STABILITY OF BARRIER BUCKETS WITH SHORT OR ZERO BARRIER SEPARATIONS*

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Abstract

A barrier bucket with very short or zero rf-barrier separation (relative to the barrier widths) has its synchrotron tune decreasing from a very large value towards the bucket boundary. As a result, chaotic region may form near the bucket center and extends outward under increasing modulation of rf voltage and/or rf phase. Application is made to those barrier buckets used in momentum mining at the Fermilab Recycler Ring.

UNPERTURBED BARRIER SYSTEM

During momentum mining at the Fermilab Recycler Ring, a barrier bucket with zero barrier separation is opened to store the unmined particles. At the barrier width $T_1 = 1.27$ μs and height $V_0 = 2$ kV, the maximum half bucket height is $\Delta E_{\rm pk} = \sqrt{2\beta^2 eV_0T_1E_0/(|\eta|T_0)}$ $=21.77$ MeV, with nominal beam energy $E_0 = 8.938$ GeV, nominal velocity $v=\beta c$, c being velocity of light, revolution period $T_0 = 11.13$ μs, and slip factor $\eta = -0.008812$. The synchrotron frequency is infinite at the center of the bucket, and decreases hyperbolically with the maximum energy offset $\widehat{\Delta E}$ to the minimum value $\nu_{s\,\rm min} = \frac{1}{4}eV_0/\Delta E_{\rm pk} = 2.297\times 10^{-5}$ (Eq. 1) at the edge. Because of the rather slowly decreasing synchrotron tune towards the bucket edge, fixed points of parametric resonances exhibit themselves rather far away from the edge in the presence of voltage and/or rf phase modulation. The implication is that, unlike a sinusoidal rf bucket, chaotic regions, if present, start from the bucket center and extend outward when the modulation strength increases as illustrated in Fig. 1. This paper serves as an extract of Ref [1].

Without voltage or phase modulation, the equations of motion of a beam particle are

$$\frac{d\tau}{d\theta}=\frac{\eta\Delta E}{\omega_0\beta^2E_0},\quad \frac{d\Delta E}{d\theta}=\frac{eV_0T_1}{2\pi}\frac{\partial f_0}{\partial\tau}=\frac{eV_0}{2\pi}\,f_1(\tau,T_1),$$

where ΔE is the energy offset, τ is the arrival time lagging

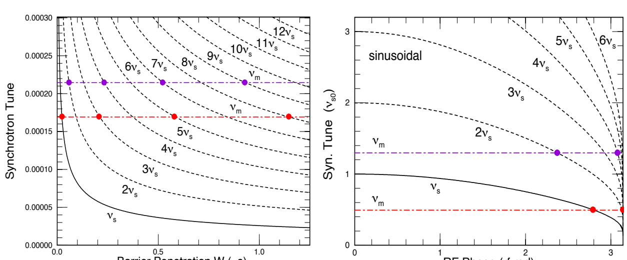

Figure 1: Left: Synchrotron tune and harmonics vs. barrier penetration showing resonances extending from bucket center towards edge. The reverse is true for the sinusoidal rf bucket on the right.

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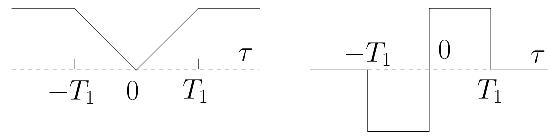

$f_0(\tau,T_1)$ $\qquad$ $f_1(\tau,T_1)$

Figure 2: Reduced rf potential $f_0(\tau,T_1)$ and rf voltage $f_1(\tau,T_1) = T_1\partial f_0(\tau,T_1)/\partial\tau$.

behind some on-energy particle. The barrier voltage and potential are V_0f_1 and $V_0T_1f_0$ as illustrated in Fig 2. They can be derived from the Hamiltonian

$$H_0=\frac{\eta(\Delta E)^2}{2\omega_0\beta^2E_0}-\frac{eV_0T_1}{2\pi}f_0(\tau,T_1),$$

from which the maximum energy offset $\widehat{\Delta E}$ corresponding to the maximum barrier penetration W is obtained:

$$H_0=-\frac{|\eta|(\widehat{\Delta E})^2}{2\omega_0\beta^2E_0}=-\frac{eV_0W}{2\pi}.$$

The action-angle variables (J,ψ) are, respectively,

$$J=\frac{1}{2\pi}\oint\Delta E d\tau=-\frac{4W\widehat{\Delta E}}{3\pi}=-\frac{4W^{3/2}}{3\pi}\sqrt{\frac{\omega_0\beta^2E_0eV_0}{\pi|\eta|}},$$

$$\psi=\frac{\partial F_2}{\partial J}=\frac{dW}{dJ}\int_0^\tau\frac{\partial\Delta E}{\partial W}d\tau=-\frac{\pi}{4\sqrt{W}}\int_0^\tau\frac{d\tau}{\pm\sqrt{W-T_1f_0}},$$

where $F_2(J,\tau) = \int_0^\tau \Delta E\, d\tau$ is the generating function. The synchrotron tune can be derived easily:

$$\nu_s(W)=\frac{\partial H_0}{\partial J}=\nu_{s\,\rm min}\frac{\Delta E_{\rm pk}}{\widehat{\Delta E}}=\nu_{s\,\rm min}\sqrt{\frac{T_1}{W}}.\qquad(1)$$

VOLTAGE MODULATION

Voltage modulation is introduced by the substitution $V_0 \longrightarrow V_0(1 + a\cos\nu_m\theta)$, where ν_m is the modulation tune and aV_0 is the modulation voltage. The Hamiltonian receives a perturbative term

$$\Delta H=-\frac{eV_0T_1}{2\pi}f_0(\tau,T_1)\,a\cos\nu_m\theta=-\frac{aeV_0W}{3\pi}\cos\nu_m\theta$$
$$+\sum_{n=1,2,\cdots}\frac{aeV_0W}{n^2\pi^3}\Big[\cos(2n\psi+\nu_m\theta)+\cos(2n\psi-\nu_m\theta)\Big],$$

where we have used the expansion

$$f_0(\tau,T_1)=\frac{2W}{3T_1}-\sum_{n=1,2,\cdots}\frac{4W}{n^2\pi^2T_1}\cos 2n\psi.$$

Picking out the $2n$:1 resonance, the Hamiltonian in a frame rotating with the perturbation becomes

$$H(J,\psi)=-p(-J)^{2/3}\left[1-\frac{2a}{n^2\pi^2}\cos 2n\psi\right]-\frac{\nu_m}{2n}J,$$

$$p=\frac{eV_0}{2\pi}\left(\frac{3\pi}{4}\right)^{2/3}\left(\frac{T_0|\eta|}{2\beta^2E_0eV_0}\right)^{1/3}.$$

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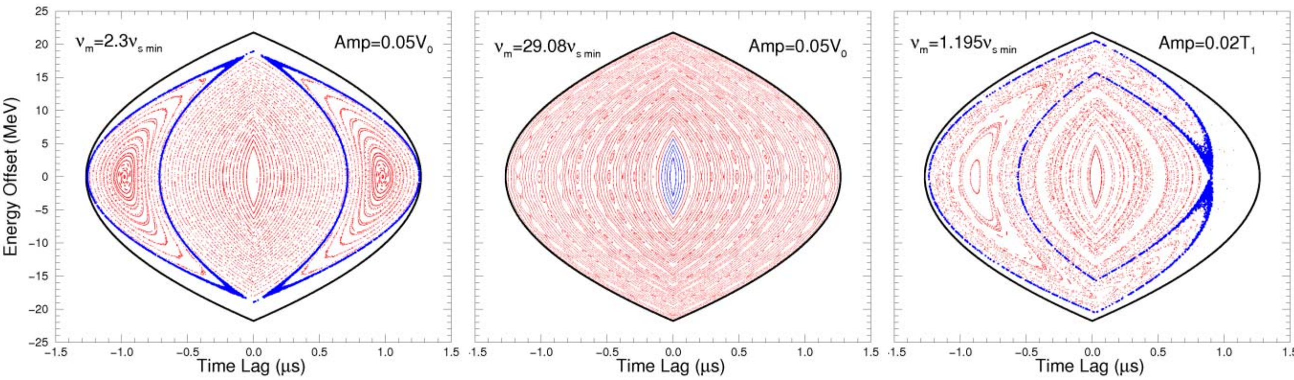

Figure 3: Poincare section showing rf voltage modulation with $aV_0 = 0.05V_0$ at modulation tune (left) $\nu_m = 2.3\nu_{s\,\rm min}$ with excitation of 2:1 resonance, (middle) $\nu_m = 2.3\nu_{s\,\rm min}$ with no chaoticity. Right: Poincare section showing rf phase modulation with $aT_1 = 0.02T_1$ at $\nu_m = 1.195\nu_{s\,\rm min}$ with 1:1 resonance excited.

When the modulation frequency is near an even harmonic of the synchrotron frequency, the particle motion is perturbed severely with possible particle loss. Left plot of Fig. 3 shows the simulation of 100 particles for 5 million turns (55.7 s) with voltage modulated at $a = 0.05$ and $\nu_m = 2.30\nu_{s\,\rm min}$. The 2:1 resonance is evident. Particles just outside the inner separatrix, $\tau \approx \pm 071\ \mu$s, can be driven to the outer separatrix and get lost. At high modulation tune ν_m, all $2n$:1 resonances with n up to $\nu_m/(2\nu_{s\,\rm min})$ will be excited. The half width of an island chain is $\delta(-J)^{2/3} = [16\sqrt{a}/(n\pi)][2p/(3\nu_m)]^2$ while the separation between two adjacent island chains is $\Delta(-J)^{2/3} = (2n+1)[\frac{2p}{3\nu_m}]^2$. Thus adjacent chains will overlap only when $|a| > \frac{\pi^2}{4}[1+\frac{1}{2n}]^2$ which is too big to be possible. Thus there will not be any chaotic region as is shown in middle plot of Fig. 3 at $\nu_m = 29.08\nu_{s\,\rm min}$ and $a = 0.02$.

To estimate the tolerance of the rf voltage modulation, we calculate the maximum stable bunch area of the rf system by tracking 5000 particles for more than 100 synchrotron oscillations (with reference to $\nu_{s\,\rm min}$). The fractional stable area in units of the bucket area is defined as the ratio of the number of survival particles to the number of initial particles. The result is depicted in Fig. 4. It appears that the bucket cannot be filled to more than 95% without encountering beam loss, when the modulation amplitude is larger than $a \sim 0.001$.

RF PHASE MODULATION

Rf phase modulation is introduced by $\tau \longrightarrow \tau + aT_1 \cos \nu_m\theta$. The perturbation term in the Hamiltonian becomes

$$\Delta H = -\sum_{m=1,3,\cdots} \frac{aeV_0T_1}{m\pi^2}\Big[\sin(m\psi+\nu_m\theta)+\sin(m\psi-\nu_m\theta)\Big],$$

where use has been made of the expansion

$$f_1(\tau, T_1) = \sum_{m=1,3,\cdots} \frac{4}{m\pi}\sin m\psi.$$

Picking out one m:1 resonance, the Hamiltonian in a frame rotating with the perturbation becomes

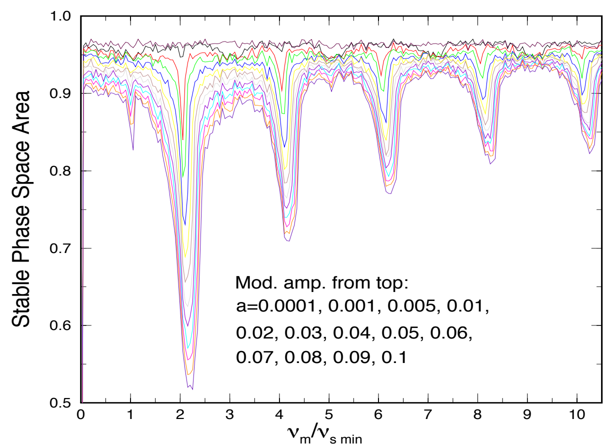

Figure 4: Fractional stable bunch area to bucket area as functions of voltage modulation tune, for various modulation amplitudes.

$$H(J,\psi) = -p(-J)^{2/3} - \frac{\nu_m}{m}J - \frac{aeV_0T_1}{m\pi^2}\sin m\psi.$$

The effect of rf phase modulation will be stronger than the effect of voltage modulation, because of the m^{-1} dependency of the resonance strengths. Simulation of the 1:1 resonance is shown in the right plot of Fig. 3 with phase modulation amplitude $aT_1 = 0.02T_1$ at the modulation tune of $\nu_m = 1.195\nu_{s\,\rm min}$. Here, the half-width of the island chain is $\delta(-J)^{2/3} = \sqrt{am/\pi}(4\nu_m/\nu_{s\,\rm min})[2p/(3\nu_m)]^2$ while the separation between 2 adjacent chains is $\Delta(-J)^{2/3} = 4(m+1)\ [2p/(3\nu_m)]^2$. Thus the condition of overlapping island chains is

$$a > \frac{\pi(m+1)^2\nu_{s\,\rm min}^2}{m\nu_m^2},$$

which is also the condition of evolution into chaos. The 3 plots in Fig. 5 are for phase modulation amplitude $a = 0.02$ at modulation tunes $\nu_m = 29.08,\ 58.13$, and $87.24\nu_{s,\rm min}$, corresponding to 60, 120, and 180 Hz. Only one particle has been used. We see that the chaotic region becomes larger with higher modulation tunes, and the particle wanders outside the bucket eventually in the last plot.

To estimate the tolerance of the rf phase modulation, we calculate the maximum stable bunch area of the rf system by tracking 1000 particles for about 1000 synchrotron

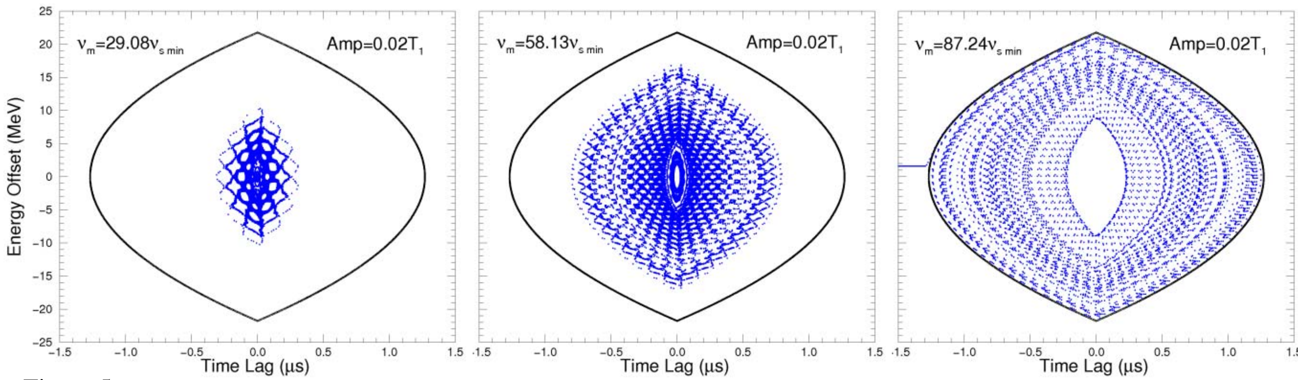

Figure 5: Poincare sections showing the chaotic region produced by tracking 1 particle with phase modulation $a = 0.02$ at modulation frequencies 60 Hz (left), 120 Hz (middle), and 180 Hz (right), corresponding to modulation tunes $\nu_m = 29.08$, 58.13, and $87.24\nu_{s\,\mathrm{min}}$.

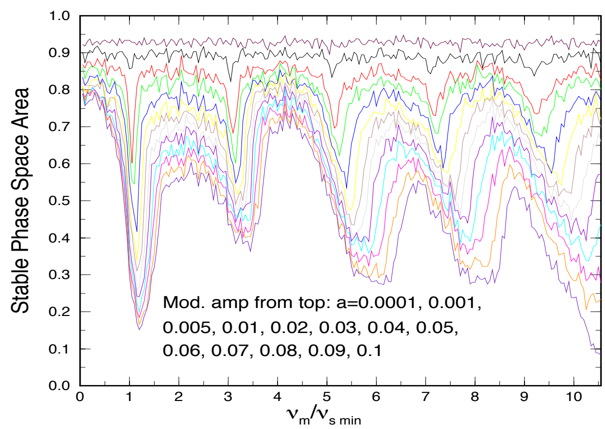

Figure 6: Fractional stable bunch area to bucket area as functions of phase modulation tune for various modulation amplitudes.

oscillations (with reference to $\nu_{s,\mathrm{min}}$), about 8.1 min. The results depicted in Fig. 6 show that the bucket cannot be stable if it is more than 92% filled at $a \gtrsim 0.0001$. We find rf phase modulation more devastating than rf voltage modulation.

SYNCHRO-BETATRON COUPLING

An extra dipole field with an extra bending angle $\Delta\theta$ results in a closed-orbit lengthening of $\Delta C = D\Delta\theta$, D being the dispersion. The beam particle will arrive at the rf cavity with a phase error $\Delta T = \Delta C T_0/C$ in time. When this extra dipole field comes from the vibration of a quadrupole with a horizontal modulation tune ν_m, the rf phase error accumulates for roughly half the modulation period before de-accumulation takes place, and the accumulation enhancement factor is $(2\pi\nu_m)^{-1}$. Thus the rf phase error will oscillate at the modulation tune with the amplitude

$$a = \frac{\Delta T}{T_1} = \frac{D\Delta\theta}{2\pi\nu_m C}\frac{T_0}{T_1}.$$

If we wish to fill the barrier bucket up to 92%, Fig. 6 indicates that the rf phase error has to be less than $a \sim 0.0001$ to avoid beam loss. This implies that the allowable orbit lengthening is $\Delta C = D\Delta\theta = 2\pi\nu_m a C T_1/T_0 = 54.7\ \mu$m, where we have used the Recycler circumference $C = 3319$ m and set $\nu_m = \nu_{s,\mathrm{min}}$.

A typical quadrupole set in the Recycle Ring consists of two half-quadrupoles each of length $\Delta\ell/2 = 0.787$ m separated by 1 m with a field gradient $K_1 = B_y'/(B\rho) = 0.0886$ m^{-2}, where $(B\rho)$ is the rigidity of the beam. The half FODO cell is 17.28 m long. If the quadrupole set has a horizontal offset $\Delta x = 1\ \mu$m from the designed orbit, the beam will see an extra dipole field $\Delta B_y = K_1\Delta x(B\rho)$, receive an extra bend of $\Delta\theta = \Delta B_y\Delta\ell/(B\rho) = K_1\Delta x\Delta\ell$, and thus lengthen the closed orbit by $\Delta C = D\Delta\theta = 3.07\ \mu$m, about 18.0 times less than the stability criterion derived above, where maximum dispersion of $D = 2.227$ m has been used.

Consider a truck driven on the service road about 22 m above the Recycler Ring. The stability criterion will be surpassed if horizontal vibrations are excited in 18 consecutive quadrupole sets with an amplitude of $\Delta x = 1\ \mu$m. The next possibility is the influence of the LCW water cooling pumps of the Main Injector which shares the same tunnel with the Recycler Ring. When all the 208 sets of quadrupoles oscillate randomly at the natural frequency of $\nu_m\omega_0/(2\pi) = 9.6$ Hz [4], the lengthening of the closed orbit will be enhanced by $\sqrt{206} = 14.4$ times that from one quadrupole. However, the stability criterion will also be enhanced by $\nu_m/\nu_{s\,\mathrm{min}} = 4.7$ times because of the higher modulation frequency. As a result, an oscillation amplitude of $\Delta x = 6\ \mu$m will surpass the stability criterion. Another possibility is the 60 Hz electrical noise corresponding to the modulation tune $\nu_m = 29.08\nu_{s,\mathrm{min}}$. If all the quadrupole sets are excited randomly, an amplitude of oscillation of $\Delta x = 36\ \mu$m is required to surpass the stability criterion.

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