IBS FOR ION DISTRIBUTION UNDER ELECTRON COOLING*

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Abstract

Standard models of the intra-beam scattering (IBS) are based on the growth of the rms beam parameters for a Gaussian distribution. As a result of electron cooling, the core of beam distribution is cooled much faster than the tails, producing a denser core. In this paper, we compare various approaches to IBS treatment for such distribution. Its impact on the luminosity is also discussed.

CORE-TAIL MODEL

Formation of a dense core due to cooling and diffusion is modeled using the macro-particle approach which allows variable with time beam distribution. The individual-particle kicks due to cooling and IBS are applied in the velocity space. Such approach was the basis of the SIMCOOL code [1] and was also recently implemented in the BETACOOL code as a "Model Beam" approach [2]. To account for a core collapse (which directly impacts luminosity in a collider) of ion distribution, the "core-tail" model for the IBS was proposed. In general, to describe dynamics of such distributions an accurate kinetic simulation is required which will be addressed in the future work. With the core-tail model we attempt to capture only basic features of the core formation in order to estimate the luminosity. In this model, the individual-particle kick in the velocity space due to IBS is applied based on diffusion coefficients which are different for particles inside and outside of the core. Cooling process for typical RHIC parameters [3] is shown in Figs. 1-4. Figure 1 shows that an rms emittance under cooling stays approximately constant for these parameters, while there is a fast formation of a distinct core in the beam profiles.

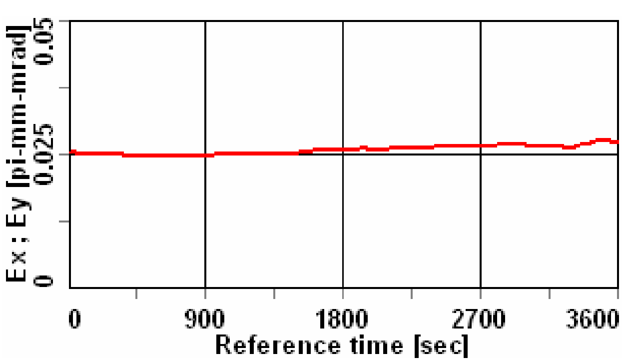

Figure 1: Time evolution of unnormalized transverse rms emittance for typical magnetized cooling parameters of Au ions at 100 GeV/u.

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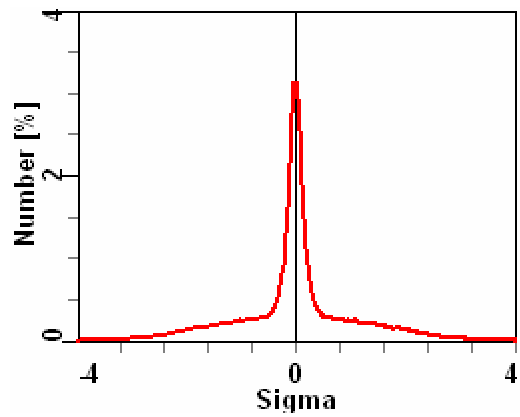

Figure 2: Transverse profile of ion beam after 1 hour of cooling with the IBS calculation based on rms values of full distribution.

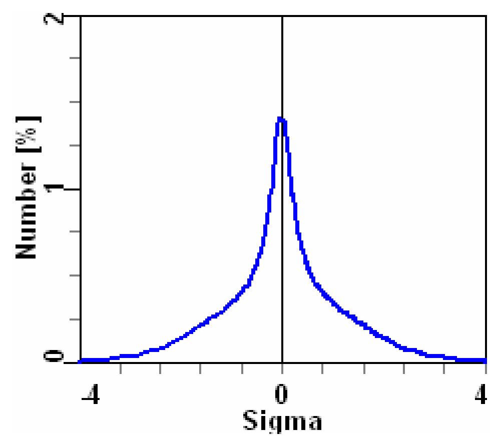

Figure 3: Transverse profile of ion beam after 1 hour of cooling with the "core-tail" IBS approach.

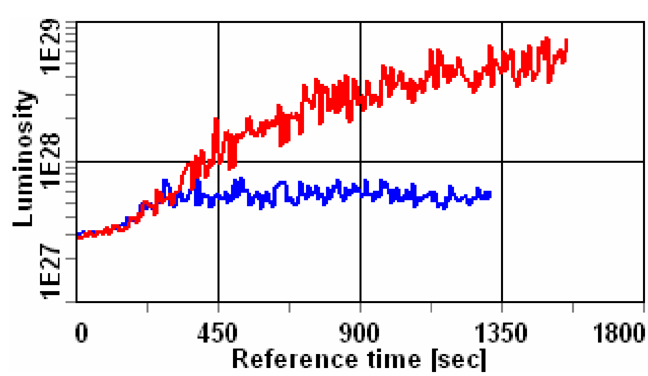

Figure 4: Luminosity growth using IBS approach based the rms of full distribution (red top curve) and using "core-tail" approach (blue), for typical parameters of the magnetized cooling for Au ions at 100 GeV/u.

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For the distribution with a pronounced core, if one calculates the rms parameters of a full distribution (with tails) and uses them to calculate the IBS kicks on all particles, the IBS for particles in the core may be strongly underestimated. Figure 2 shows formation of a "collapsed" core when the IBS kicks are applied based on the rms parameters calculated for a full distribution (which are approximately constant for chosen parameters, see Fig. 1). This calls for a different treatment of particles in the core and tails. Figure 3 shows beam distribution when IBS kicks are applied according to the "core-tail" model. The predicted luminosity based on these two approaches is very different, as shown in Fig. 4 (with logarithmic scale on the vertical axis).

To demonstrate how the core-tail model works we use simplified expressions for the diffusion coefficients based on the gas-relaxation formula [4,5]. In an approximation that the transverse temperature of the ion beam is much higher than the longitudinal, the longitudinal coefficient can be easily derived [6]. Such approximation is valid for energies much higher than the transition energies as can be seen from the flatness parameter which is defined as a ratio of beam temperatures in the beam moving frame. As a result, this parameter describes a degree of flatness of the distribution function in the velocity space:

$$g_f = \left(\frac{v_{longitud.}}{v_{transverse}} \right)^2 = \frac{\sigma_p^{\ 2}}{\gamma^2 (\varepsilon / \beta_a)}, \qquad (1)$$

where σ_p is an rms momentum spread, ε is an rms emittance, and β_a is the beta function averaged over the ring. For typical parameters of Au ions in RHIC at 100 GeV/u the flatness parameter is 0.1-0.2, and assumption that the distribution is flat ($v_{transverse} >> v_{longitudinal}$) may be used. In such an approximation, the diffusion coefficient in the longitudinal direction can be written for a bunched beam as [5]:

$$D_{zz} = \frac{N r^2 c}{8(\gamma\beta)^3} \frac{\Lambda_{ibs}}{\varepsilon_x^{\ 3/2} \sqrt{\beta_a} \sigma_s} \sqrt{\frac{2}{\pi}} = N \frac{C}{\varepsilon_x^{\ 3/2} \sigma_s}, \qquad (2)$$

where ε_x is the horizontal rms beam emittance, σ_s is an rms bunch length, β_a is an average beta-function over the ring lattice, r is the classical radius, Λ_{ibs} is the Coulomb logarithm for IBS, and N is the total number of particles in a bunch. Note, that one gets exactly the same coefficient apart from a factor $(2/\pi)^{1/2}$, using the high-energy approximation of Bjorken-Mtingwa [7] formulas for the IBS with an assumption of a smooth lattice [5]. In the "core-tail" model one finds rms parameters separately for the core (ε_c, σ_{sc}) and tails (ε, σ_s) of the distribution. These rms parameters are then used to apply different diffusion kicks for particles which are in the core and tails according to the expressions

$$D_{zz,core} = N_{core} \frac{C}{\varepsilon_c^{3/2} \sigma_{sc}} + \left(N - N_{core}\right) \frac{C}{\varepsilon^{3/2} \sigma_s}, \qquad (3)$$

$$D_{zz,tails} = N \frac{C}{\varepsilon^{3/2} \sigma_s}. \qquad (4)$$

The accuracy of the algorithm depends on finding a number of particle in the core of the distribution N_{core} and an rms parameters for the core and tails. In the first simplified approach, the core parameters were determined through the FWHM of the distribution while an rms parameters of the full distribution were used for the tails. This model was later improved with a numerical procedure which fits two Gaussian distributions to a real distribution observed in simulations for each time step of the calculation. The amplitude and width of fitted Gaussians (in all three directions) provide more accurate parameters which are used instead of ε, ε_c, σ_s, σ_{sc} in the diffusion coefficients. In the same approximation of the high energy, when heating is dominated by a longitudinal degree of freedom, the transverse diffusion rate can be expressed through the longitudinal one using the H-function of the ring (for a smooth lattice it is $\langle D_x^2/\beta_x \rangle$):

$$\tau_\perp^{-1} = \frac{\sigma_p^{\ 2}}{\varepsilon_x} \left\langle \frac{D_x^{\ 2} + (D_x^{'} \beta_x + \alpha_x D_x)^2}{\beta_x} \right\rangle \tau_{II}^{\ -1}, \qquad (5)$$

where σ_p is the rms momentum spread, D_x, D'_x, α_x, β_x are the lattice functions, $\langle\ \rangle$ stands for averaging over the ring, and the longitudinal growth rate is defined as

$$\tau_{II}^{\ -1} = \frac{1}{\sigma_p^{\ 2}} D_{zz}. \qquad (6)$$

More general relations, which are valid for all energies and include derivatives of the lattice functions are summarized in [7,8,9]. In fact, for accurate simulation of the IBS in RHIC we do use one of the expressions from [7,8,9], all of which are implemented in the BETACOOL code, and were recently cross-checked vs. one another [10]. Experimental verification of these models vs. dedicated experiments of IBS at RHIC [11] with good agreement, increased our confidence in numeric models being used. The "core-tail" approach is also implemented in the BETACOOL code. As a result, it allows us to use more accurate diffusion coefficients (rather than the one in Eq. (2)) which can take into account correct ratio between the temperatures of the ion beam, as well as realistic RHIC lattice (including derivatives). In addition, the standard IBS theory was recently reformulated for the growth rates of a bi-Gaussian distribution [12].

DIFFUSION COEFFICIENTS

The "core-tail" model described in previous section is just an application of standard IBS theory [7,8,9] for the distribution which has a pronounced core with an attempt to have an estimate for expected luminosity. Also, it does not take into account dependence of the diffusion coefficients on particle amplitudes within the core. Below we try to explore the accuracy of such assumption.

A detailed treatment of the IBS, which depends on individual particle amplitudes, was recently proposed by Burov [13], with an analytic formulation done for a

Gaussian distribution in approximation that transverse rms velocity of the ion beam is much higher than the longitudinal. In general, for accurate dependence of the diffusion coefficients on both the transverse and longitudinal velocities, which are needed to describe IBS for different beam parameters at different energies (which is one of the tasks for RHIC since cooling at various energies is considered [14]), the integrals over the distribution function should be performed numerically. Similar algorithm was recently implemented for numerical calculation of the non-magnetized friction force [2], for example. For accurate IBS calculation one has to calculate such integrals at each lattice element which would make calculations too slow. However, similar approach is presently being considered for a simplified lattice structure in order to perform needed benchmarking of the core-tail model.

In the "core-tail" model, the diffusion coefficients are found at each lattice element for the particles in the core and tails of beam distribution. However, the assumption is made that all particle in the core get the same kick compared to the amplitude-dependent coefficient within the core. To understand the accuracy of this approximation one can have a look at the dependence of the diffusion coefficient on amplitude. For anisotropic Maxwellian distribution written in the form:

$$f(v) = \frac{n}{\pi\sqrt{2\pi}\Delta^2{}_\perp \Delta_{II}} e^{-v^2{}_\perp / \Delta^2{}_\perp - v_{II}^2/(2\Delta^2{}_{II})}, \quad (7)$$

the longitudinal component of the diffusion tensor is given by

$$D_{zz} = \frac{4\pi n(Ze)^4}{m^2} \Lambda_{ibs} \int d^3 v f(\vec{v}) \frac{w^3 - w^2{}_z}{w^2}, \quad (8)$$

with $\vec{u} = u\hat{x}$ and $\vec{w} = v - u\hat{x}$. When one assumes that an rms transverse velocity is much larger than the longitudinal one (flattened distribution), the integrals can be easily evaluated, as shown, for example, by Sorensen [5], with the result:

$$D_{zz} = \frac{4\pi n(Ze)^4}{m^2 \Delta_\perp} \Lambda_{ibs} \left[\sqrt{\pi} e^{-u^2/2\Delta^2{}_\perp} I_0\left(\frac{u^2}{2\Delta^2{}_\perp}\right)\right]. \quad (9)$$

A generalization of this expression to include also dependence on spatial amplitudes was done by Burov [13]. The function in square brackets in Eq. (9) decreases very slowly with the amplitude u. For u=0, it gives a factor $\sqrt{\pi}$, while for amplitudes equal to an rms value $\Delta_\perp$, the expression in the square brackets is close to unity, which allows us to approximate the amplitude dependence by an rms values with a reasonably good accuracy. The expression in Eq. (9) is then simply becomes:

$$D_{zz} = \frac{4\pi n(Ze)^4}{m^2 \Delta_\perp} \Lambda_{ibs}. \quad (10)$$

When rewritten in terms of beam parameters for the bunched beam, Eq. (10) gives Eq. (2). As a result, it seems that an assumption of a constant-amplitude kick based on the rms values should give reasonably accurate estimate. At the same time it allows us to do efficient computer calculations and include (in an accurate way) the ratio between the transverse and longitudinal beam velocities without approximations. Also, such an assumption allows us fast calculation of the IBS at each lattice element with a realistic lattice rather than assuming a smooth approximation. The formulas for "detailed" IBS by Burov [13] were also included in the BETACOOL. Cooling dynamics results based on the "detailed" and "core-tail" approach were found qualitatively similar. However, it is not clear whether observed differences are related to the amplitude dependence or to the approximations being made in [13]. At this point, we are evaluating various scenarios, including the use of formalism in [12] for the diffusion coefficients, as well as a possibility of an accurate numerical calculation, to understand what is the most efficient and at the same time sufficiently accurate approach. Benchmarking vs. 3D "IBS map" approach [15], as well as vs. experiments is also being considered.

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