggest light with helical phase structure can be generated in an FEL.

* ehemsing@physics.ucla.edu

PARAXIAL OAM MODES

The analogy between the eigenstates of the L_z operator from quantum mechanics and the Laguerre-Gaussian TEM modes from paraxial optics was first discussed by Allen et al[1]. It was observed that the exp($-il\phi$) azimuthal dependence of a Laguerre-Gaussian (LG) optical mode yields a mode that is not strictly transverse, and gives rise to an azimuthal component of the linear momentum. Thus, in addition to the *spin* angular momentum ($\pm\hbar$ per photon), the LG modes also have $l\hbar$ units of *orbital* angular momentum per photon. The time-harmonic field amplitudes of LG modes are expressed as,

$$\tilde{u}_{p,l}(\mathbf{r}) \propto e^{-i\Phi(\mathbf{r})}e^{-r^2/w(z)^2}\Big(\frac{r\sqrt{2}}{w(z)}\Big)^{|l|}L_p^{|l|}\Big(\frac{2r^2}{w(z)^2}\Big), \tag{1}$$

where the phase is

$$\Phi(\mathbf{r}) = l\phi - kz + \frac{kr^2}{2R(z)} - (2p+l+1)\Psi(z) \tag{2}$$

and $L_p^{|l|}$ is a Laguerre polynomial with radial and azimuthal indices p and l, respectively. The term $\Psi(z)$ =tan$^{-1}(z/zR)$ is part of the Gouy phase, $R(z) = (z_R^2 + z^2)/z$ is the radius of curvature, $w(z) = w_0\sqrt{1+(z/z_R)^2}$ is the beam waist and z_R=$kw_0^2/2$ is the Rayleigh range. It is clear from Eqn (2) that the total phase evolves helically along the propagation coordinate z. By exploiting relationships between the field amplitude and phase, several simple techniques can be used to resolve the mode structure. Laguerre-Gaussian modes are used as an example basis to model the results of various VISA diagnostics.

VISA RESULTS

The VISA FEL operates as a high-gain SASE FEL, and has achieved saturation within the 4 m undulator at 840 nm, facilitated by energy chirped bunch compression through the dispersive dog-leg section[2]. Several examples of the transverse intensity profiles from VISA are shown in Fig. (1). Clearly evident are both the hollow structures and the

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spirals that resemble the higher-order spatial structures of LG modes. The images were taken using a CCD camera viewing a screen 4 m from the undulator exit. Relevant operational parameters are listed in Table1.

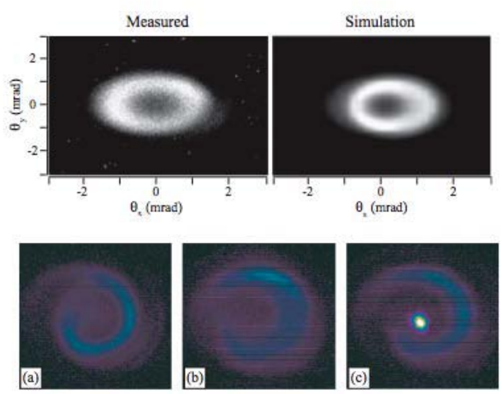

Figure 1: Spiral intensity profiles from VISA. A HeNe alignment laser identifies the e-beam axis in the right, lower image.

Table 1: VISA parameters

Parameter	Symbol	Value
Undulator period	λ_w	1.8 cm
Number of periods	N	220
Undulator Parameter	K	1.26
e-beam relativistic factor	γ	138.5
Signal wavelength	λ	840 nm
e-beam rms radius	r_0	70 μm
bunch charge	e	300 nC
beam current	I_0	100 A

PHASE MEASUREMENTS

A few techniques and diagnostics designed to reveal phase structures in the VISA FEL are presented here in brief. See Ref. [3] and [4] and the references therein for an expanded, comprehensive review of these and other phase recovery methods.

Interference

Several different techniques involving the interference of a single mode, or of many modes together, can be used to easily identify the presence of a phase discontinuity. In some cases, a singularity is revealed as lobes or as forked, bifurcated fringes that are predictably related to the azimuthal mode number. The resulting interference patterns of several input beams are shown in Figure 2.

Pepperpot

A periodic array of holes called a pepperpot can be used to perform a 2D transform on the coherent input field and

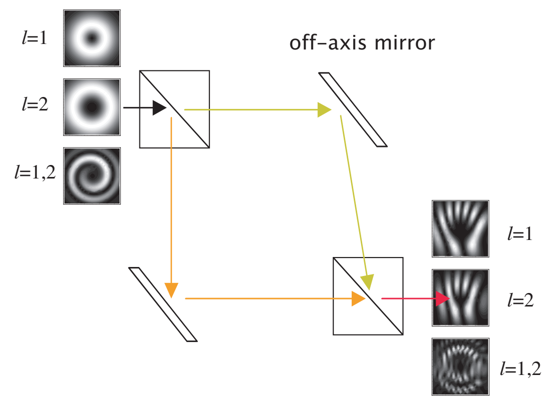

Figure 2: Schematic representation of a simple skew interference measurement used to reveal the azimuthal mode content OAM modes. Each input beam (left) interferes with itself and produces the signature fringe patterns (right).

can be used to reconstruct broad features of the wavefront. As noted in Figure 3, the pattern generated by a simple, phase symmetric, annular Gaussian beam is distinguished from that of an l=1 LG beam by the intensity on axis at the detector. Since the phase of higher-order OAM modes varies periodically about the axis, the pepperpot method can be used to measure both the transverse coherence, and the transverse phase structure.

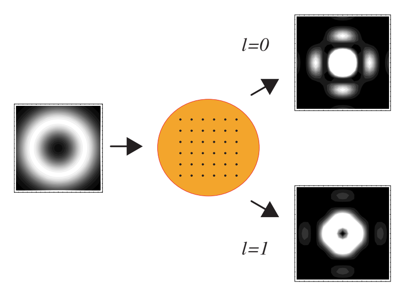

Figure 3: The far field patterns of a pepperpot can be used to distinguish different phase topologies of similar intensity patterns.

Microlens Array

A basic component of many wavefront phase measurement schemes, a microlens array provides a 2D map of the transverse phase gradient, weighted by the intensity. The relative position of focal points on the detector changes according to the curvature of the incoming wavefront. Figure 4 shows a simplified schematic of the setup, and the resultant phase components (superimposed on the intensity) of an $l \geq 1$ input mode.

SIMULATIONS

Preliminary simulations on Genesis 1.3 suggest that it is possible to generate fields from SASE with spiraling inten-

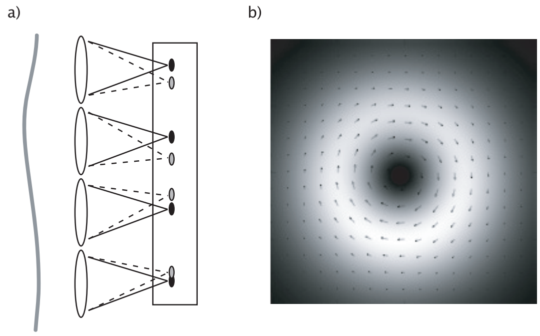

Figure 4: a) A basic microlens array. The focal points of an incoming signal wave with a curved phase front are compared with those of a reference wave. The shift in the focal point is related to the phase curvature over each lens. b) An example l=1 mode (courtesy A. Mancuso).

sity and phase using a helically perturbed electron beam distribution. That is, the perturbation on the bunch distribution is like that of a spring coaxial with the direction of propagation. The results from one such simulation are shown in Figure 5, where the transversely gaussian e-beam had approximately one "helical twist" per coherence length ($\approx 200\ \mu m$).

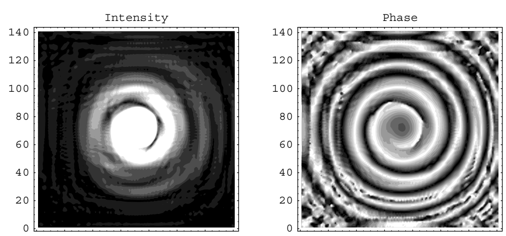

Figure 5: The complex field from VISA SASE at a single longitudinal slice from a Genesis 1.3 simulation with a helically perturbed e-beam.

OAM MODE COUPLING

The generation of OAM modes is examined in a recent formulation wherein the FEL signal field is expanded into a set of LG eigenmodes of a weakly guiding quadratic index medium[5]. In this model the coupled excitation equations for the slowly-varying mode amplitudes and the small-signal source current are derived and model the high-gain regime prior to the onset of saturation. The coupling to orbital angular momentum modes is shown to occur if the e-beam has a helical density perturbation. The general form for the perturbation is

$$\tilde{\mathsf{n}}_1(\mathbf{r}) = \mathsf{n}_0 \sum_{\eta} \tilde{\epsilon}_\eta e^{-i\eta\phi} \qquad (3)$$

where $|\tilde{\epsilon}_\eta| < 1$. Figure 6 shows the transverse intensity and phase for a numerical solution of the excitation equations with an initial helical density perturbation of ϵ_0=10^{-5} and ϵ_1=8×10^{-3} on a transversely gaussian e-beam with rms radius r_0. It is clear that the structure is that of a dominant (p, l)=(0,1) LG mode, and that the field is optically guided by the appearance of inward curvature superimposed on the helical phase front.

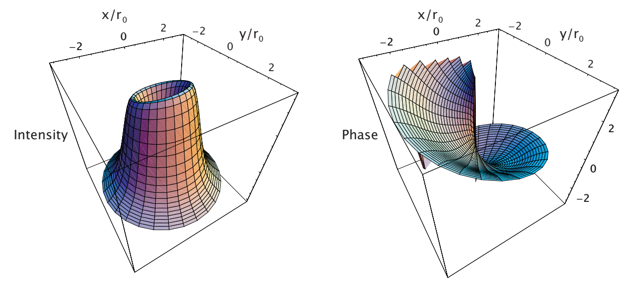

Figure 6: Transverse field at undulator exit for helical density perturbation.

CONCLUSIONS

Electromagnetic radiation modes that possess orbital angular momentum may provide a novel tool for future sources of coherent radition. The generation of such modes in an FEL is suggested by recent data, and results from simulations and theory suggest these structures can arise from coupling to existing helical perturbations in the source electron beam. Several diagnostics are discussed and are in preparation for phase measurements of hollow and spiral intensity light observed at the VISA FEL.

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