NONLINEAR δf PARTICLE SIMULATIONS OF COLLECTIVE EFFECTS IN HIGH-INTENSITY BUNCHED BEAMS*

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Abstract

The collective effects in high-intensity bunched beams are described self-consistently by the nonlinear Vlasov-Maxwell equations. The nonlinear δf method, a particle simulation method for solving the nonlinear Vlasov-Maxwell equations, is being used to study the collective effects in high-intensity bunched beams. The δf method, as a nonlinear perturbative scheme, splits the distribution function into equilibrium and perturbed parts. The perturbed distribution function is represented as a weighted summation over discrete particles, where the particle orbits are advanced by equations of motion in the focusing field and self-generated fields, and the particle weights are advanced by the coupling between the perturbed fields and the zero-order distribution function. The nonlinear δf method exhibits minimal noise and accuracy problems in comparison with standard particle-in-cell simulations. A self-consistent kinetic equilibrium is first established for high intensity bunched beams. Then, the collective excitations of the equilibrium are systematically investigated using the δf method implemented in the Beam Equilibrium Stability and Transport (BEST) code.

INTRODUCTION

Collective effects in high intensity charged particle beams often manifest as collective excitations with certain interesting dynamical properties such as instabilities and Landau damping. To understand the collective effects, it is necessary to study the equilibria of the beams and the characteristics of linear and nonlinear perturbations of the equilibria. A self-consistent theoretical framework based on the nonlinear Vlasov-Maxwell equations has been established for this purpose [1]. A corresponding numerical method, the δf particle simulation method, has also been developed [2]. This theoretical and numerical framework has been successfully applied to study stable beam propagation [3], electron-ion two-stream instabilities [4–6], and temperature anisotropy instabilities [7, 8]. However, previous studies were carried out for long coasting beams with arbitrary nonlinear space-charge field in the transverse direction. In this paper, we apply the Vlasov-Maxwell equations and the δf simulation method to bunched beams with nonlinear space-charge fields in both the longitudinal and transverse directions. For bunched beams, the equilibrium and collective excitation properties are qualitatively different from those for coasting beams. First of all, due to the coupling between the transverse and longitudinal dynamics induced by the 2D nonlinear space-charge field, there exists no exact kinetic equilibrium which has anisotropic temperature in the transverse and longitudinal directions. Secondly, even in a thermal equilibrium with isotropic temperature, particles' trajectories on constant energy surfaces are non-integrable [9, 10], which implies that it is impossible to perform an integration along unperturbed orbits to analytically calculate the linear eigenmodes. This paper is organized as follows. After a brief summary of the theoretical model and simulation method, the self-consistent equilibrium of a bunched beam is solved, and then, one case of linear collective excitations for the bunched beam is examined numerically.

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THEORETICAL MODEL AND δF SIMULATION METHOD

To simplify the problem, in the present study we consider a single species bunched beam confined in both the $r-$ and $z-$ directions by external smooth focusing force in the beam frame

$$\mathbf{F}_{foc} = -m_b\omega_{\beta b}^2\mathbf{x}_\perp - m_b\omega_z^2 z\mathbf{e}_z\,. \tag{1}$$

In the beam frame, the dynamics of the bunched beam is described by the nonlinear Vlasov-Maxwell equations

$$\begin{aligned}&\left\{\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} - \left[m_b\left(\omega_{\beta b}^2\mathbf{x}_\perp + \omega_z^2 z\mathbf{e}_z\right)\right.\right.\\ &\left.\left. + e_b(\nabla\phi - \frac{v_z}{c}\nabla_\perp A_z)\right]\cdot\frac{\partial}{\partial \mathbf{p}}\right\} f(\mathbf{x},\mathbf{p},t) = 0,\end{aligned} \tag{2}$$

$$\nabla^2\phi = -4\pi e_b\int d^3p f(\mathbf{x},\mathbf{p},t), \tag{3}$$

$$\nabla^2 A_z = -\frac{4\pi}{c}e_b\int d^3p v_z f(\mathbf{x},\mathbf{p},t). \tag{4}$$

This set of equations is a simplified version of the nonlinear Vlasov-Maxwell equations in the general cases [1,6]. For the boundary conditions, a perfect conducting cylindrical pipe is located at the radius $r = r_w$. To numerically solve the Vlasov-Maxwell equations, we use the low-noise δf method [2, 4, 5], where the total distribution function is divided into two parts, $f = f_0 + \delta f$. Here, f_0 is a *known* equilibrium solution ($\partial/\partial t = 0$) to the nonlinear Vlasov-Maxwell equations (2)-(4), and the numerical simulation is carried out to determine the detailed nonlinear evolution of the perturbed distribution function δf. This is accomplished by advancing the weight function defined

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by $w \equiv \delta f/f$, together with the particles' positions and momenta. The dynamical equation for w_i is given by [5]

$$\begin{aligned} \frac{dw_i}{dt} &= -(1-w_i)\frac{1}{f_0}\frac{\partial f_0}{\partial \mathbf{p}} \cdot \delta\left(\frac{d\mathbf{p}_i}{dt}\right), \\ \delta\left(\frac{d\mathbf{p}_i}{dt}\right) &\equiv -e_b\left(\nabla\delta\phi - \frac{v_{zi}}{c}\nabla_\perp \delta A_z\right), \end{aligned} \tag{5}$$

where the subscript "i" labels the i'th simulation particle, $\delta\phi = \phi - \phi_0$, and $\delta A_z = A_z - A_{z0}$. Here, the equilibrium solutions (ϕ_0, A_{z0}, f_0) solve the steady-state Vlasov-Maxwell equations (2)-(4). A detailed description of the nonlinear δf method can be found in Ref. [5]. For a single species beam, we neglect A_z in the beam frame because $|A_z| \ll |\phi|$.

EQUILIBRIUM AND NON-INTEGRABLE ORBITS

Collective excitations or eigenmodes of charged particle beams are perturbations around the self-consistent equilibrium. The first step in the investigation is to identify the equilibrium (ϕ_0, f_0) satisfying

$$\left\{\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} - \left[m_b\left(\omega_{\beta b}^2 \mathbf{x}_\perp + \omega_z^2 z \mathbf{e}_z\right)\right.\right. \tag{6}$$

$$\left.\left. + e_b\nabla\phi_0\right]\cdot\frac{\partial}{\partial \mathbf{p}}\right\} f_0(\mathbf{x},\mathbf{p},t) = 0, \tag{7}$$

$$\nabla^2\phi_0 = -4\pi e_b \int d^3p f_0(\mathbf{x},\mathbf{p},t)\,. \tag{8}$$

Equation (6) implies that f_0 is an invariant of the particle dynamics in the equilibrium space-charge potential ϕ_0 and the external focusing field. Therefore, f_0 is a function of all of the independent invariants. Even for the simple model adopted here for bunched beams, there are only two invariants of the single particle dynamics in the equilibrium field, the total energy H and angular momentum defined by

$$H = \frac{p^2}{2m_b} + e_b\phi + \frac{1}{2}m_b\left(\omega_{\beta b}^2 r^2 + \omega_z^2 z^2\right), \tag{9}$$

$$p_\theta = r m_b v_\theta\,. \tag{10}$$

We choose f_0 to be a function of H only as

$$f_0 = f_0(H) = \frac{\hat{n}_b}{(2\pi m_b T)^{3/2}}\exp\left(\frac{-H}{T}\right), \tag{11}$$

which gives an isotropic temperature T in all directions. Here, $\hat{n}_b$ is the beam number density at $(r,z) = (0,0)$. To model bunched beams in accelerators, it is desirable to have anisotropic temperature in the transverse and longitudinal directions. However, rigorously speaking, such equilibria do not exist for bunched beams. Approximate kinetic equilibria with anisotropic temperature can be constructed for long bunches, or other cases where the coupling induced by the nonlinear space-charge field is weak. Results on this topic will be reported elsewhere. In the present study, we will adopt the equilibrium specified by Eq. (11) for simplicity. Under this assumption, the Poisson equation (8) becomes

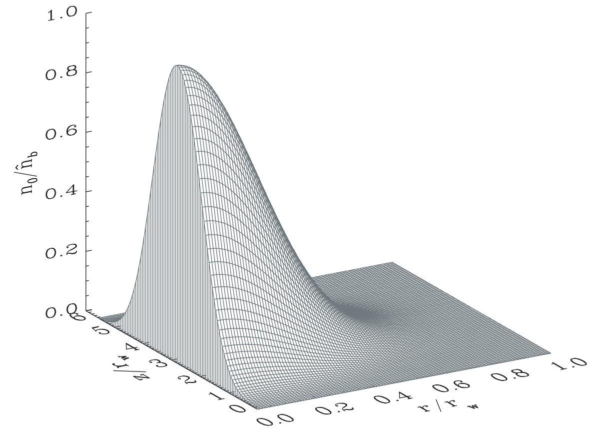

Figure 1: Equilibrium beam density at a function of (r, z).

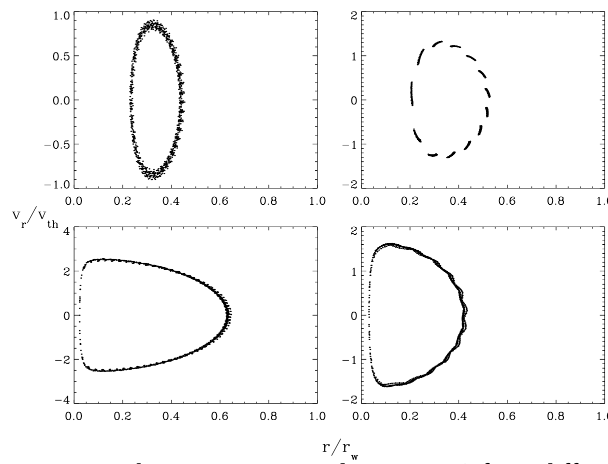

Figure 2: The r-v_r Poincare plot at $z = 0$ for 4 different particles.

$$\nabla^2\phi_0 = -4\pi e_b \hat{n}\exp\left[-\frac{m_b\left(\omega_{\beta b}^2 r^2 + \omega_z^2 z^2\right)}{2T} - \frac{e_b\phi_0}{T}\right], \tag{12}$$

which can be solved numerically for ϕ_0 in a perfect cylindrical conducting pipe with radius $r = r_w$. As an example, let's consider the case of a proton beam with $s_b = 0.079$, $\omega_z/\omega_{\beta b} = 0.1$, and $v_{th}/c = 1.6 \times 10^{-3}$, and $r_w\omega_{\beta b}/c = 6.75 \times 10^{-3}$. Here, $s_b \equiv 4\pi\hat{n}_b e_b^2/2m_b\omega_{\beta b}^2$ measures the relative strength of the space-charge force compared with the applied focusing force. Equation (12) is numerically solved for ϕ_0. Plotted in Fig. 1 is the normalized equilibrium density $n_0/\hat{n}_b \equiv \exp\left[-m_b\left(\omega_{\beta b}^2 r^2 + \omega_z^2 z^2\right)/2T - e_b\phi_0/T\right]$ as a function of (r, z). Even though the kinetic equilibrium is taken to be the well-behaved thermal equilibrium in Eq. (11), the dynamics of a single particle on the constant energy surface is nonintegrable. Figure 2 shows the r-v_r Poincaré plots at $z = 0$ for 4 different particles. Clearly, the $r - v_r$ cross-

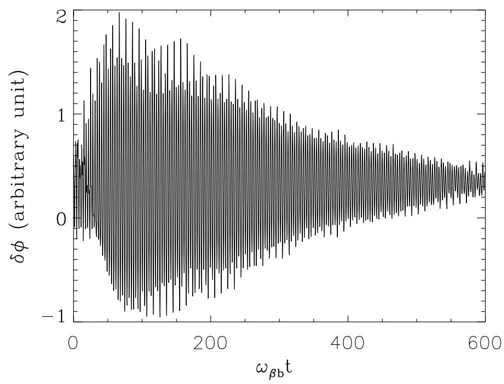

Figure 3: Time history of perturbed potential at a fixed spatial location.

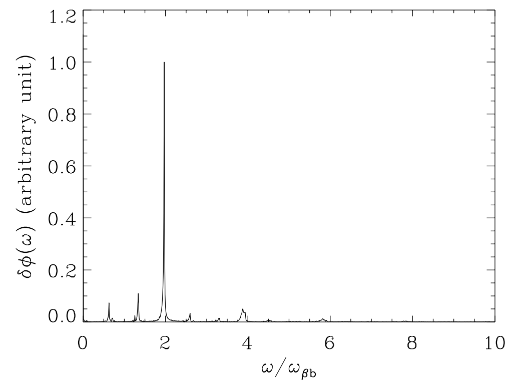

Figure 4: FFT spectrum of the perturbation.

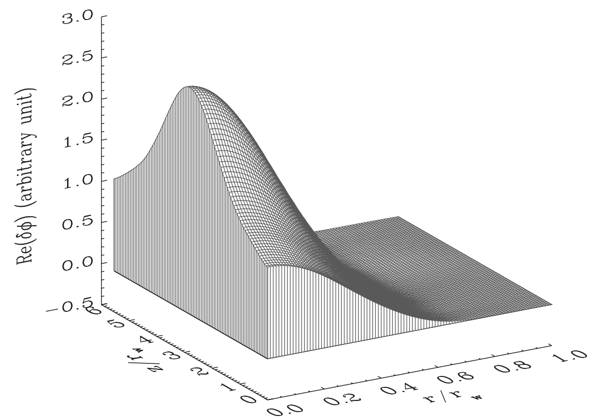

Figure 5: Real part of the mode structure for $\delta\phi$ at $\omega = 1.970\omega_{\beta b}$

sections are not curve-forming at $z = 0$. The nonintegrability is a result of lacking a third invariant of the dynamics, which is fundamentally due to the coupling between the transverse and longitudinal dynamics induced by the 2D nonlinear space-charge field. Previous studies on this subject can be found in References [9, 10]. It is also clear from Fig. 2 that the nonintegrability and the corresponding nonlinear space-charge-induced coupling are weak for this case, which permits us to construct an asymptotically approximate kinetic equilibrium with anisotropic temperature.

COLLECTIVE LINEAR EXCITATIONS

Once the equilibrium is determined, we can apply the δf particle simulation method to examine the linear and nonlinear evolution of perturbations in the system. In the present paper, we will only focus on linear perturbations. Because the particles' unperturbed orbits are nonintegrable, it is impossible to carry out the conventional analytical procedure of integrating along unperturbed orbits in an eigenmode calculation. From the point of view of particle simulations, the nonintegrability does not present any difficulty. Linear perturbations can be simulated in the same way as for coasting beams, where the particles' unperturbed orbits are integrable. Numerically, an arbitrary initial perturbation is imposed at $t = 0$, and the system is evolved using the δf method. Figure 3 shows the perturbed potential $\delta\phi$ as a function of time at a fixed spatial location. The spectrum (FFT) of the same data is displayed in Fig. 4. Clearly, the linear perturbation has several distinct eigenmode excitations. The dominate collective excitation is located at $\omega = 1.970\omega_{\beta b}$. From the detailed spectrum near $\omega = 1.970\omega_{\beta b}$, we can identify a small but finite-size width of the spectrum peak, which indicates that the mode is weakly damped. This fact is obvious from Fig. 3, which indeed shows a linear damping with a damping rate measured to be $\gamma = -0.02\omega_{\beta b}$. We can also extract the corresponding mode structure at $\omega = 1.970\omega_{\beta b}$. The real part of the mode structure is plotted in Fig. 5 as a function of (r, z). The imaginary part of $\delta\phi$ has a similar structure. The mode structure indicates that this is a body mode and has no node in either the r or z directions.

REFERENCES

[1] R. C. Davidson and H. Qin, *Physics of Intense Charged Particle Beams in High Energy Accelerators*, World Scientific, Singapore, 2001.

[2] H. Qin, Physics of Plasmas **10**, 2078 (2003).

[3] R. C. Davidson, Physical Review Letters **81**, 991 (1998).

[4] H. Qin, E. A. Startsev, and R. C. Davidson, Physical Review Special Topics-Accelerators and Beams **6**, 014401 (2003).

[5] H. Qin, R. C. Davidson, and W. W. Lee, Physical Review Special Topics Accelerators and Beams **3**, 084401 (2000).

[6] R. C. Davidson, H. Qin, P. H. Stoltz, and T. S. Wang, Phys. Rev. ST Accel. Beams. **2**, 054401 (1999).

[7] E. A. Startsev, R. C. Davidson, and H. Qin, Physics of Plasmas **9**, 3138 (2002).

[8] E. A. Startsev, R. C. Davidson, and H. Qin, Physical Review Special Topics Accelerators and Beams **6**, 084401 (2003).

[9] C. Bohn and I. Sideris, Physical Review Special Topics Accelerators and Beams **6**, 034203 (2003).

[10] S. R. Hudson, H. Qin, and R. C. Davidson, Nuclear Instruments and Methods in Physics Research A , in press (2005).

 0-7803-8859-3/05/$20.00