APPLYING FREQUENCY MAP ANALYSIS TO THE AUSTRALIAN SYNCHROTRON STORAGE RING

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Abstract

The technique of frequency map analysis has been applied to study the transverse dynamic aperture of the Australian Synchrotron Storage Ring. The results have been used to set the strengths of sextupoles to optimise the dynamic aperture. The effects of the allowed harmonics in the quadrupoles and dipole edge effects are discussed.

INTRODUCTION

The Australian Synchrotron storage ring is a 3 GeV machine with 14 identical cells and a circumference of 216 meters. Each of the unit cells are double bend achromats (DBA) [4] with the optical functions and layout of the elements shown in Figure 1.

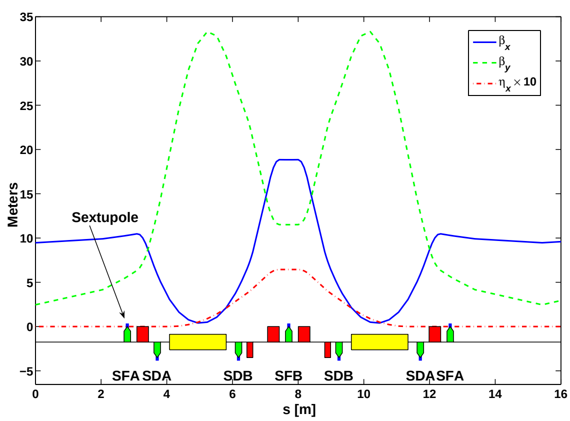

Figure 1: The above plot shows the location of the sextupoles in the lattice and the corresponding optical functions. In this case the optics has been configured for zero dispersion in the straight sections.

It is well known that the sextupoles that are used to correct for chromatic effects also introduce geometric aberrations that limit the dynamic aperture (DA). To compensate for these aberrations and maximise the DA, two extra families of sextupoles located in the outer regions of the cell (SFA and SDA) were added. The "chromatic" sextupoles (SFB and SDB), primarily used to control the linear chromaticities, are located between the dipoles. In a DBA style lattice the interleaving families of sextupoles make it difficult to completely compensate for aberrations. However some measure of correction can be achieved with the careful tuning of the sextupole families.

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In this paper we will report the results of the use of Frequency Map Analysis (FMA) to improve the DA by tuning the outer sextupoles (SFA and SDA) and to show the effects of the allowed harmonics of the quadrupoles on the DA.

For simplicity the following analysis will be conducted with zero dispersion in the straight sections. In doing so, the function of the outer and inner sextupoles are clearly separated so that the outer sextupoles can be tweaked optimise the DA without affecting the chromaticity. The optics for the ideal lattice was initially set for a tune of 13.3 and 5.2 in the horizontal and vertical, and zero chromaticity in both planes. The respective magnet strengths were fitted in DIMAD and converted in to Accelerator Toolbox (AT) [8] input files.

SURVIVAL PLOT

The first study conducted to determine the optimal strengths for the outer sextupoles was a survival plot as a function of the two sextupole strengths. The method involved launching 1000 particles in a grid evenly distributed in the transverse plane and tracked in DIMAD for 1000 turns. This was done for all combinations of $k_2^{SFA} = 0, \ldots, 20$ and $k_2^{SDA} = -20, \ldots, 0$, where $k_2 = \frac{2B_{pole}}{a^2 B\rho}$, a is the pole-tip radius, B_{pole} the pole-tip field and $B\rho$ the beam stiffness. Rather than plotting the number of surviving particles, those that were lost were plotted, as shown in Figure 2.

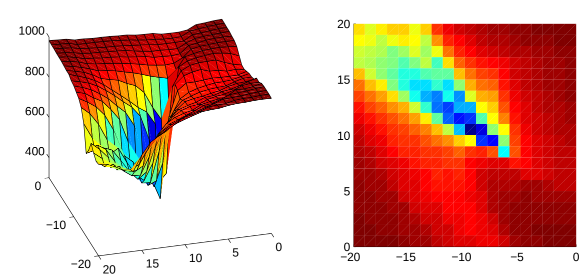

Figure 2: The above plot shows the number of particles that were lost at 1000 particles were tracked for 1000 turns. A global minumum is observed around $k_2^{SFA} = 10$ and $k_2^{SDA} = -10$.

The results indicate that the optimal values lie in a narrow valley such that $|k_2^{SFA} + k_2^{SDA}| < 2$ for $|k_2^{SDA}|$ and $|k_2^{SFA}| > 9$. This is good method to approximate the effective dynamic aperture however it gives no information as to the exact nature and size of the DA. For this information Frequency Maps (FMs) were used.

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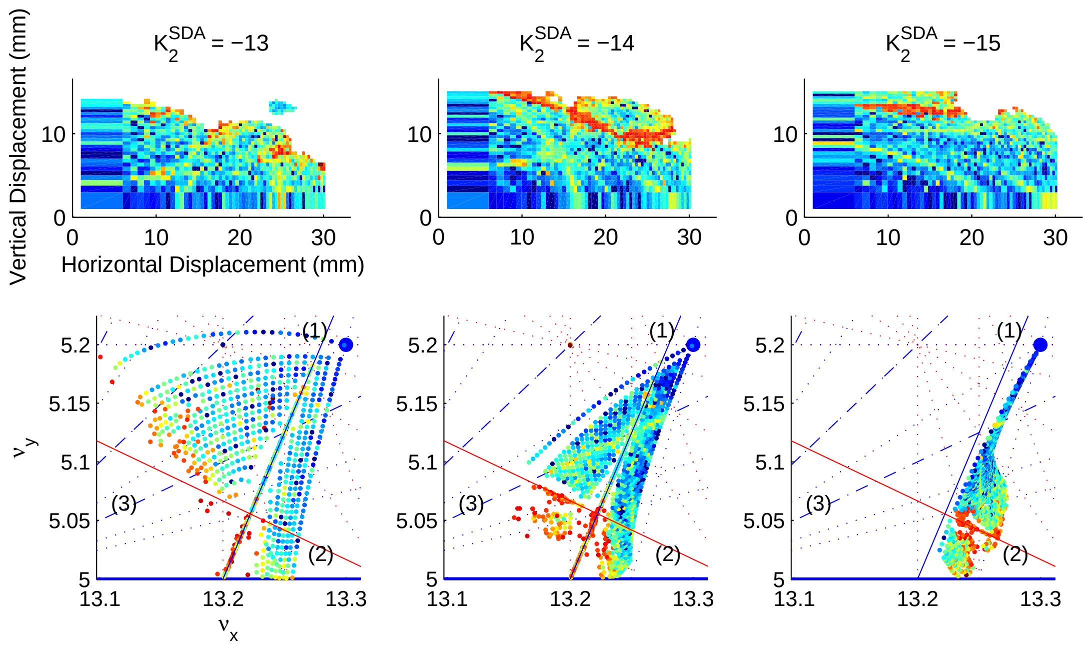

Figure 3: In the cases above $k_2^{SFA} = 14$ and k_2^{SDA} was varied. The plotted lines show both sum (red) and difference (blue) resonances up to 5th order. The labelled resonances are (1) $5\nu_x - 2\nu_y$, (2) $3\nu_x + 6\nu_y$ and (3) $\nu_x - 2\nu_y$. The vertical tuneshift with amplitude is along the side of the triangle closest to the label (1) and the horizontal tuneshift with amplitude along the side closest to (2).

FREQUENCY MAP ANALYSIS

Frequency map analysis (FMA) is a technique that can be used to determine the underlying resonances that affect the transverse and momentum apertures [7]. The method and technique of FMA is detailed in Ref. [2], [5] and [3] along with the maps and studies conducted on various third generation light sources. For all the maps in this study the diffusion index is used to colour code plots and is given by

$$D = log_{10}(\sqrt{(\nu_x^2 - \nu_x^1)^2 + (\nu_y^2 - \nu_y^1)^2}), \qquad (1)$$

where ν_x and ν_y are the transverse tunes, measured over two different time intervals. All the tracking was done using AT without radiation.

To learn more about the results of the survival plot, a number or maps were calculated such that $k_2^{SFA} \in \{11, 12, \ldots, 16\}$ and $|k_2^{SFA} + k_2^{SDA}| \leq 1$. These are values that lie along the “valley” observed in Figure 2.

The results showed how the sextupoles affect the shape and extent of the spread in frequency space. Figure 3 gives one such example where changes to the strength of the SDA sextupole, relative to SFA, changes the way the tune propagates through tune space. In the case where $k_2^{SDA} = -13$ we find that the vertical displacement mainly effects the horizontal tune while the horizontal displacement effects the vertical tune. The result (left most plots in Figure 3) is a tune spread that covers a relatively large area in tune space and a decreased DA. As the strength of SDA increases, this begins to change and the result (right most plots in Figure 3) is a more compact tune spread with fewer resonances affecting the DA. This behaviour is the same for all k_2^{SFA} between 11 and 16 m^{-2}. The effect of increasing k_2^{SFA} is an increase in the overall the vertical tuneshift with amplitude. In all cases the most problematic resonances are the three that have been marked in Figure 3. It has yet to be determined what drives these resonances and how to minimise its impact.

The results from these maps have shown that the “valley” in Figure 2 does exist and more specifically that larger apertures that are less affected by resonances require $k_2^{SDA} = -(k_2^{SFA} + 1)$. To further minimise the impact of the resonances, a careful study of alternative working points needs to be conducted and the possibility of implementing other optimisation techniques [6, 9] to get better results.

HIGHER ORDER MULTIPOLES

Results of the multipole content in the first few production quadrupole magnets were recently made available. To see how this would affect the DA we used FMA. The typical measured strengths of the allowed harmonics of the quadrupoles were added to the model and the map calculated. The results are shown in Figure 4.

The FM indicates that the additional multipoles lead to a reduced overall aperture, mainly in the horizontal. The 9th order resonance is excited, beyond which very few particles survive. The horizontal tuneshift with amplitude also appear to increase dramatically with larger amplitudes, indicating that highter order terms are being excited by the multipoles.

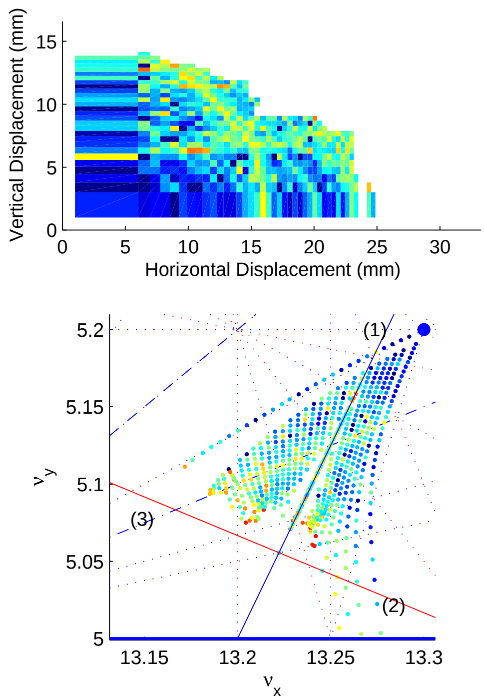

Figure 4: In the cases above $k_2^{SFA} = 14$ and $k_2^{SDA} = -14$ with the measured harmonics (allowed) of the quadrupoles. This should be compared to the centre plots in Figure 3.

DIPOLE EDGE EFFECTS AND OFF MOMENTUM STUDIES

When this model was converted into an AT model the chromaticity measured in AT was -0.23 and 3.06 in the horizontal and vertical. This was the result of AT only having first order terms when calculating the edge effects in dipoles. This was also verified with DIMAD and Elegant [1]. To correct the chromaticity, thin sextupoles ($k_2 = 0.107m^{-2}$) were added to either side of the dipoles which reduced the chromaticity to -0.016 and -0.001 in the horizontal and vertical respectively.

At this stage further investigation is required to determine if this correction is sufficiently accurate for the purpose of doing off momentum studies. Figure 5 shows the effect the sextupoles have on the transverse DA. It is fortunate that it seems to be a positive effect, improving the stability of the DA by reducing the overall spread of the vertical tunes.

CONCLUSION

FMA has been used here to successfully demonstrate the effect the outer sextupoles have on the nonlinear dynamics in the transverse plane which supports the results from the survival plots. It has also been used to show the nonlinear effects of the allowed harmonics in the quadrupoles.

To correct the chromaticity due to the edge effects in AT, sextupoles are needed. These additional sextupoles have shown to change in the transverse dynamics and it has yet

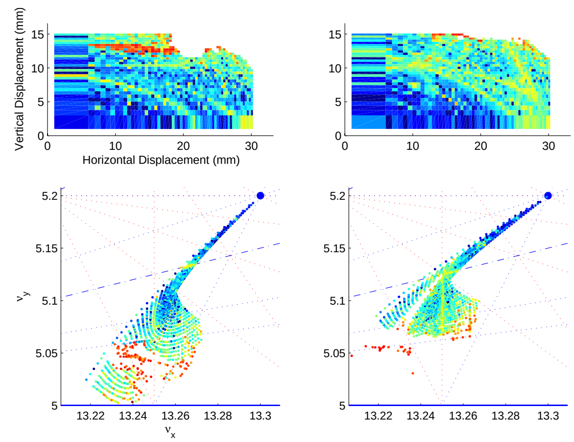

Figure 5: In the cases above $k_2^{SFA} = 14$ and $k_2^{SDA} = -15$. The plots on the left do not have thin sextupoles while those on the right do.

to be determined if this form of correction is accurate.

FMA is a useful tool for understanding underlying structure of the DA. However a more direct method also needed to correlate the parameters, such as the strengths of the sextupoles, and the constraints, amplitude or momentum dependent terms, in order build a more complete picture of the nonlinear dynamics of the system.

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