HAMILTONIAN ANALYSIS OF TRANSVERSE DYNAMICS IN AXISYMMETRIC RF PHOTOINJECTOR*

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Abstract

A general Hamiltonian that governs the beam dynamics in an rf photoinjector is derived from first principles. With proper choice of coordinates, the resulting Hamiltonian has a simple and familiar form, while taking into account the rapid acceleration, rf focusing, magnetic focusing, and space-charge forces. From the linear Hamiltonian, beam-envelope evolution is readily obtained, which better illuminates the theory of emittance compensation. Preliminary results on the third-order nonlinear Hamiltonian will be given as well.

HAMILTONIAN WITH ACCELERATING REFERENCE PARTICLE

Electrons are rapidly accelerated in an rf injector, thus a Hamiltonian with accelerating reference particle is needed for perturbative analysis. Since commonly used Hamiltonians assume a reference particle with (quasi) constant momentum, we start from the basic relativistic Hamiltonian for a particle of mass m and charge q moving under the influence of an external electromagnetic field, i.e.,

$$H = q\phi(\mathbf{X}, t) + \sqrt{m^2c^4 + [\mathbf{P} - q\mathbf{A}(\mathbf{X}, t)]^2 c^2}, \quad (1)$$

where $\mathbf{X}$ is the laboratory-frame Cartesian coordinate of the particle and $\mathbf{P}$ is the canonical momentum. The electromagnetic field is given by the scalar potential ϕ and vector potential $\mathbf{A}$. Time t is the independent variable.

For convenience we use the longitudinal position s of a reference particle as the independent variable and replace t with the time of the reference particle $t_r(s)$. Furthermore, we normalize the momentum by mc. In other words, we use $\hat{\mathbf{X}}(s) = \mathbf{X}(t_r)$ and $\hat{\mathbf{P}}(s) = \mathbf{P}(t_r)/mc$ as the canonical variables. The new Hamiltonian is given by

$$\hat{H}(\hat{\mathbf{X}}, \hat{\mathbf{P}}, s) = \frac{1}{ds/dt}\frac{H}{mc} = \frac{H(\mathbf{X}, \mathbf{P}, t_r(s))}{\beta_r(s)\, mc^2}. \quad (2)$$

Normalizing the 4-potentials of the E.M. field by the 4-momentum of the reference particle as

$$\hat{\phi} = \frac{q\phi}{\gamma_r mc^2}, \quad \hat{\mathbf{A}} = \frac{q\mathbf{A}}{\beta_r\gamma_r mc}, \quad (3)$$

then the Hamiltonian can be written as

$$\hat{H} = \frac{\gamma_r}{\beta_r}\,\hat{\phi} + \frac{1}{\beta_r}\sqrt{1 + \left[\hat{\mathbf{P}} - \hat{P}_r^k\hat{\mathbf{A}}\right]^2}, \quad (4)$$

where $\hat{P}_r^k = \beta_r\gamma_r$ is the amplitude of the normalized kinematic momentum of the reference particle.

Using the new coordinate $\mathbf{x} = \hat{\mathbf{X}} - \hat{\mathbf{X}}_r = \mathbf{X} - \mathbf{X}_r$ and the new momentum $\mathbf{p} = \hat{\mathbf{P}} - \hat{\mathbf{P}}_r = (\mathbf{P} - \mathbf{P}_r)/mc$, a new Hamiltonian can be obtained with the generating function $F_2(\hat{\mathbf{X}}, \mathbf{p}, s) = (\hat{\mathbf{X}} - \hat{\mathbf{X}}_r)\cdot(\mathbf{p} + \hat{\mathbf{P}}_r)$ as

$$\begin{aligned} H \;=\; & \frac{\gamma_r}{\beta_r}\,\hat{\phi} + \frac{1}{\beta_r}\sqrt{1 + \left[\mathbf{p} + \hat{\mathbf{P}}_r - \hat{P}_r^k\hat{\mathbf{A}}\right]^2} \\ & - \mathbf{X}_r'\cdot(\mathbf{p} + \hat{\mathbf{P}}_r) + \mathbf{x}\cdot\hat{\mathbf{P}}_r', \end{aligned} \quad (5)$$

where the prime means differentiation with respect to s.

To expand the square root into a series, we note that $\hat{\mathbf{P}}_r - \hat{P}_r^k\hat{\mathbf{A}} = \hat{\mathbf{P}}_r^k - \hat{P}_r^k\Delta\hat{\mathbf{A}}$, where $\Delta\hat{\mathbf{A}} \equiv \hat{\mathbf{A}} - \hat{\mathbf{A}}_r$ is the difference in vector potential seen by a particle and the reference particle, which should be small. Thus the terms under the square root can be written as $1 + (\hat{P}_r^k)^2 + 2\hat{\mathbf{P}}_r^k\cdot(\mathbf{p} - \hat{P}_r^k\,\Delta\hat{\mathbf{A}}) + (\mathbf{p} - \hat{P}_r^k\,\Delta\hat{\mathbf{A}})^2$, where the first two terms are dominating and sum to γ_r^2. Taking γ_r^2 out of the square root, the rest becomes $1 + 2\beta_r^2\,\boldsymbol{\delta}_\parallel + \beta_r^2\,\boldsymbol{\delta}^2$, where

$$\boldsymbol{\delta} = \frac{\mathbf{p}}{\hat{P}_r^k} - \Delta\hat{\mathbf{A}} = \frac{\mathbf{P}^k - \mathbf{P}_r^k}{P_r^k} \quad (6)$$

is the relative deviation in kinematic momentum. The subscript $_\parallel$ indicates the longitudinal component. This form is suitable for Taylor expansion since both $\mathbf{p}/\hat{P}_r^k$ and $\Delta\hat{\mathbf{A}}$ and thus $\boldsymbol{\delta}$ are small quantities. Furthermore, when the momentum of the reference particle is not much larger than the momentum deviation, i.e., $\boldsymbol{\delta}$ is not small, β_r will be small. Expanding the square-root term up to the third order yields $1+\beta_r^2\boldsymbol{\delta}_\parallel+\beta_r^2(\boldsymbol{\delta}^2-\beta_r^2\boldsymbol{\delta}_\parallel^2)/2-\beta_r^4\boldsymbol{\delta}_\parallel(\boldsymbol{\delta}^2-\beta_r^2\boldsymbol{\delta}_\parallel^2)/2+O(\boldsymbol{\delta}^4)$. Ignoring both the zeroth-order terms, which do not play a role in the Hamiltonian equations, and the first-order terms, which cancel out since the reference particle follows the first-order solution, the Hamiltonian reduces to $H = H_2 + H_3 + \cdots$, where

$$H_2 \;=\; \left(\frac{\gamma_r}{\beta_r}\,\hat{\phi} - \hat{\mathbf{P}}_r^k\cdot\hat{\mathbf{A}}\right)_2 + \frac{\hat{P}_r^k}{2}\left(\boldsymbol{\delta}_\perp^2 + \frac{1}{\gamma_r^2}\,\boldsymbol{\delta}_\parallel^2\right)_2 \quad (7)$$

$$\begin{aligned} H_3 \;=\; & \left(\frac{\gamma_r}{\beta_r}\,\hat{\phi} - \hat{\mathbf{P}}_r^k\cdot\hat{\mathbf{A}}\right)_3 + \frac{\hat{P}_r^k}{2}\left(\boldsymbol{\delta}_\perp^2 + \frac{1}{\gamma_r^2}\,\boldsymbol{\delta}_\parallel^2\right)_3 \\ & - \frac{\beta_r^2}{2}(\hat{\mathbf{P}}_r^k\cdot\boldsymbol{\delta}_1)\left(\boldsymbol{\delta}_\perp^2 + \frac{1}{\gamma_r^2}\,\boldsymbol{\delta}_\parallel^2\right)_2. \end{aligned} \quad (8)$$

The integer subscripts indicate which order to keep.

To simplify the linear dynamics given by H_2, we make a linear canonical transformation generated by

$$\begin{aligned} F_2 = x & \left[\sqrt{\hat{P}_r^k}\,\hat{p}_x - \frac{1}{2}\sqrt{\hat{P}_r^k}\sqrt{\hat{P}_r^k}'x + \frac{1}{2}\hat{P}_r^k(\partial_x\hat{A}_x)_r x\right] \\ & + (x \leftrightarrow y) + z\left[\hat{p}_z + \frac{1}{2}\hat{P}_r^k(\partial_z\hat{A}_z)_r\,z\right]. \end{aligned} \quad (9)$$

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The variables are transformed as

$$\hat{x} = \sqrt{\hat{P}_r^k}\, x = \sqrt{\beta_r\gamma_r}\, x, \tag{10}$$

$$p_x = \sqrt{\hat{P}_r^k}\, \hat{p}_x - \sqrt{\hat{P}_r^k}'\, \hat{x} + \sqrt{\hat{P}_r^k}\, (\partial_x \hat{A}_x)_r\, \hat{x}, \tag{11}$$

$$\hat{z} = z, \tag{12}$$

$$p_z = \hat{p}_z + \hat{P}_r^k (\partial_z \hat{A}_z)_r \hat{z}, \tag{13}$$

and the y dimension is similarly transformed. Note that $\hat{x}$ and $\hat{y}$ are the so-called reduced coordinates. Under the conditions

$$(\partial_z A_\perp)_r = 0 \quad \text{and} \quad (\partial_y A_x)_r = -(\partial_x A_y)_r \tag{14}$$

the new Hamiltonian reduces to

$$\begin{aligned} H_2 &= \frac{\gamma_r}{\beta_r}\,\hat{\phi}_2 - \hat{\mathbf{P}}_r^k \cdot \hat{\mathbf{A}}_2 \\ &+ \frac{\hat{p}_x^2}{2} + \left[-\frac{\sqrt{\hat{P}_r^k}''}{\sqrt{\hat{P}_r^k}} + \frac{1}{\hat{P}_r^k}\, \frac{\partial^2(\hat{P}_r^k \hat{A}_x)}{\partial s\, \partial x}\bigg|_r + (\partial_x \hat{A}_y)_r^2 \right] \frac{\hat{x}^2}{2} \\ &+ (x \leftrightarrow y \text{ in the previous 2 terms}) \\ &+ (\partial_y \hat{A}_x)_r\, (\hat{x}\hat{p}_y - \hat{y}\hat{p}_x) + \frac{\beta_r}{2\gamma_r} (\mathbf{x}_\perp \cdot \nabla_\perp \hat{A}_z)_r^2 \\ &+ \frac{\hat{p}_z^2}{2\beta_r\gamma_r^3} - \frac{\hat{p}_z}{\gamma_r^2} (\mathbf{x}_\perp \cdot \nabla_\perp \hat{A}_z)_r + \frac{\partial^2(\hat{P}_r^k \hat{A}_z)}{\partial s\, \partial z}\bigg|_r \frac{\hat{z}^2}{2}. \end{aligned} \tag{15}$$

The advantages of this transformation become clear now. The complication due to acceleration is reduced to a pseudo focusing. The kinematic and potential terms are separated, and the Hamiltonian resumes a familiar form. Note that, until now, the treatment is generally valid under the conditions in Eq. (14).

LINEAR FOCUSING AND HAMILTONIAN

A typical axisymmetric injector consists of TM_{01} accelerating field with vector potential (using the real part)

$$A_z^{\rm rf} = \mathsf{E}_0 \sum_{n=-\infty}^{\infty} \frac{a_n}{\omega} J_0(k_{rn} r) e^{i[\omega t - k_{zn}(s+z) + \varphi_0]}\,, \tag{16}$$

$$A_r^{\rm rf} = i\mathsf{E}_0 \sum_{n=-\infty}^{\infty} \frac{k_{zn}}{k_{rn}} \frac{a_n}{\omega} J_1(k_{rn} r) e^{i[\omega t - k_{zn}(s+z) + \varphi_0]} \tag{17}$$

and solenoid focusing field

$$\hat{\mathbf{A}}^{\rm mag} = \left(-\hat{b}_s\, y,\ \hat{b}_s\, x,\ 0 \right). \tag{18}$$

Here E_n is the amplitude of the space harmonic of index n and $a_n = \mathsf{E}_n/\mathsf{E}_0$. ω is the rf frequency. φ_0 is the initial rf phase from the zero-crossing at the origin. The longitudinal and transverse wave numbers of the n-th space harmonic (k_{zn} and k_{rn}, respectively) are given by $k_{zn} = k_{z0} + 2\pi n/d$ and $k_{rn}^2 + k_{zn}^2 = k^2 = (\frac{\omega}{c})^2$, where d is the period of rf structure. J_0 and J_1 are the Bessel functions. r is the radial coordinate. The normalized solenoid strength $\hat{b}_s = (q/2\beta_r\gamma_r mc) B_s(0,0,s)$.

The internal space-charge force is modeled by an average static potential ϕ_0^{sc} in the beam frame. In the lab frame,

$$\phi^{sc} = \gamma_r \phi_0^{sc},\ A_z^{sc} = \frac{\beta_r}{c}\,\phi^{sc},\ \text{i.e.,}\ \hat{A}_z^{sc} = \hat{\phi}^{sc} = \frac{q\phi_0^{sc}}{mc^2}. \tag{19}$$

Note that, using these relations, the space-charge contribution to the potential terms $\frac{\gamma_r}{\beta_r}\hat{\phi} - \hat{\mathbf{P}}_r^k \cdot \hat{\mathbf{A}}$ in Hamiltonian reduces to $\hat{\phi}^{sc}/\beta_r\gamma_r$, showing the well-known cancellation of electro and magnetic forces in ultra-relativistic limit.

Under these forces, the linear Hamiltonian becomes

$$\begin{aligned} H_2 &= \frac{\hat{p}_x^2 + \hat{p}_y^2}{2} + K\frac{\hat{x}^2 + \hat{y}^2}{2} - \hat{b}_s(\hat{x}\hat{p}_y - \hat{y}\hat{p}_x) \\ &+ \frac{\hat{p}_z^2}{2\beta_r\gamma_r^3} + \left[\frac{\partial^2(\hat{P}_r^k \hat{A}_z^{\rm rf})}{\partial s\, \partial z} - \frac{\partial^2(\hat{P}_r^k \hat{A}_z^{\rm rf})}{\partial z^2} \right]_r \frac{\hat{z}^2}{2}, \end{aligned} \tag{20}$$

where the focusing strength K is given by

$$\begin{aligned} K(z,s) &= -\frac{\sqrt{\hat{P}_r^k}''}{\sqrt{\hat{P}_r^k}} + \hat{b}_s^2 + \frac{1}{2(\hat{P}_r^k)^2}\, \partial_x^2 \hat{\phi}^{sc}\Big|_{r=0} \\ &+ \frac{1}{\hat{P}_r^k}\, \frac{\partial^2(\hat{P}_r^k \hat{A}_r^{\rm rf})}{\partial s\, \partial r}\bigg|_r - (\partial_r^2 \hat{A}_z^{\rm rf})_r. \end{aligned} \tag{21}$$

Due to rotational symmetry, the angular momentum term in H_2 can be dropped with the understanding that the dynamical variables refer to the Larmor frame.

The last two terms in the focusing strength K are due to the rf ponderomotive focusing. They contain fast oscillation of rf period that can be smoothed out as

$$\bar{K}^{\rm rf} = \langle K^{\rm rf} \rangle + \left\langle \left[\int^s d\bar{s}\, \left(K^{\rm rf} - \langle K^{\rm rf} \rangle \right) \right]^2 \right\rangle = \frac{\hat{E}_0^2}{8}\,\eta. \tag{22}$$

Here $\langle\ \rangle$ means averaging over rf period. For standing wave structure, $\hat{E}_0 \equiv qE_0/2\beta_r\gamma_r mc^2$, $E_0 = 2\mathsf{E}_0$, and $\eta(\varphi_0) = \sum_{n=0}^{\infty} \left(a_n^2 + a_{n+1}^2 - 2a_n a_{n+1} \cos 2\varphi_0 \right)$ [1]. The total external electromagnetic focusing reduces to [2]

$$K^{\rm em} \simeq \hat{E}_0^2 \left[\frac{\eta}{8} + \left(\frac{cB_s}{E_0} \right)^2 \right]. \tag{23}$$

The pseudo focusing due to acceleration can be written as

$$-\frac{\sqrt{\hat{P}_r^k}''}{\sqrt{\hat{P}_r^k}} = \frac{1}{4} \left(1 + \frac{2}{\gamma_r^2} \right) \left(\frac{\gamma_r'}{\beta_r^2\gamma_r} \right)^2 - \frac{\gamma_r''}{2\beta_r^2\gamma_r}. \tag{24}$$

For synchronized acceleration, the energy gain per unit length is approximately constant with

$$\gamma_r' = \frac{qE_0 \sin\varphi_0}{mc^2} = \beta_r\gamma_r \hat{E}_0 \sin\varphi_0. \tag{25}$$

Thus the total external focusing strength becomes

$$K^{\rm ext} \simeq (\hat{E}_0 \sin\varphi_0)^2 \left(\Omega^2 + 1/4 \right), \tag{26}$$

where

$$\Omega^2 = \frac{1}{\sin^2\varphi_0} \left[\frac{\eta}{8} + \left(\frac{cB_s}{E_0} \right)^2 \right].$$

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The linear space-charge defocusing can be written as [2]

$$K^{\mathrm{s.c.}} = -\frac{Ig(z,s)}{2I_A}\frac{1}{\beta_r^3\gamma_r^3\sigma_r^2}, \tag{27}$$

where Ig is the slice current whose s dependence is usually ignored, I_A is the Alfén current, and σ_r is the transverse beam size. The total focusing strength $K = K^{\mathrm{ext}} + K^{\mathrm{s.c.}}$.

TRANSVERSE BEAM ENVELOPE

Ignoring the longitudinal dynamics, the transverse dynamics of each z-slice is governed by the simple Hamiltonian $(\hat{p}_x^2+\hat{p}_y^2)/2+K(s,z)(\hat{x}^2+\hat{y}^2)/2$, whose behavior is well known and the beam envelope is described by the standard Courant-Snyder parameters with the β-function satisfying $\sqrt{\beta}\,'' + K\sqrt{\beta} - 1/\sqrt{\beta}\,^3 = 0$. The normalized emittance ϵ_n is conserved for each slice under this Hamiltonian. The rms beam size in reduced variable is $\hat{\sigma}_r = \sqrt{\epsilon_n\beta} = \sqrt{\beta_r\gamma_r}\,\sigma_r$. Using $\hat{\sigma}_r$, the beta-function equation becomes the beam-envelope equation $\hat{\sigma}_r'' + K\hat{\sigma}_r - \epsilon_n^2/\hat{\sigma}_r^3 = 0$. Inserting the above focusing strength we have

$$\hat{\sigma}_r'' + \left(\frac{\gamma_r'}{\beta_r\gamma_r}\right)^2\left(\Omega^2+\frac{1}{4}\right)\hat{\sigma}_r - \left(\frac{\gamma_r'}{\beta_r\gamma_r}\right)^2\frac{S}{\hat{\sigma}_r} - \frac{\epsilon_n^2}{\hat{\sigma}_r^3} = 0, \tag{28}$$

where $S = Ig/2I_A(\gamma_r')^2$. For a "space-charge dominated" beam, which is considered here, the last emittance term can be neglected. This equation is equivalent to the envelope equation used in [2] and others. The difference is that we are using the reduced coordinates, which significantly simplifies the particle dynamics. When Ω and S are independent of s, an obvious solution of the reduced envelope equation is given by $\hat{\sigma}_r'' = \hat{\sigma}_r' = 0$ and

$$\hat{\sigma}_r = \hat{\sigma}_{\mathrm{inv}} \equiv \sqrt{\frac{S}{\Omega^2+1/4}}. \tag{29}$$

This is the so-called invariant envelope discussed obscurely in [2]. See [3] for further discussions.

For small deviations from the invariant envelope, we can linearize the envelope equation around the invariant envelope with $\hat{\sigma} = \hat{\sigma}_{\mathrm{inv}} + \delta\hat{\sigma}$ and get

$$\delta\hat{\sigma}'' + 2(\gamma'/\beta\gamma)^2(\Omega^2+1/4)\delta\hat{\sigma} = 0. \tag{30}$$

Solving this equation, we obtain the solution of the envelope equation around the invariant envelope as

$$\hat{\sigma} \;=\; \hat{\sigma}_{\mathrm{inv}} + \sqrt{\frac{\gamma}{\gamma_0}}\frac{\delta\hat{\sigma}(0)}{\cos\theta}\cos(u+\theta), \tag{31}$$

$$\hat{\sigma}' \;=\; -\sqrt{\frac{\gamma'^2}{\gamma_0\gamma}\left(\omega^2+\frac{1}{4}\right)}\,\frac{\delta\hat{\sigma}(0)}{\cos\theta}\sin(u+\theta-\theta_0), \tag{32}$$

where $\omega = \sqrt{2\Omega^2+1/4}$, $\delta\hat{\sigma}_r(0)$ and $\delta\hat{\sigma}_r'(0)$ are the initial envelope deviations, and $\theta_0 = \tan^{-1}(1/2\omega)$ and $\theta = \tan^{-1}[\frac{1}{2\omega} - \frac{\gamma_0\delta\hat{\sigma}'(0)}{\omega\gamma'\delta\hat{\sigma}(0)}]$ are phase angles determined by the initial values. Estimating the emittance with the commonly used two-slice approximation $\epsilon = \frac{1}{2}\left|\hat{\sigma}_+\hat{\sigma}_-' - \hat{\sigma}_-\hat{\sigma}_+'\right|$, and assuming one slice is the rms-matched invariant envelope $\hat{\sigma}_{\mathrm{rms}}$ with $\hat{\sigma}_{\mathrm{rms}}' = 0$ and the other is the slightly mismatched edge slice oscillating around its own invariant envelope according to the above expression, then $\epsilon = \frac{1}{2}\left|\hat{\sigma}_{\mathrm{rms}}\hat{\sigma}_{\mathrm{edge}}'\right|$ and we have

$$\begin{aligned}\epsilon =\; & \hat{\sigma}_{\mathrm{rms}}\sqrt{\frac{\gamma'^2}{\gamma_0\gamma}\left(\omega^2+\frac{1}{4}\right)}\left|\frac{\delta\hat{\sigma}_{\mathrm{edge}}(0)}{\cos\theta}\sin(u+\theta-\theta_0)\right| \\ \propto\; & \frac{1}{\sqrt{\gamma}}\left|\sin\left(\omega\ln\frac{\gamma}{\gamma_0}+\theta-\theta_0\right)\right|. \end{aligned}\tag{33}$$

It clearly shows that the correlated emittance is damped by $\sqrt{\gamma}$ and periodically returns to zero, which is the behavior of an emittance-compensated beam [2]. The focusing solenoid controls the emittance oscillation through ω.

THIRD-ORDER HAMILTONIAN

The third-order transverse Hamiltonian can be worked out as $H_3 = H_G + H_C$. The geometric part reads

$$H_G = \frac{\hat{\phi}_3^{\mathrm{sc}}}{\hat{P}_r^k} + w_g\frac{(\hat{x}^2+\hat{y}^2)\hat{z}}{2} - \left(\frac{\partial^2\hat{A}_r}{\partial z\partial r}\right)_r(\hat{x}\hat{p}_x+\hat{y}\hat{p}_y)\hat{z},$$

where

$$w_g = \frac{(\hat{P}_r^k)'}{\hat{P}_r^k}\left(\frac{\partial^2\hat{A}_r}{\partial z\partial r}\right)_r - \left(\frac{\partial^3\hat{A}_z^{\mathrm{rf}}}{\partial z\partial r^2}\right)_r.$$

The chromatic part reads

$$\begin{aligned}H_C \;=\; & -\frac{\beta_r^2}{2\hat{P}_r^k}\,\hat{p}_z\left[(\hat{p}_x^2+\hat{p}_y^2)+w_c(\hat{x}^2+\hat{y}^2)\right] \\ & +\frac{\beta_r^2}{\hat{P}_r^k}\,\hat{p}_z\left[\frac{\sqrt{\hat{P}_r^k}\,'}{\sqrt{\hat{P}_r^k}}(\hat{x}\hat{p}_x+\hat{y}\hat{p}_y) - (\partial_y\hat{A}_x)_r\,(\hat{x}\hat{p}_y-\hat{y}\hat{p}_x)\right],\end{aligned}$$

where

$$w_c = \left(\frac{\sqrt{\hat{P}_r^k}\,'}{\sqrt{\hat{P}_r^k}}\right)^2 + (\partial_x\hat{A}_y)_r^2 + \frac{(\partial_r^2\hat{A}_z)_r}{(\hat{P}_r^k)^2}.$$

In emittance-compensated guns, the quasi-laminar beam propagates around the invariant envelope, on which there are no transverse momenta, and thus most of these nonlinear terms could become insignificant. More detailed analysis will be reported elsewhere.

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