LUMINOSITY OPTIMIZATION WITH OFFSET, CROSSING ANGLE, AND DISTORTION

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Abstract

In a linear collider, sources of beam jitter due to kicker noise, quadrupole vibration and long-range transverse wakefields will lead to beam offsets and tilts at the Intersection Point (IP). In addition, sources of emittance dilution such as short-range transverse wakefields or dispersive errors will lead to internal beam distortions. When the IP disruption parameter is large, these beam imperfections will be amplified by a single bunch kink instability which will lead to luminosity loss. In this paper, we study the luminosity loss and then the optimization required to partially cancel the luminosity loss both analytically and with direct simulation.

Introduction

To achieve the desired luminosity in a future linear collider, the beams are focused to small spot sizes and the resulting beam-beam forces can be very large. With oppositely charged beams, the beam-beam forces will lead to a mutual focusing or pinch which further increases the beam densities and the luminosity and is referred to as the luminosity enhancement. In addition, if the beams are offset from each other, the attractive beam-beam force can bring the beams closer together possibly recovering some of the lost luminosity. Unfortunately, if the beam-beam force is too large, this attraction can lead to an instability much like a plasma two-stream instability which is referred to as a single bunch kink instability [1, 2], which can be parameterized with the disruption parameter: $D_{x,y}(y) = \sigma_z / [f_{x,y}(y)] = 2N_b r_c \sigma_z / [\gamma \sigma_x(y) \sigma_y(y)]$, where $f_{x,y}(y)$ is the focal length due to the beam-beam force; $\sigma_x(y,z)$ is the rms beam size, $N_b$ the number of particle per beam, $r_c$ the electron classical radius, and $\gamma$ the Lorentz factor. We will follow closely to the approach in Ref. [2]

Equations of Motion

Suppose that two beams move towards each other with velocity $v$, the equations of motion read [1]

$$\left( \frac{\partial}{\partial t} + v \frac{\partial}{\partial s} \right)^2 y_{l,r}(s,z) = -2 \lambda r_c e^2 \left[ y_{l,r}(s,z) - y_{r,l}(s,z) \right] / \sigma_y(\sigma_x + \sigma_y) \gamma,$$

where, $y_{l,r}(s,z)$ is the centroid displacements of the electron (positron) beam from the reference axis and $\lambda$ is the line density. The readers may refer to Ref. [2] for details.

Initial Value Problem

The internal coordinate $z$ is introduced to label the slice at a distance $z$ from the head of the beam, and $0 < z < l$, where $l$ is the beam length. We define $t = 0$ when the heads of the two beams collide. We also define $s = 0$ as the IP where the two beams first collide. The positive $s$ direction is to the right. Hence in the left-coming beam, the slice $z$ will be located at $z = vt - z$ at time $t$ when the beam head is at the location $s = vt$. In the right-coming beam, we also introduce $s$ to describe the distance between a certain slice and the head of the beam, and again $0 < z < l$. Hence, when the head of the right-coming beam is at location $s = -vt$ at time $t$, the slice $z$ is at location $s = -vt + z_t$. We now use $(s, z)$ as the independent variable pair, and we define $k_0^2 = 2 \lambda r_c / [\sigma_y(\sigma_x + \sigma_y) \gamma]$. The coordinate system is shown in Fig. 1. Now let us study two cases. For the first case, the right-coming beam has an initial offset $y_{r,0}$, and the left-coming beam is undistorted and on-axis. The second case, the right-coming beam is grabbed, and the left-coming beam is perfect. The initial conditions are then

$$y_l(0, z) = 0 \text{ and } \frac{\partial y_l(s, z)}{\partial s} \bigg|_{s=0} = 0 , \quad (2)$$

for the left-coming electron beam. In the first case,

$$y_r(0, z) = y_{r,0} \text{ and } \frac{\partial y_r(s, z)}{\partial s} \bigg|_{s=0} = 0 , \quad (3)$$

for the right-coming positron beam. In the second case,

$$y_r(0, z) = \theta_{r,0} z \quad \text{and } \frac{\partial y_r(s, z)}{\partial s} \bigg|_{s=0} = 0 . \quad (4)$$

The equations of motion together with the initial conditions yield the following integral representation of the solution

$$y_l(s, z) = y_l(z/2, z) \cos[k_0(s - z/2)] + \frac{\partial y_l(s, z)}{\partial s} \bigg|_{s=z/2} \sin[k_0(s - z/2)] k_0$$

Similarly, for the right-coming beam, we have

$$y_r(s, z) = y_r(-z/2, z) \cos[k_0(s + z/2)] + \frac{\partial y_r(s, z)}{\partial s} \bigg|_{s=-z/2} \sin[k_0(s + z/2)] k_0$$

The readers may refer to Ref. [2] for details.

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Series Solution

Let us solve the above set of Eqs. (5) and (6) via a series solution approach [2, 3, 4]. We expand \(y_{r,(j)}(s, z)\) in a series of powers of \(k_0\)

\[
y_{r,(j)}(s, z) = \sum_{n=0}^{\infty} y^{(n)}_{r,(j)}(s, z),
\]

and obtain the \(n\)th-order term from the \((n-1)\)th-order term. According to Eq. (5), for \(n = 1, 2, 3, \ldots\), \(y_r(s, z)\) would be

\[
y^{(n)}_r(s, z) = k_0 \int_{z/2}^{s} ds' y^{(n-1)}_r(-s', 2s'-z) \sin[k_0(s-s')];
\]

and similarly, according to Eq. (6), for \(y_s(s, z)\), we have

\[
y^{(n)}_s(s, z) = k_0 \int_{z/2}^{s} ds' y^{(n-1)}_s(-s', 2s'-z) \sin[k_0(s-s')].
\]

According to Eqs. (5) and (6) with the initial conditions in Eqs. (2), (3), and (4), we can get series solution as Ref. [2]. Here, we only give explicit asymptotic solution.

Offset For the first case, \(i.e.,\) the right coming beam has an initial offset, we have

\[
y_r(s, z) \approx -\frac{iy_0}{2} \sqrt{\frac{2s+z}{z}} J_1 \left(ik_0 \sqrt{z(2s+z)/2} \right) \times \sin[k_0(s+z)],
\]

\[
\approx -\frac{y_0}{2} \sqrt{\frac{2s+z}{z}} \left(\pi k_0 \sqrt{z(2s+z)/2} \right)^{-1/2} \exp \left(k_0 \sqrt{z(2s+z)/2} \right) \sin[k_0(s+z)],
\]

for \(-z/2 > s > -(l+z)/2\); and

\[
y_s(s, z) \approx -\frac{y_0}{2} J_0 \left(ik_0 \sqrt{z(2s-z)/2} \right) \cos[k_0(s-z)] \times \exp \left(k_0 \sqrt{z(2s-z)/2} \right) \cos[k_0(s-z)],
\]

for \((l+z)/2 > s > z/2\).

Crabbed beam For the second case, \(i.e.,\) the right coming beam is crabbed, we have

\[
y_r(s, z) \approx \frac{id_0}{k_0} \sqrt{\frac{2s+z}{z}} J_1 \left(ik_0 \sqrt{z(2s+z)/2} \right) \times \cos[k_0(s+z)],
\]

\[
\approx -\frac{\theta_0}{k_0} \sqrt{\frac{2s+z}{z}} \left(\pi k_0 \sqrt{z(2s+z)/2} \right)^{-1/2} \exp \left(k_0 \sqrt{z(2s+z)/2} \right) \cos[k_0(s+z)],
\]

for \(-z/2 > s > -(l+z)/2\); and

\[
y_s(s, z) \approx -\frac{\theta_0}{k_0} J_0 \left(ik_0 \sqrt{z(2s-z)/2} \right) \sin[k_0(s-z)] \times \exp \left(k_0 \sqrt{z(2s-z)/2} \right) \sin[k_0(s-z)],
\]

for \((l+z)/2 > s > z/2\).

Luminosity loss

Now let us illustrate how the beam-beam disruption leads to a large luminosity loss. We study the ILC US Cold

\[
L = 2N_0^2 \int dx dy ds dt n_l(x, y, z_l, t)n_r(x, y, z_r, t),
\]

where \(z_l = vt - s\) and \(z_r = vt + s\) and we assume the same number population \(N_0\) in each beam and head-on collisions. The distribution function is normalized to unit, \(i.e.,\) \(\int dx dy ds dt n_l(x, y, z_l, t) = 1\). Assuming Gaussian transversely and uniform longitudinally, and ignoring the luminosity enhancement due to beam-beam pinch, the ‘geometric’ luminosity is \(L_{00} = N_0^2 / [4\pi \sigma_x \sigma_y]\).

Figure 1: Schematic of the initial condition of the two beams and also the definition of the notations: \(s, z_{r,l}, l, and y_{r,0}\). In Fig. 1, \(\theta_{r,0} < 0\).

for \((l+z)/2 > s > z/2\).

Luminosity loss

Now let us illustrate how the beam-beam disruption leads to a large luminosity loss. We study the ILC US Cold

\[
\begin{array}{|c|c|c|}
\hline
E (GeV) & N_0 (10^{10}) & \sigma_x (\mu m) \\
\hline
250 & 2.0 & 0.543 \\
\hline
36 & 5.7 & 14 \\
\hline
0.3 & 0.1 & 22.0 \\
\hline
\end{array}
\]

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Luminosity optimization

We now study possible optimization via partial cancellation among various imperfection. As an example, imaging that the two beams come in with an offset $y_{r0}$; naively, we would crab the right-coming beam with a negative angle, so that the beam-beam attraction will bend the beam opposite to its original angle. The configuration is shown in Fig. 1. Now, in Fig. 6, the right-coming beam has an initial offset of $y_{r0}/\sigma_y = 1/3$ (black), 1 (green), 2 (red), and 5 (blue). We vary the initial angle $\theta_{r0}$. It is clearly shown that the optimization for each case comes at a negative $\theta_{r0}$.

Discussion

Due to the strong beam-beam disruption, initial imperfection is exponentially amplified, which leads to substantial luminosity loss for $D_y > 10$. In reality, beams have offset, crossing angle, and also modulation [2], then a detailed optimization is necessary, besides feedback approach.

REFERENCES