

# THREE-DIMENSIONAL DESIGN OF A NON-AXISYMMETRIC PERIODIC PERMANENT MAGNET FOCUSING SYSTEM\*

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## Abstract

A three-dimensional (3D) design is presented of a non-axisymmetric periodic permanent magnet (PPM) focusing system which will be used to focus a large-aspect-ratio, ellipse-shaped, space-charge-dominated electron beam. In this design, an analytic theory is used to specify the magnetic profile for beam transport. The OPERA3D code is used to compute and optimize a realizable magnet system. Good beam transport is simulated with the designed magnetic fields, using both 2D particle-in-cell (PIC) and 3D ray trajectory codes.

## INTRODUCTION

High-intensity ribbon (thin sheet) beams [1] are of great interest because of their applications in particle accelerators and vacuum electron devices. Recently, an equilibrium beam theory has been developed for an elliptic cross-section space-charge-dominated beam in a non-axisymmetric periodic magnetic focusing field [2,3].

In the equilibrium beam theory [2,3], a paraxial cold-fluid model is employed to derive generalized envelope equations which determine the equilibrium flow properties of ellipse-shaped beams with negligibly small emittance. The magnetic field is expanded to the lowest order in the direction transverse to beam propagation. A matched envelope solution is obtained numerically from the generalized envelope equations, and the results show that the beam edges in both transverse directions are well confined, and that the angle of the beam ellipse exhibits a periodic small-amplitude twist. Two-dimensional (2D) particle-in-cell (PIC) simulations with our Periodic Focused Beam 2D (PFB2D) code show good agreement with the predictions of equilibrium theory as well as beam stability.

In this paper, we present a three-dimensional (3D) design of a non-axisymmetric periodic permanent magnet (PPM) focusing field for a ribbon-beam amplifier (RBA) that is being developed at Massachusetts Institute of Technology (MIT) in collaboration with Beam Power Technology (BPT) and Communication Power Industries (CPI).

The 3D design of the non-axisymmetric PPM focusing field is performed with OPERA3D [4]. In this design, the magnet material SmCo 2:17TC-16 is chosen for the magnets. Results from the 3D magnet design are imported into an OMNITRAK [5] simulation of a ribbon electron

beam, which shows good beam transport [6].

## DESIGN

For the beam transverse dimensions small relative to the characteristic scale of magnetic variations, i.e.,  $(k_{0x}x)^2/6 \ll 1$  and  $(k_{0y}y)^2/6 \ll 1$ , a three-dimensional (3D) non-axisymmetric PPM focusing field can be described to the lowest order in the transverse dimension as [2,3]

$$\mathbf{B}^{ext}(\mathbf{x}) \cong B_0 \left[ \frac{k_{0x}^2}{k_0} \cos(k_0 s) x \hat{\mathbf{e}}_x + \frac{k_{0y}^2}{k_0} \cos(k_0 s) y \hat{\mathbf{e}}_y - \sin(k_0 s) \hat{\mathbf{e}}_z \right], \quad (1)$$

where  $k_0 = 2\pi/S$ ,  $k_{0x}^2 + k_{0y}^2 = k_0^2$ , and  $S$  is the axial periodicity length.

The 3D magnetic field in Eq. (1) is fully specified by the following three parameters:  $B_0$ ,  $S$  and  $k_{0y}/k_{0x}$ . In order to achieve good beam transport, it is important to design the magnets which yield a three-dimensional magnetic field profile whose paraxial approximation assumes the form given by Eq. (1). In the design, we adjust the dimensions of the magnets to achieve the three parameters specified by the equilibrium beam theory [2,3].

For the MIT RBA, the parameters for the ellipse-shaped electron beam and non-axisymmetric PPM focusing field are listed Table 1. The ellipse-shaped electron beam has a current of 0.11 A, a voltage of 2.29 kV, a semi major axis (envelope) of 0.373 cm, an aspect ratio of 6.0, and a maximum twist angle of 10.4 degrees. Here, the aspect ratio is defined as the semi major axis relative to the semi minor axis of the ellipse.

Table 1: System parameters for MIT RBA

Parameter (unit)	Value
Current (A)	0.11
Voltage (kV)	2.29
$S$ (cm)	1.912
$k_{0y}/k_{0x}$	1.60
$B_0$ (G)	336.5
$b/a$	6.0
$b$ (cm)	0.373
$\theta_{max}$	10.4°

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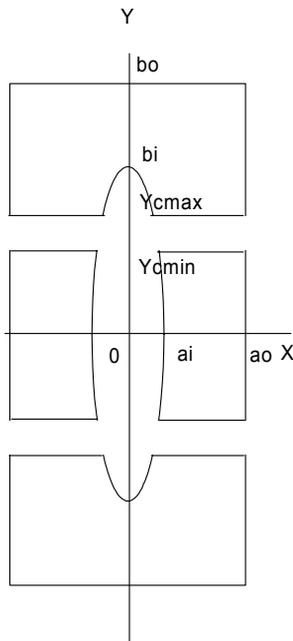


Figure 1: Cross-sectional view of one set of the permanent magnets that form a one-half period of a non-axisymmetric PPM focusing field.

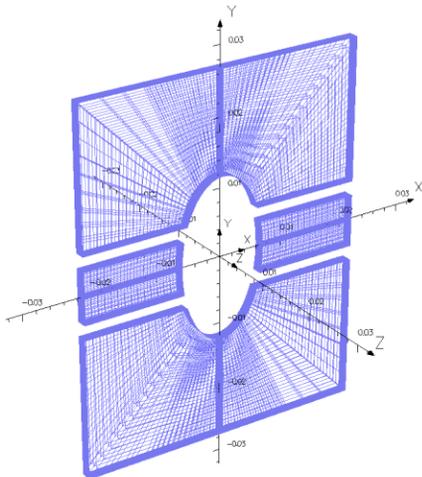


Figure 2: One set of the permanent magnets in a one-half period used in the OPERA3D calculation.

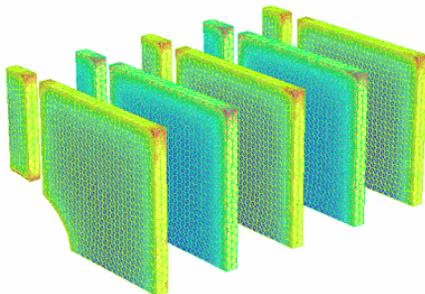


Figure 3: One quarter section of two and one-half periods of the non-axisymmetric PPM in the OPERA3D calculation.

Table 2: Non-axisymmetric PPM design

Parameter (unit)	Value
$S$ (mm)	19.12
$a_0$ (mm)	6.1
$a_1$ (mm)	18.2
$b_0$ (mm)	11.5
$b_1$ (mm)	22.0
$Y_{c\ min}$ (mm)	3.82
$Y_{c\ max}$ (mm)	6.52
Thickness (mm)	1.488
$k_{0x}/k_{0y}$ (computed)	1.598
$B_0$ (G) (computed)	336.3

In addition to assuring that parameters  $B_0$ ,  $S$  and  $k_{0x}/k_{0y}$  meet the design requirement, an important design consideration for the MIT RBA is that the non-axisymmetric PPM must be compatible with the corrugated slow-wave structure. This limits the range of magnet thickness we can work with.

Figure 1 shows a cross-sectional view of one set of the permanent magnets that form a one-half period of non-axisymmetric PPM focusing field. In this calculation, the major axis is in the  $y$ -direction. Figure 2 shows the corresponding 3D drawing of the magnet set used in our OPERA3D calculation. Figure 3 show an example of one quarter section of two and one-half periods of the non-axisymmetric PPM in the OPERA3D calculation. The magnetization is in the  $z$ -direction, but changes its sign from one set of the magnets to another, forming a periodic magnetic field [see Eq. (1)]. Because of the periodicity and symmetry, we only need to compute the field distribution in a one-half period from  $z = -S/4$  to  $S/4$ , and apply an anti-symmetric boundary condition in the OPERA3D calculations.

For the design parameters listed in Table 2, the maximum magnetic field on the  $z$ -axis calculated from the OPERA3D calculation is  $B_0 = 336.3$  G, which is within 0.06% of the design goal (see Table 1). The parameter  $k_{0x}/k_{0y}$  from the OPERA3D calculation is 1.598, which is within 0.13% of the design goal.

Figure 4 shows the comparison of the transverse magnetic fields at  $z = S/4$  from the OPERA3D calculation with those from the paraxial approximation in Eq. (1). Within the beam envelope with  $|x| < a = 0.622$  mm and  $|y| < b = 3.73$  mm, the magnetic fields from the OPERA3D calculation are well approximated by Eq. (1).

We have used the designed magnetic fields shown in Fig. 3 in our 2D and 3D simulations of beam transport, using our two-dimensional particle-in-cell (PIC) code, PFB2D, and a commercial 3D ray trajectory code, OMNITRAK, respectively. Results of our beam transport simulations are presented elsewhere [6].

## CONCLUSION

A three-dimensional (3D) design was presented of a non-axisymmetric periodic permanent magnet focusing system which will be used to focus a large-aspect-ratio, ellipse-shaped, space-charge-dominated electron beam. In this design, the beam equilibrium theory [2,3] was used to specify the magnetic profile for beam transport. The OPERA3D code was used to compute and optimize a realizable magnet system. We have also used the designed magnetic fields shown in Fig. 3 in our 2D and 3D simulations of beam transport, using our two-dimensional particle-in-cell (PIC) code, PFB2D, and a commercial 3D ray trajectory code, OMNITRAK, respectively. Results of our beam transport simulations are presented elsewhere [6].

## REFERENCES

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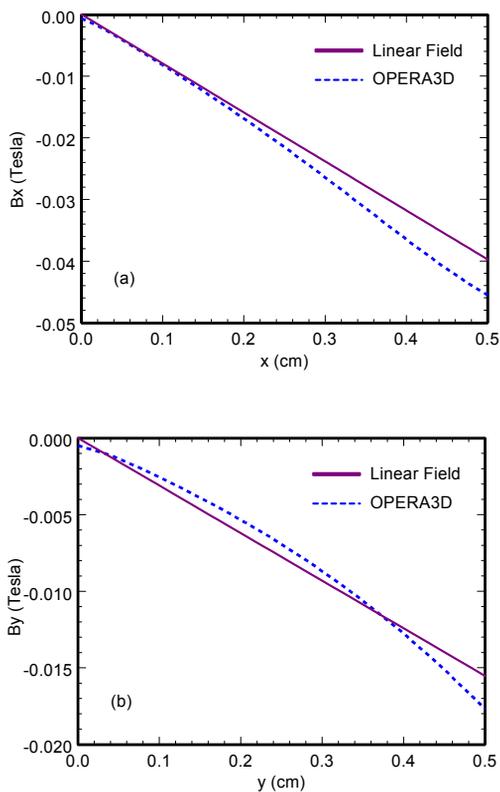


Figure 4: Plots of the magnetic field in (a)  $x$ -direction and (b)  $y$ -direction. The dashed curves are from the OPERA3D calculation, whereas the solid curves are from Eq. (1).