

# IMPEDANCE ANALYSIS OF LONGITUDINAL BUNCH SHAPE MEASUREMENTS AT PLS

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## Abstract

We measured the longitudinal bunch shape by a streak camera at 2.5 GeV Pohang Light Source (PLS). The impedances estimated by a series R+L model indicate a resistance  $R=0.96$  k $\Omega$ , an inductance  $L=80$  nH and a longitudinal impedance  $|Z/n|_{eff}=0.53$   $\Omega$ . The scaling law for the bunch lengthening is expressed as  $I^{0.22}$ . The effects of insertion device in the ring on the impedance, particularly the effect of the vertical height of the in-vacuum undulator currently placed in the PLS ring is also presented.

## INTRODUCTION

The stored beam quality in a storage ring depends on details of the electromagnetic interaction of the beam bunch with its surrounding geometry such as RF cavity and various discontinuities of the vacuum chamber. The interaction is described by wake fields in the time domain or impedance in the frequency domain. As a result, the electromagnetic interaction causes the parasitic energy loss, the energy spread of electrons in a bunch, and transverse forces which increase the effective emittance of a beam, etc. A simple model which is a resistor and an inductor connected in series can describe the potential-well distortion and the parasitic mode losses[1]. In this paper we present bunch profile measurements at PLS to estimate the impedances. The effect of the revolver undulator is also presented.

## R+L IMPEDANCE MODEL

The impedance of the storage ring is approximated by a series R(resistance) +L(inductance) impedance model [1]. The steady-state bunch distribution for a series R+L impedance can be obtained from the Haïssinski equation:

$$\lambda(z) = \frac{\exp\left(\frac{-z^2}{2\sigma_{z0}^2} + \frac{1}{V'_{rf}\sigma_{z0}} \int_0^z V_{ind}(z')dz'\right)}{\int_{-\infty}^{\infty} \exp\left(\frac{-z'^2}{2\sigma_{z0}^2} + \frac{1}{V'_{rf}\sigma_{z0}} \int_0^{z'} V_{ind}(z'')dz''\right)}, \quad (1)$$

where the induced voltage is given by

$$V_{ind}(z) = - \int_0^{\infty} W(z')\lambda(z-z')dz', \quad (2)$$

with  $\lambda$  the bunch-position distribution,  $z$  the longitudinal position ( $z < 0$  is toward the front of the bunch),  $V'_{rf}$  the slope of the rf voltage,  $\sigma_{z0}$  the zero-current bunch length,

and  $W(z)$  the wakefield. For the series R+L impedance, it is given by

$$V_{ind} = -eNc(R\lambda + cL\lambda'). \quad (3)$$

For this case, Eq. (1) can be written in non-linear differential equation with normalized units as

$$y' = -y \frac{x + ry}{1 + ly}, \quad (4)$$

where  $x = z/\sigma_{z0}$  and  $y(x) = \lambda\sigma_{z0}$ . The normalized resistance and inductance are:

$$r = ecNR/(V'_{rf}\sigma_{z0}^2), \quad (5)$$

$$l = ec^2NL/(V'_{rf}\sigma_{z0}^3). \quad (6)$$

From the numerical results with the PLS parameters, we can treat the resistance R and the inductance L linearly and separately.

Table 1: PLS parameter

Parameter	Value
Energy	2.5 GeV
Circumference	280.56 m
Revolution Frequency	1.068 MHz
Momentum Compaction	0.00181
Tune (z/x/y)	0.01/14.28/8.18
Natural Bunch Length	9 mm
Energy Spread	$8.5 \times 10^{-4}$
Damping Time (z/x/y)	4.3/8/8 ms
Harmonic Number	468
Stored Beam Current	170 mA

## MICROWAVE INSTABILITY

As the bunch current increases, the shift of each azimuthal mode becomes so big that two adjacent modes overlap each other [2,3]. This phenomenon has been referred to as "mode-mixing", "strong head-tail", "turbulence", and/or "microwave instability" in the literature. A rough estimate of the threshold is given by

$$\eta_2 = \frac{4\pi^2 eI_b \eta}{3\beta^2 E_0 \omega_s^2 \tau_L^3} \left. \frac{Z_0^{\parallel}}{w} \right|_{eff} \approx 1, \quad (7)$$

where the effective longitudinal impedance for mode  $m$  is defined as

$$\left. \frac{Z_0^{\parallel}}{m} \right|_{eff} = \frac{\int d\omega \frac{Z_0^{\parallel} \omega}{w} h_m(\omega)}{\int d\omega h_m(\omega)}. \quad (8)$$

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If the bunch samples the impedance at a frequency range where  $Z_0^{\parallel} \propto \omega^a$ , the effective impedance is proportional to  $\sigma_z^{1-a}$ . Thus the bunch length obeys the Chao-Gareyte scaling law

$$\sigma_z \propto \xi^{1/(2+a)}, \quad (9)$$

where

$$\xi = \frac{\eta I_b}{\nu_s^2 E_0} \quad (10)$$

is the scaling parameter.

## BEAM PROFILE MEASUREMENT

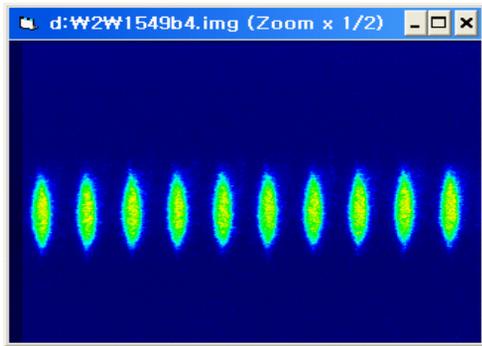


Figure 1: An Example of the streak camera image in the synchro-scan mode. The single bunch of 15.49 mA is stored. The vertical length represents the longitudinal bunch length, and the horizontal range is  $10 \mu\text{s}$ .

### Streak Camera

A dual-axis streak camera, Hamamatsu model C5680, has been used to measure the longitudinal profile of the electron bunch. We can see the bunch profile in the synchro-scan mode as shown in Fig. 1. With the horizontal sweeping time range of  $10 \mu\text{s}$ , we took five snapshots per the same single bunch current.

### Image Data Processing Results

The images were fitted with the normal and asymmetric gaussian curve. The asymmetric gaussian curve is better than normal one because the solution of the Haïssinski equation is generally not symmetric. But the fitting results are almost the same at the point of the average and the standard deviation. The resistance was then estimated from the bunch center shift and the inductance from the bunch length.

Fig. 2 shows the bunch-center shift due to the resistive impedance. As the bunch-current increases, the bunch loses more energy due to the resistance and moves forward to supply for the lost energy. Above the threshold current, the energy spreading and the bunch lengthening make the center shift slow down. As shown in Fig. 3, the zero current length is measured to be 28 ps. The bunch lengthening is

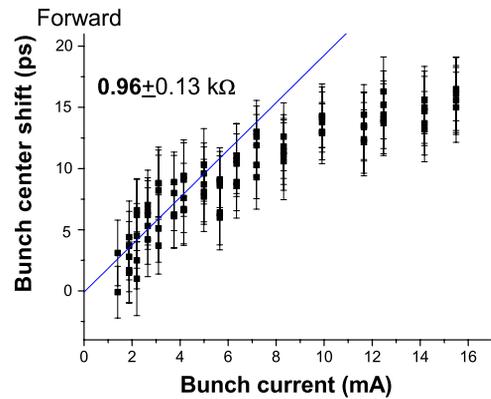


Figure 2: The linear resistance below the threshold current is measured as  $0.96 \pm 0.13 \text{ k}\Omega$  at March 23, 2004.

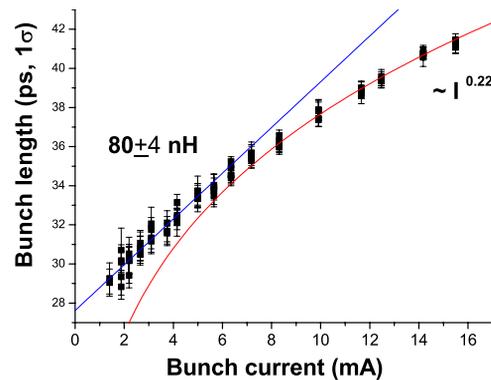


Figure 3: The linear inductance is measured as  $80 \pm 4 \text{ nH}$  at March 23, 2004. Above 8 mA, the scaling law for the bunch lengthening is expressed as  $I^{0.22}$ .

proportional to the current below 6 mA and is estimated as  $I^{0.22}$  above 8 mA. This scaling factor  $a = 2.5$  is quite different from the boussard criterion. Because the microwave instability causes the microwave fluctuation in the longitudinal density, the bunch profile fluctuates above the threshold

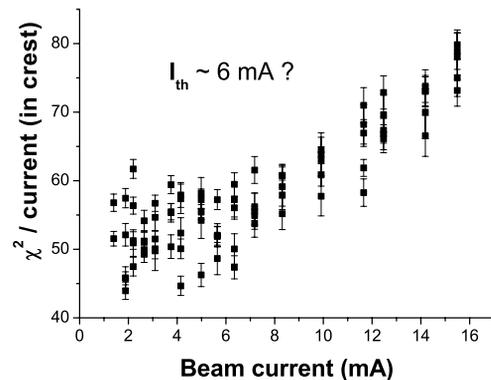


Figure 4: The fitting error normalized by the current. The longitudinal density fluctuates due to the microwave instability.

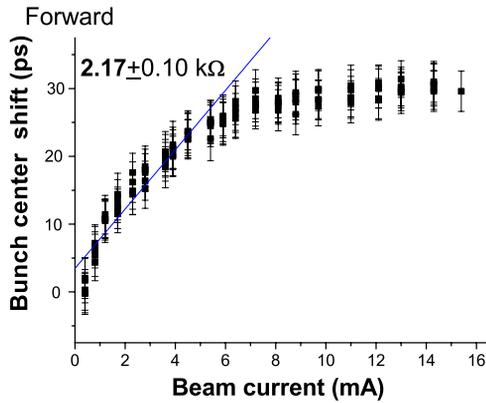


Figure 5: The linear resistance below the threshold current is measured as  $2.17 \pm 0.10 \text{ k}\Omega$  at December 19, 2004.

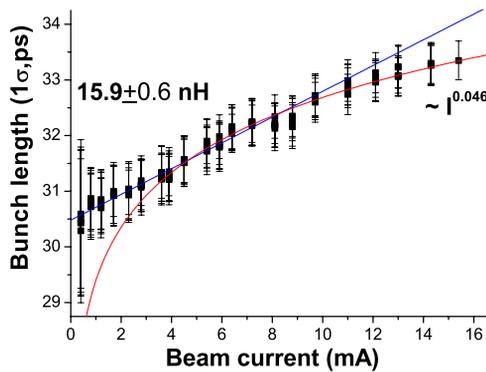


Figure 6: The linear inductance is measured as  $15.9 \pm 0.6 \text{ nH}$  at December 19, 2004. Above 6 mA, the scaling law for the bunch lengthening is expressed as  $I^{0.046}$ .

current. As shown in Fig. 4 the fitting error can be regarded as the profile fluctuation. After 2004 summer shutdown the ring impedance was notably changed as in Fig. 5, 6. The resistance increased to  $2.17 \pm 0.10 \text{ k}\Omega$  while the inductance decreased to  $15.6 \pm 0.6 \text{ nH}$ . Above 6 mA, the scaling law for the bunch lengthening is expressed as  $I^{0.046}$ . The new insertion device's resistive wall impedance could be a source of

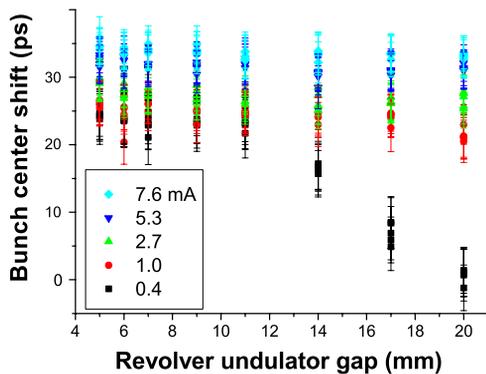


Figure 7: The bunch center shift due to the revolver undulator.

the resistance change. The reduction of the inductance requires some further explanations such as a new damping mechanism. There is no definite threshold in the longitudinal distribution. The threshold current of microwave instability is roughly estimated to be 4~6 mA. The energy spread measurement is required for more accurate threshold. With the PLS parameters and the threshold current 4 mA, the effective impedance is about  $0.5 \Omega$  from Eq. (1). The longitudinal impedance design budget of the PLS is about  $|Z/n| \approx 2 \Omega$  [4].

The bunch lengthening is independent of the revolver undulator gap but the narrower undulator gap causes the bunch center shift as shown in Fig. 7. It can be explained by the resistive-wall impedance of the undulator. The center shift is saturated at 11 mm gap.

## CONCLUSION

The effective impedances for the bunch deformation are obtained from the analysis of the streak camera images when the single bunch stored. The effective impedances is estimated by a series R+L model indicate a resistance  $R = 0.96 \pm 0.13 \text{ k}\Omega$ , an inductance  $L = 80 \pm 4 \text{ nH}$  and a longitudinal impedance  $|Z/n|_{eff} = 0.53 \Omega$  in spring 2004. Two methods (the effective linear impedance from the profile and the threshold current for microwave instability) agreed with the each other. The scaling law for the bunch lengthening is expressed as  $I^{0.22}$ . The resistive wall impedance of the revolver undulator cause the center shift that means the energy loss. In winter 2004, The resistance increased to  $2.17 \pm 0.10 \text{ k}\Omega$  and the inductance decreased to  $15.6 \pm 0.6 \text{ nH}$ . This requires some further investigation.

## REFERENCES

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