PRINCIPLE OF GLOBAL DECOUPLING WITH COUPLING ANGLE MODULATION*

Y. Luo, F. Pilat, S. Peggs, D. Trbojevic, T. Roser
Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

The global betatron decoupling on the ramp is an important issue for the operation of the Relativistic Heavy Ion Collider (RHIC). A new scheme, coupling angle modulation, was found. By modulating two orthogonal skew quadrupole families, an extra rotating coupling is introduced into the coupled machine. The skew quadrupole modulation frequency is about 0.2Hz for the RHIC ramp, and 0.5Hz at injection and store. The eigentune changes are tracked with a high resolution phase lock loop (PLL) tune measurement system. The global coupling correction strengths are determined by the modulation skew quadrupole strengths at the minimum eigentune split multiplied by a factor k. k is determined by the uncoupled eigentune split and the maximum and the minimum tune split during the skew quadrupole modulation. This decoupling scheme is fast and robust. It had been verified at the RHIC and has been applied for the RHIC global decoupling on the ramp. In this article, the principle of the coupling angle modulation is presented in detail. Simulation results are also shown.

INTRODUCTION

A fast and robust global decoupling scheme, coupling angle modulation, is found and reported in this article. It modulates two orthogonal skew quadrupole families with the same modulation amplitude and modulation frequency, but with a 90° initial phase difference. This skew quadrupole modulation introduces an extra rotating coupling coefficient into the coupled optics. Presently, the eigentune split is used as the global coupling observable [1, 2]. The two eigentunes are tracked with a high-resolution phase lock loop (PLL) tune measurement system [3, 4]. The global coupling correction strengths are directly obtained from the modulating skew quadrupole strengths at the minimum tune split during the modulation multiplied by a factor k. k is given by the minimum and maximum tune splits, together with that without the modulation.

Coupling angle modulation is a fast and robust decoupling scheme, compared to other coupling measurement or correction schemes [5, 6]. According to the beam experiments at the Relativistic Heavy Ion Collider (RHIC), the modulation frequency for RHIC is chosen as 0.5 Hz at injection and store, 0.2 Hz on the ramp. Since the correction strengths can be determined in one modulation period, the decoupling strengths can be found in seconds. This advantage makes this scheme viable for the global decoupling on the non-stop energy ramp of superconducting accelerators [7, 6].

The correction skew quadrupole strength combination comes from the skew quadrupole modulation currents at the minimum tune split. And the exact modulation amplitude combination of the skew quadrupole families is proved to be not strictly required. Therefore, this scheme is less connected to the optics model. And since only the minimum and maximum tune splits are used to calculate the factor k, the scheme is robust. The data processing is simple, too.

COUPLING ANGLE MODULATION

Tune split

From the linear difference coupling's Hamiltonian perturbation theory [8, 9, 10, 11], the two eigentunes $Q_{1,2}$ in the coupled situation are given by

$$Q_1 = Q_{x,0} - \frac{\Delta}{2} + \frac{1}{2}\sqrt{\Delta^2 + |C^-|^2}, \tag{1}$$

$$Q_2 = Q_{y,0} + \frac{\Delta}{2} - \frac{1}{2}\sqrt{\Delta^2 + |C^-|^2}, \tag{2}$$

where $Q_{x,0}$, $Q_{y,0}$ are the uncoupled tunes when all coupling sources are removed. Δ is the uncoupled tune split,

$$\Delta = Q_{x,0} - Q_{y,0} - p, \tag{3}$$

where p is the integer tune split. C^- is the coupling coefficient, which normally is a complex number. It is defined as

$$C^- = |C^-|e^{i\chi} = \frac{1}{2\pi}\oint \sqrt{\beta_x\beta_y}k_s e^{i(\Phi_x - \Phi_y - \Delta\frac{2\pi s}{L})}dl. \tag{4}$$

$|C^-|$ is the coupling amplitude, χ is the angle of the coupling. β_x and β_y are the unperturbed betatron amplitude functions, Φ_x and Φ_y are the unperturbed betatron phase advances, k_s is the skew quadrupole strength, L is the ring circumference, and s is the distance between the skew quadruple and the reference point to calculate the coupling coefficient.

From Eqs. (1) and (2), the eigentune split ΔQ is given by

$$\begin{aligned} |\Delta Q| &= |Q_1 - Q_2 - p| \\ &= \sqrt{\Delta^2 + |C^-|^2}. \end{aligned} \tag{5}$$

This tune split ΔQ can be measured experimentally.

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Rotating coupling coefficient

Coupling angle modulation modulates two orthogonal skew quadrupole families. The coupling coefficients contributed by the orthogonal families differ by $90°$. If their coupling coefficient modulation amplitudes and modulation frequencies are same, and there is a $90°$ difference in their initial modulation phases, the total introduced coupling coefficient is

$$C^-_{mod} = |C^-_{mod,amp}| \cdot e^{i2\pi ft}, \tag{6}$$

where f is the modulation frequency, and $|C^-_{mod,amp}|$ is the coupling modulation amplitude.

FIG. 1 shows the schematic plot of the coupling angle modulation. The horizontal and vertical axises are the real and imaginary parts of the complex coupling coefficient C^-. The blue line represents the rotating coupling given by Eq. (6). The red line represents the residual coupling.

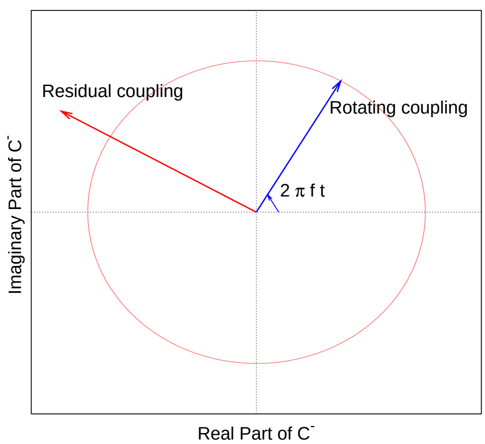

Figure 1: Schematic plot of coupling angle modulation.

Correction strengths

The total coupling coefficient C^-_{total} during the coupling angle modulation is

$$C^-_{total} = C^-_{res} + C^-_{mod}, \tag{7}$$

where the residual coupling C^-_{res} is

$$C^-_{res} = |C^-_{res,amp}|e^{i\phi_{res}}. \tag{8}$$

According to Eq. (5), the tune split's square during the modulation is

$$\begin{aligned} |\Delta Q|^2 &= \Delta^2 + |C^-_{res,amp}|^2 + |C^-_{mod,amp}|^2 \\ &\quad +2|C^-_{res,amp}||C^-_{mod,amp}|\cos{(2\pi ft - \phi_{res})}. \end{aligned} \tag{9}$$

Assuming the rotating coupling's amplitude $|C^-_{mod,amp}|$ is constant during the coupling angle modulation, we define

$$|C^-_{res,amp}| = k\;|C^-_{mod,amp}|. \tag{10}$$

k is a non-negative number.

From Eq. (9) and FIG. 1 , we obtain the maximum tune split when the rotating coupling has the same direction as the residual coupling, and the minimum tune split when the rotating coupling has the opposite direction to the residual coupling.

Therefore, according to Eqs. (9) and (10), the maximum and minimum tune split's squares are

$$\Delta Q^2_{min} = \Delta^2 + (k-1)^2 \cdot |C^-_{mod,amp}|^2, \tag{11}$$

$$\Delta Q^2_{max} = \Delta^2 + (k+1)^2 \cdot |C^-_{mod,amp}|^2. \tag{12}$$

Considering the tune split ΔQ_0 without modulation,

$$\Delta Q^2_0 = \Delta^2 + k^2 \cdot |C^-_{mod,amp}|^2, \tag{13}$$

the factor k can be determined,

$$k = [4(\frac{\Delta Q^2_{max} - \Delta Q^2_0}{\Delta Q^2_{max} - \Delta Q^2_{min}} - \frac{1}{2})]^{-1}. \tag{14}$$

The factor k has a significant role in determining of the strengths of the global coupling correction. The minimum tune split is obtained when the rotating coupling takes the opposite direction to the residual coupling. Therefore, the right decoupling skew quadrupole strengths' combination is given at the minimum tune split time stamp.

If the skew quadrupole modulation currents are recorded during the coupling modulation, from which the skew quadrupole modulation strengths at the minimum tune split time stamp can be determined. Then, according to Eq. (10) and Eq. (14) , the global decoupling strengths are obtained from the modulating skew quadrupole strengths at the minimum tune split multiplied by the factor k.

Discussion

The advantage of the coupling angle modulation is very clear. It doesn't need FFT or FIT to the PLL tune data. Only ΔQ^2_0 under no introduced coupling situation, ΔQ^2_{max} and ΔQ^2_{min} during the coupling angle modulation are needed. The correction strengths are also easily obtained. They are just the modulation strengths at the minimum tune split square time point multiplied by the positive factor k. We doesn't need to determine the residual coupling first. The correction strengths are directly obtained in the modulation procedure.

The time period occupied by the coupling angle modulation is much shorter. In principle, one modulation period is enough to find the correction strengths. Since the power supplies' currents need to rise from zero current, the cosinuous function modulation is obtained operationally by starting a sinuous function modulation $1/4$ period ahead the coupling angle modulation, as shown in Fig. 2. So the time occupied in the modulation is about 2 modulation periods.

This correction scheme is also robust since only ΔQ^2_0, ΔQ^2_{max} and ΔQ^2_{min} are needed. It doesn't care too much about the spikes in the detailed tune data from PLL. In

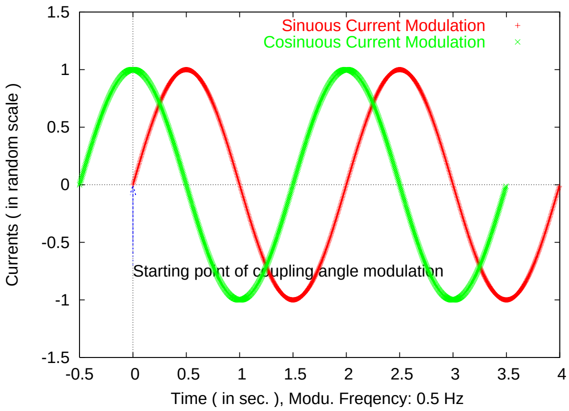

Figure 2: Skew quadrupoles' Currents in the operational coupling angle modulation

order to get rid off the large spikes, the fitting according Eq.(9) can be used to find the ΔQ^2_{max} and ΔQ^2_{min}.

It is also easy to prove that the orthogonal modulations are not strictly demanded. The above solution to factor k is still valid even if the two modulation is not strictly out-of-phase, or the two modulation amplitudes are not exactly the same. Under that situation, the rotating coupling traces out an ellipse instead a circle in Fig. 1. Considering the radial symmetry of the ellipse, the rotating coupling at the maximum and the minimum tune split are 180° different, the rotating coupling amplitudes at the minimum and the maximum tune splits are the same. So we still can use Eq. (14) to compute k. So the coupling angle modulation scheme has less connections to the optics.

SIMULATION

Each RHIC ring has three correction skew quadrupole families, F1, F2 and F3, each having 12 skew quadrupole magnets, with four activating power supplies.To produce the rotating coupling coefficient, we use skew quadrupole families F1 and F3 to generate a new skew quadrupole family (F1F3) that is orthogonal to the coupling contribution only from family F2. To equalize families (F1F3) and F2's coupling modulation amplitudes, the amplitudes of modulation strengths for families F1 and F3 are set to $\frac{1}{\sqrt{3}}$ of that of family F2. To get a 90° difference in the initial modulation phases, family F2 modulation starts a quarter period earlier than families F1 and F3.

Here we give an simulation correction example based on the smooth accelerator model. The residual coupling is introduced by setting the constant integrated strengths of F1,F2,F3 to 0.m^{-1}, -0.001 m^{-1}, -0.0015 m^{-1}. Please be reminded that there are only three skew quadrupoles in the tracking model to represent the three families. The integrated strengths given here is not the same to the individual skew quadrupole integrated strength in the real lattice model.

We modulate F3 with integrated strength amplitude 0.0005 m^{-1}, modulate F1 and F2 with $0.0005/\sqrt{3}$ m^{-1}. The modulation frequency is 0.2 Hz. And F3's strength is modulated like co-sinuous function, F1 and F2 's strengths are modulated like sinuous function. The correction strengths are given by the modulation strengths at that time point multiplied by the factor $k = 5.0$. After correction, we see in Fig. 3 that the tunes restore to the design tunes.

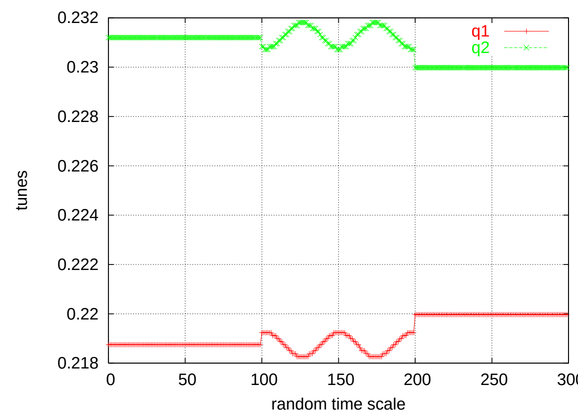

Figure 3: The tunes in the whole procedure of global decoupling in the simulation.

CONCLUSION

Coupling angle modulation is a fast and robust global decoupling scheme. In this article, the principle is presented and the simulation is done. Coupling angle modulation correction has been verified at RHIC injection, store, and on the ramp. It has been applied to RHIC global decoupling on the ramp.

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