LONG TIME SIMULATION OF LHC BEAM PROPAGATION IN ELECTRON CLOUDS*

B. Feng, A. Ghalam, E. Benedetto*, F. Zimmermann*, V. Decyk**, W. Mori**, T. Katsouleas
University of Southern California,*CERN, ** University of California at Los Angeles.

Abstract

In this report we show the simulation results of single-bunch instabilities caused by interaction of a proton beam with an electron cloud for the Large Hadron Collider (LHC) using the code QuickPIC [1]. We describe three new results: 1) We test the effect of the space charge of the beam on itself; 2) we add the effect of dispersion in the equation of motion in the x direction, and 3) we extend previous modeling by an order of magnitude (from 50ms to 500ms) of beam circulation time. The effect of including space charge is to change the emittance growth by less than a few percent. Including dispersion changes the plane of instability but keeps the total emittance approximately the same. The longer runs indicate that the long term growth of electron cloud instability of the LHC beam cannot be obtained by extrapolating the results of short runs.

SPACE CHARGE EFFECT

In previous models of beam dynamics in an electron cloud based on the plasma wakefield code QuickPIC[1], the effect of the space charge of the cloud on the beam and itself were self consistently modelled. However, the effect of the beam's own space charge on itself was neglected. Although this effect is expected to be small, we explicitly add it here and check this assumption.

To include the effect of beam space charge on itself, we re-derive the field and force equations used in QuickPIC for beam velocity v_z less than c. For the field equations, we have,

$$\nabla_\perp^2 \phi = \rho_e + \rho_b \tag{1}$$

and,

$$\nabla_\perp^2 A_\parallel \cong J_z/c \approx \rho_b e v_z/c \tag{2}$$

where ρ_e is the electron cloud density and ρ_b is the beam charge density. ϕ is the electric potential, ψ is the pseudo potential and $A_\parallel$ ($\equiv \phi - \psi$) is the vector potential along the direction of the motion. Potentials have been normalized by factor of -4π.

For the force equations on the beam, we have,

$$\frac{F_{b\perp}}{e} = E_\perp - \frac{v_z}{c} B_\perp = E_\perp - B_\perp + (1 - \frac{v_z}{c}) B_\perp \tag{3}$$

and,

$$F_{e\perp} = -e \nabla_\perp \phi \tag{4}$$

Where $E_\perp = \nabla_\perp \phi$, $B_\perp = \nabla_\perp A_\parallel$ are transverse electric field and magnetic field respectively. $F_{b\perp}$ and $F_{e\perp}$ are the transverse forces on the beam and on the electrons respectively. Equations for ψ can then be recast as:

$$F_{\perp b} = e \nabla_\perp \psi' \tag{5}$$

and,

$$\nabla_\perp^2 \psi' = \rho_e + \frac{\rho_b}{\gamma^2}(1 - \frac{1}{4\gamma^2}) \tag{6}$$

where $\gamma = (1 - v_z^2/c^2)^{-1/2}$ is the relativistic factor, and

$$\psi' = \psi + \frac{\phi - \psi}{2\gamma^2} \tag{7}$$

The result of horizontal spot size with and without space charge is shown in Fig.1 for an LHC example. The parameters for the simulations are given in Table 1. We use a ten-kick per turn approximation for convenience. Although the ten-kick approximation is not valid, it is adequate to verify that the space-charge effect is small.

Table 1: Parameters used for LHC simulation [3]

Electron cloud density	ρ_e	$6*10^{11}m^{-3}$
Bunch population	N_b	$1.1*10^{11}$
Beta function	$\beta_{x,y}$	100m
rms bunch length	σ_z	0.115m
rms beam size	$\sigma_{x,y}$	0.884mm
rms momentum spread	δ_{rms}	$4.68*10^{-4}$
Synchrotron tune	Q_s	0.0059
Momentum compaction fact	α_c	$3.47*10^{-4}$
circumference	C	26.659km
Nominal tunes	$Q_{x,y}$	64.28, 59.31
chromaticity	$Q'_{x,y}$	2, 2
Space charge		Yes/no
Magnetic field		no
Linear coupling		no
dispersion	D	0m
Relativistic factor	γ	479.6

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#bfeng@usc.edu

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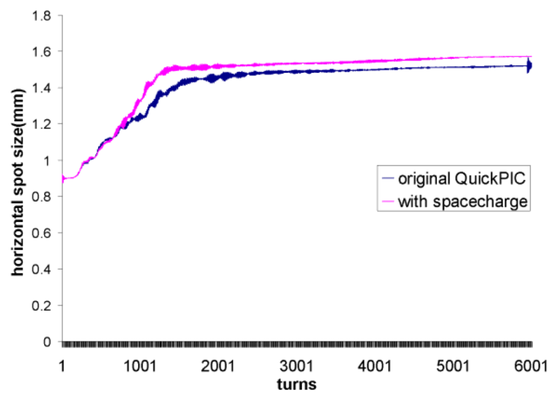

Fig. 1 Horizontal spot size of the beam at CERN-LHC with/without space charge using ten-kick method

As shown in Fig. 1, in the first 6000 turns (about 0.5s), space charge has a very small effect on horizontal emittance for the LHC (less than 3%).

DISPERSION EFFECT

The equation of motion of a single beam particle in the x direction previously implemented in QuickPIC was as follows [2]:

$$\frac{d^2x}{dt^2}+\left[Q_x+\Delta Q_x\left(\frac{\delta p}{p_0}\right)\right]^2\omega_0^2 x=\frac{1}{m_p\gamma}F_{b\_x} \quad (8)$$

In this equation Q_x is the horizontal tunes, ΔQ_x represents the chromatic shift proportional to the particle relative momentum offset $(\delta p/p_0)$; ω_0 is the angular revolution frequency of the beam in the circular accelerator (which for ultra-relativistic beams can be written as c/R_0, R_0 being the average machine radius); F_{b_x} is the force exerted by the cloud on each beam particle in x direction; p_0 is the nominal beam particle momentum; γ is its relativistic factor [2].

However, because of the momentum spread, the energy of the particles is not exactly p_0. Low energy particles fall toward the center of the ring, while Eq. (8) indicates that all particles oscillate about x=0. Thus we neglected a term due to dispersion in the x equation.

When the beam is moving in the circular ring, there are three forces exerted on the beam: the quadrupole focusing or betatron force F_β, the average dipole magnet force F_B and the centrifugal force F_C. These three forces balance at position $x=\delta x$ for a particle with velocity v:

$$F_\beta=\gamma m_p\omega_0^2 Q_x^2\delta x$$

$$F_B=qvB$$

$$F_C=\frac{\gamma m_p v^2}{R}$$

$$F_\beta+F_B=F_C$$

With energy spread, we have

$$\gamma m_p\omega_0^2 Q_x^2\delta x+qvB=\frac{\gamma m_p v^2}{R_0+\delta x} \quad (9)$$

δx is calculated as:

$$\delta x=\gamma^2\frac{\delta v}{v}\frac{R}{Q_x^2}=\frac{\delta p}{p}\frac{R}{Q_x^2} \quad (10)$$

Adding the missing term due to the dispersion, the adjusted x equation is

$$\frac{d^2x}{dt^2}+\left[Q_x+\Delta Q_x\left(\frac{\delta p}{p_0}\right)\right]^2\omega_0^2 x=\frac{1}{m_p\gamma}(F_{b\_x}+F_{disp}) \quad (11)$$

where,

$$F_{disp}/m_p\gamma=\frac{\delta p}{p}\frac{c^2}{R}$$

Another way to change the code is to keep the original form of the x equation (Eq. 8), and solve from that equation for x_0. Then the effect of dispersion can be added to that solution via the following expression

$$x=x_0+\delta x=x_0+\eta\frac{\delta p}{p} \quad (12)$$

where η is the dispersion factor and has the value R/Q_x^2 from Eq. (10).

We now evaluate δx for two cases – the LHC at injection and the SPS (parameters and results are shown in Table 2). δx is on the order of 10^{-4}m for the LHC case and is on the order of 10^{-3}m for SPS. Since the beam size is on the order of mm, we conclude that the dispersion effect is significant for both the LHC injection and SPS.

The results of simulation with and without dispersion are shown in Fig. 2 and Fig. 3. Parameters are given in Table 2.

Table 2: Parameters used for δx calculation [3]

	LHC at injection	**SPS**
Nominal tunes Q_x	64.28	26.185
rms momentum spread	$4.68*10^{-4}$	0.002
Circumference C	26.659km	6.9km
Radius R_0	4.25km	1.10km
Velocity v	$3*10^8$m/s	$3*10^8$m/s
Angular revolution frequency ω_0	$7.06*10^4$s^{-1}	$2.73*10^5$s^{-1}
Relativistic factor γ	479.6	27.728
δx	$4.81*10^{-4}$m	$3.21*10^{-3}$m

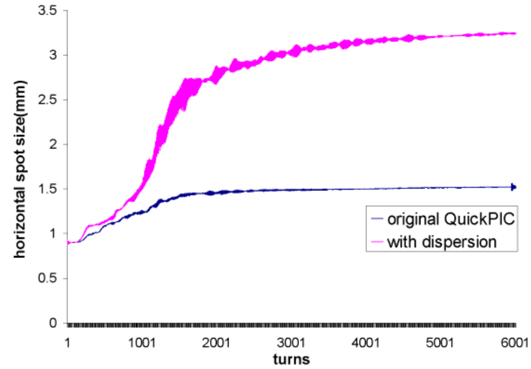

Fig. 2 Horizontal spot size of the beam at CERN-LHC with/without dispersion effect using ten-kick method.

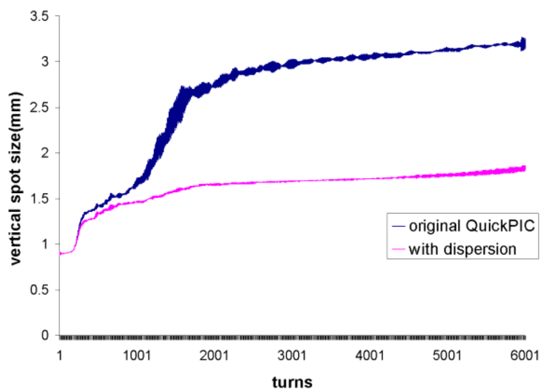

Fig. 3 Vertical spot size of the beam at CERN-LHC with/without dispersion effect using ten-kick method.

In the simulation of Fig. 2 and 3, dispersion changes the plane of instability, and the total emittance is approximately unchanged. Further work is in progress with continuous rather than ten-kick approximation.

LONG TIME SIMULATION

E.Benedetto in Ref. [3] simulated LHC beam propagation for 50ms using the code HEADTAIL, and extrapolated the result to 30minutes. We take advantage of parallel computing using QuickPIC to model the same problem over a period of 500ms. We then check the validity of the extrapolation of HEADTAIL results. One thing should be noted is that the multi-kick method is only qualitatively correct until there are more than 30 kicks per betatron wavelength [4].

We simulate the LHC case for 6000 turns which corresponds to about 500ms. The parameters are given in table 1. Fig. 4 shows the results of comparisons of QuickPIC and HEADTAIL both using ten kicks per turn.

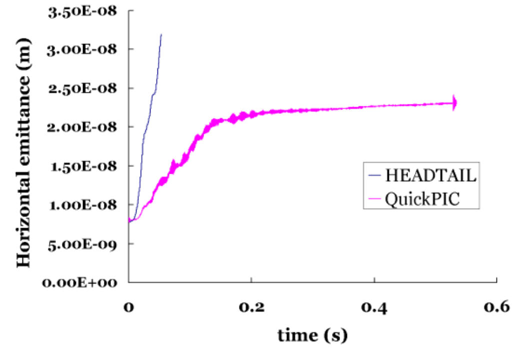

Fig. 4 Comparison of horizontal emittance of the beam at CERN-LHC using HEADTAIL and QuickPIC methods.

From the above, Fig. 4 illustrates it is not possible to extrapolate emittance growth from 0.05s to 30min or even 1s. At any moment an upward trend can saturate or a plateau can begin to grow anew. The results indicate the need for developing faster algorithms with the capability of modelling for 30 minutes.

We do not quite understand the difference between QuickPIC results and HEADTAIL results in the first 0.05s. One possible reason can be that multi-kick approximation exhibits an aspect of chaotic nonlinear behave. It is numerically very sensitive to initial conditions.

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