IONIZATION COOLING USING A PARAMETRIC RESONANCE*

Yaroslav Derbenev, Jefferson Lab, Newport News, Virginia
Rolland P. Johnson#, Muons, Inc, Batavia

Abstract

Muon collider luminosity depends on the number of muons in the storage ring and on the transverse size of the beams in collision. Ionization cooling as it is presently envisioned will not cool the beam sizes sufficiently well to provide adequate luminosity without large muon intensities. A new idea to combine ionization cooling with parametric resonances has been developed that will lead to beams with much smaller sizes so that high luminosity in a muon collider can be achieved with fewer muons. In the linear channel described here, a half integer resonance is induced such that the normal elliptical motion of particles in x-x' phase space becomes hyperbolic, with particles moving to smaller x and larger x' as they pass down the channel. Thin absorbers placed at the focal points of the channel then cool the angular divergence of the beam by the usual ionization cooling mechanism where each absorber is followed by RF cavities. We discuss the elementary theory of Parametric-resonance Ionization Cooling (PIC), including the need to start with a beam that has already been cooled adequately.

CONCEPT

In general, a parametric resonance is induced in an oscillating system by using a perturbing frequency that is the same as or a harmonic of a parameter of the system. Physicists are often first introduced to this phenomenon in the study of a rigid pendulum, where a periodic perturbation of the pivot point can lead to stable motion with the pendulum upside down. Half-integer resonant extraction from a synchrotron is another example familiar to accelerator physicists, where larger and larger radial excursions of particle orbits at successive turns are induced by properly placed quadrupole magnets that perturb the beam at a harmonic of the betatron frequency. In this case, the normal elliptical motion of a particle's horizontal coordinate in phase space at the extraction septum position becomes hyperbolic, $xx' = const$, leading to a beam emittance which has a wide spread in x and very narrow spread in x'.

In PIC, the same principle is used but the perturbation generates hyperbolic motion such that the emittance becomes narrow in x and wide in x' at certain positions as the beam passes down a line or circulates in a ring. Ionization cooling is then used to damp the angular spread of the beam. Figure 1 shows how the motion is altered by the perturbation.

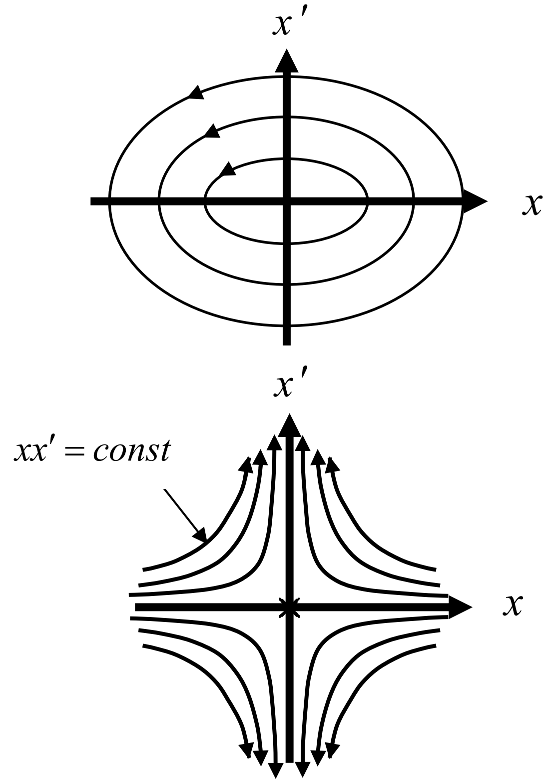

Figure 1: Comparison of particle motion at periodic locations along the beam trajectory in transverse phase space for: TOP ordinary oscillations and BOTTOM hyperbolic motion induced by perturbations at a harmonic of the betatron frequency.

The principle of ionization cooling [1] is well known, where a particle loses momentum in all three coordinates as it passes through some energy absorbing material and only the longitudinal component is replaced by RF fields. The angular divergence x' of the particle is thereby reduced until it reaches equilibrium with multiple Coulomb scattering in the material.

Thus in PIC the phase space area is reduced in x due to the dynamics of the parametric resonance and x' is reduced or constrained by ionization cooling.

For PIC to work, however, the beam must be cooled first by other means. For this analysis the initial conditions for PIC are assumed to be those as might be attained using a helical cooling channel [2].

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# rol@muonsinc.com

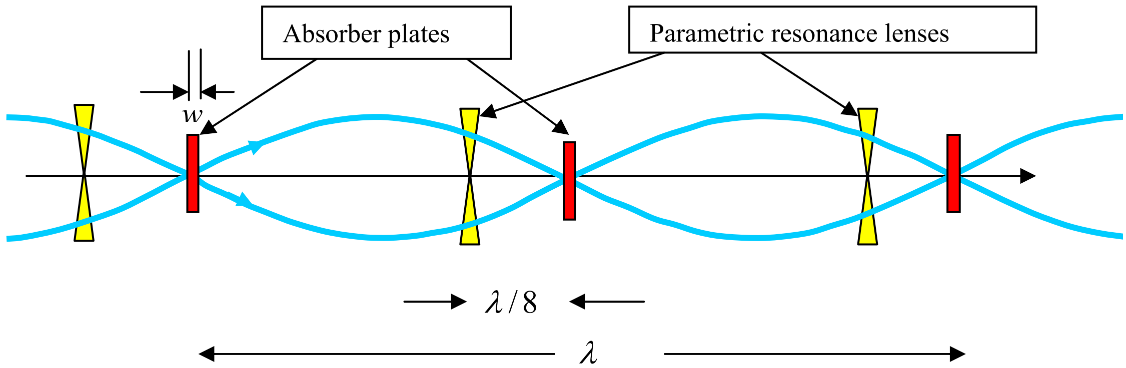

Figure 2: Conceptual diagram of a beam cooling channel in which hyperbolic trajectories are generated in transverse phase space by perturbing the beam at the betatron frequency, a parameter of the beam oscillatory behavior. Neither the focusing magnets that generate the betatron oscillations nor the RF cavities that replace the energy lost in the absorbers are shown in the diagram. The blue trajectories indicate the betatron motion of particles that define the beam envelope.

BASIC EQUATIONS

Let there be a periodic focusing lattice of period λ along the beam path with coordinate z. Particle tracking or mapping is based on a single period transformation matrix, M (between two selected points, z_0 and $z_0+\lambda$), for particle transverse coordinate and angle,

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{z_0+\lambda} = M_x \begin{pmatrix} x \\ x' \end{pmatrix}_{z_0},$$

with a similar expression for the y coordinate.

The matrices M_x and M_y are symplectic or canonical, which means each has determinant equal to one. Otherwise, the matrix elements are arbitrary in general. Thus, each can be represented in a general form convenient for later discussions as follows:

$$M = \begin{pmatrix} e^{-\lambda\Lambda_d}\cos\psi & g\sin\psi \\ -\frac{1}{g}\sin\psi & e^{\lambda\Lambda_d}\cos\psi \end{pmatrix}$$

In particular, the optical period can be designed in a way that $\sin\psi = 0$, (i.e. $\psi=\pi$ or $\psi = 2\pi$), then the evolving particle coordinate and angle (or momentum) appear uncoupled:

$$(x)_{z_0+\lambda} = \pm e^{-\lambda\Lambda_d}(x)_{z_0} \text{ and } (x')_{z_0+\lambda} = \pm e^{\lambda\Lambda_d}(x')_{z_0}.$$

Thus, if the particle angle at point z_0 grows ($\Lambda_d > 0$), then the transverse position experiences damping, and vice versa. Liouville's theorem is not violated, but particle trajectories in phase space are hyperbolic ($xx' = const$); this is an example of a parametric resonance. Exactly between the two resonance focal points the opposite situation occurs where the transverse particle position grows from period to period, while the angle damps.

Stabilizing Absorber Effect

If we now introduce an energy absorber plate of thickness w at each of the resonance focal points as shown in Figure 2, ionization cooling damps the angle spread with a rate Λ_c. Here we assume balanced 6D ionization cooling, where the three partial cooling decrements have been equalized using emittance exchange techniques as described in reference [2]:

$$\Lambda_c = \frac{1}{3}\Lambda, \quad \Lambda = 2\frac{\langle \gamma'_{abs}\rangle}{\gamma} = 2\frac{\gamma'_{acc}}{\gamma}, \quad \langle \gamma'_{abs}\rangle = \gamma'_{abs}2w/\lambda,$$

where γ'_{abs} and γ'_{acc} are the intrinsic absorber energy loss and the RF acceleration rate, respectively. If $\Lambda_d = \Lambda_c/2$, then the angle spread and beam size are damped with decrement $\Lambda_c/2$:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{z_0+\lambda} = e^{-\lambda\Lambda_c/2}\begin{pmatrix} x \\ x' \end{pmatrix}_{z_0}.$$

Phase Diffusion and Equilibrium Emittance

The rms angular spread is increased by scattering and decreased by cooling,

$$\frac{d}{dz}\overline{(x')^2} = \frac{(Z+1)}{2\gamma\beta^2}\frac{m_e}{m_\mu}\Lambda - \Lambda_c\overline{(x')^2},$$

which leads to the equilibrium angular spread at the focal point:

$$\overline{(x')^2_{eq}} = \frac{1}{\Lambda_c}\frac{d}{dz}\overline{(x')^2_{z_0}} = \frac{3}{2}\frac{(Z+1)}{\gamma\beta^2}\frac{m_e}{m_\mu}.$$

The rms product $\left[\overline{(x^2)}\cdot\overline{(x')^2}\right]^{\frac{1}{2}}_{z_0}$ determines the effective 2D beam phase space volume, or emittance.

Taking into account the continuity of x in collisions, the diffusion rate of particle position at the focus is a function of $s = z - z_0$, the local position of the beam within the absorber:

$$\delta(x)_{z_0} = -s\delta x', \quad -\frac{w}{2} \le s \le \frac{w}{2}, \quad \frac{d}{dz}\overline{(\delta x)^2_{z_0}} = \frac{w^2}{12}\frac{d}{dz}\overline{(\delta x')^2}.$$

Thus, in our cooling channel with resonance optics and correlated absorber plates, the equilibrium beam size at the plates is determined not by the characteristic focal parameter of the optics, $\lambda/2\pi$, but by the thickness of absorber plates, w. Hence, the equilibrium emittance is equal to

$$(\varepsilon_x)_{eq} = \gamma\beta\left[\overline{(x^2)\cdot(x')^2}\right]_{z_0}^{\frac{1}{2}} = \gamma\beta\frac{w}{2\sqrt{3}}\overline{(x')^2_{z_0}} = \frac{\sqrt{3}}{4\beta}(Z+1)\frac{m_e}{m_\mu}w.$$

The emittance reduction by PIC is improved compared to a conventional cooling channel by a factor

$$\frac{\pi}{\sqrt{3}}\frac{w}{\lambda} = \frac{\pi}{2\sqrt{3}}\frac{\gamma'_{acc}}{\gamma'_{abs}}.$$

Using the well-known formula for the instantaneous energy loss rate in an absorber, we find an explicit expression for the transverse equilibrium emittance that can be achieved using PIC:

$$\varepsilon_x = \frac{\sqrt{3}}{16}\beta\left(1+\frac{1}{Z}\right)\frac{(\lambda/2\pi)}{nr_e^2\log}\gamma'_{acc}.$$

Here Z and n are the absorber atomic number and concentration, m_μ the muon mass, r_e the classical electron radius, and β is the muon velocity. Here log is a symbol for the Coulomb logarithm of ionization energy loss for fast particles:

$$\log \equiv \ln\left(\frac{2p^2}{h\nu\, m_\mu}\right) - \beta^2,$$

with $h\nu$ the effective ionization potential [3]. A typical magnitude of the log is about 12 for our conditions. The equilibrium emittance in the resonance channel is primarily determined by the absorber atomic concentration, and it decreases with beam energy in the non-relativistic region.

Transverse PIC With Emittance Exchange

PIC must be used with longitudinal cooling in order to maintain the energy spread at the level achieved by the basic 6D cooling from the helical cooling channel. Emittance exchange must therefore be used for longitudinal cooling, which requires the introduction of bends and dispersion. Since the beam has already undergone basic 6D cooling, its transverse sizes are so small that the absorber plates can have a large wedge angle to provide balanced 6D cooling with even a small dispersion. Since the beam size at the absorber plates decreases as it cools, the wedge angle can increase along the beam path while the dispersion decreases. In this way, longitudinal cooling is maintained while the straggling impact on transverse emittance is negligible. Thus there is no conceptual contradiction to have simultaneous maximum transverse PIC with optimum longitudinal cooling.

Beam Optics Design and Tuning Requirements

For simultaneous PIC in the two transverse planes, the focusing channel must be designed such that the two transverse oscillations have equal tunes yet are uncoupled at the absorber plate positions. These conditions can be easily realized in a solenoidal channel. The conceptual design of simultaneous 6D PIC has yet to be addressed, however.

The resonant nature of PIC requires for hyperbolic dynamics that $2\pi\,\Delta\lambda/\lambda << \lambda\Lambda_c$ and $\Delta\lambda/\lambda < w\Lambda_c$ as conditions for maximum PIC, where $\Delta\lambda/\lambda$ is the beam tune spread. Chromaticity, non-linear aberrations, RF field, and space charge all affect the tune spread. The first three can be compensated, so it is most important that the resonance focusing and PIC do not cause space charge induced tune spread. This is another issue that is yet to be addressed.

EXAMPLE AND CONCLUSIONS

Table 1: Transverse PIC Example

Parameter	Unit	Initial	Final
Beam momentum, p	MeV/c	100	100
$\lambda/2\pi$	cm	3	3
$\lambda/2$	cm	9.5	9.5
Plate thickness, w	mm	1.6	1.6
dE/dx (Be)	MeV/m	600	600
Average energy loss	MeV/m	10	10
RF field amplitude	MV/m	30	30
Transverse emittance	mm-mr	600	24
Beam size at plates	mm	4.0	0.17
Angle spread at plates	mr	140	200
Channel length	m	70	
Integrated energy loss	GeV	0.7	
Loss due to μ decay	%	10	

The table above contains results of a calculation for an example of a transverse PIC channel using the equations shown above. The final transverse emittance for each plane is typically an order of magnitude less than what is normally assumed the best possible in an ionization cooling channel with conventional focusing. The initial values are somewhat arbitrary, where smaller initial values would allow a shorter channel.

Studies of the effects of aberrations and chromaticity corrections are underway as are simulation efforts using quadrupole, solenoidal, and helical dipole transport lines. Longitudinal PIC is also possible, although conceptually more difficult, and will be described in a future paper.

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