THE EFFECT OF INHOMOGENEOUS MAGNETIC FIELD ON BUDKER-CHIRIKOV INSTABILITY

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Abstract

The two-beams electron - ion system consists of a nonrelativistic ion beam propagating co-axially with a high-current relativistic electron beam in a longitudinal inhomogeneous magnetic field. The effect of the longitudinal inhomogeneous magnetic field on instability Budker-Chirikov (BCI) in the system is investigated by the method of a numerical simulation in terms of the kinetic description of both beams. The investigations are development of investigations in [1,2,3].

Is shown, when the inhomogeneity magnetic field results in the decreasing of an increment of instability Budker-Chirikov and the increasing of length of propagation of a electron beam. Also is shown, when take place the opposite result.

BASIC EQUATIONS

We investigate a two-beam electron—ion system consisting of a nonrelativistic ion beam propagating co-axially with a high-current relativistic electron beam. The both beams are injected in equilibrium into drift tube. The kinetic description- of both beams is provided by means of solutions of the Vlasov equations for the electron and ion distributions functions, $f_{e,i}\ (t, z, r, p_z, p_r, p_\theta)$:

$$\frac{\partial f_\alpha}{\partial t}+\frac{p_{z\alpha}}{m_\alpha\gamma_\alpha}\frac{\partial f_\alpha}{\partial z}+\frac{p_{r\alpha}}{m_\alpha\gamma_\alpha}\frac{\partial f_\alpha}{\partial r}+$$
$$F_{z\alpha}\frac{\partial f_\alpha}{\partial p_{z\alpha}}+F_{r\alpha}\frac{\partial f_\alpha}{\partial p_{r\alpha}}+F_{\vartheta\alpha}\frac{\partial f_\alpha}{\partial p_{\vartheta\alpha}}=0, \qquad (1)$$

where

$$\gamma_\alpha=\sqrt{1+\sum_\beta p_{\beta\alpha}^2/(m_\alpha c)^2}$$

The equations for the scalar potential and the three component of the vector potential are used for finding the electromagnetic fields. The equations are solved in the long-wave ($\partial^2/\partial z^2 << \Delta_\perp$), low-frequency ($\partial^2/\partial t^2 << c^2 \Delta_\perp$), axial-symmetric ($\partial/\partial\theta \equiv 0$) case. where $\Delta_\perp$ is the transverse part of the Laplace operator. Boundary conditions for the potentials fellow from the system's axial symmetry, the presence of conducting tube with radius R and the gauge condition div $\boldsymbol{A}$ = 0. The Vlasov equations are solved by the macroparticle method. It is assumed that the steady-slate process is periodic in time set with a frequency ω. In this case it is convenient to use the longitudinal coordinate z as the independent variable, using the relation d/dt = (l/ v_z) d/dz, where v_z , is the velocity of a given macroparticle. The problem is then reduced to the evolution of a periodic-in-time system on z.

The periodic in time (with frequency w) potential function is of the form

$$G(t,z,r)=\overline{G}(z,r)+$$
$$\mathrm{Re}\left\{\sum_j \tilde{G}_j(z,r)\cdot e^{ij\omega t}\right\} \qquad (2)$$

which is substituted into the equations for the potential components and integrated over a time period. The equations for the four components of the 4-poternial A_i , ($\Delta_{ri} A_i = 4\pi\rho_i$, i= 1, 2, 3, 4 Δ_{ri} is the radial parts of the D'Alambertian, and ρ_i are the components of the 4-density) are solved at every z--cross-section by the grid method.

We obtain the equations for the macroparticles:

$$\frac{dp_{z\alpha}}{d\xi}=K_\alpha\frac{\gamma_\alpha}{p_{z\alpha}}\left(-\varepsilon_z-\frac{p_{r\alpha}}{\gamma_\alpha}b_\vartheta+\frac{p_{\vartheta\alpha}}{\gamma_\alpha}b_r\right),$$

$$\frac{dp_{r\alpha}}{d\xi}=K_\alpha\frac{\gamma_\alpha}{p_{z\alpha}}\left(-\varepsilon_r-\frac{p_{\vartheta\alpha}}{\gamma_\alpha}b_z+\frac{p_{z\alpha}}{\gamma_\alpha}b_\vartheta\right)$$
$$+\frac{p_{\vartheta\alpha}^2}{\rho_\alpha p_{z\alpha}}, \qquad (3)$$

$$\frac{dp_{\vartheta\alpha}}{d\xi}=\frac{K_\alpha}{p_{z\alpha}}\left(p_{r\alpha}b_z-p_{z\alpha}b_r\right)-\frac{p_{\vartheta\alpha}p_{r\alpha}}{\rho_\alpha p_{z\alpha}},$$

where we have written K_e =1, $K_i = -m_e Z_i/m_i$,and Z_i is the ion charge state, α = e, i.

When the electric field equation:

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$$\varepsilon_\beta(\tau,\xi,\rho) = -\nabla_\beta\left\{\overline{\psi}(\xi,\rho) + \mathrm{Re}\left[\sum_j \tilde{\psi}_j(\xi,\rho)e^{ij\tau}\right]\right\} - \mathrm{Re}\left[\sum_j ij\tilde{\omega}\tilde{a}_\beta(\xi,\rho)e^{ij\tau}\right], \quad (4)$$

and the magnetic field equation:

$$b_\beta(\tau,\xi,\rho) = rot_\beta\left\{\overline{a}(\xi,\rho) + \mathrm{Re}\left[\sum_j ij\tilde{\omega}\tilde{a}(\xi,\rho)e^{ij\tau}\right]\right\} + b_{\beta 0}, \quad (5)$$

The scalar potential equations as equation (2):

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial\overline{\psi}}{\partial\rho} = -\frac{1}{2}\sum_\alpha\left\{\nu_\alpha Z_\alpha \sum_{l=0}^{L_\alpha}\frac{p_{zl}(0)}{\gamma_l(0)}\frac{\gamma_l(\xi)}{p_{zl}(\xi)}\frac{\delta(\rho-\rho_l(\xi))}{\rho_l(\xi)}\right\}, \quad (6)$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial\tilde{\psi}_j}{\partial\rho} = -\sum_\alpha\left\{\nu_\alpha Z_\alpha \sum_{l=0}^{L_\alpha}\frac{p_{zl}(0)}{\gamma_l(0)}\frac{\gamma_l(\xi)}{p_{zl}(\xi)}\frac{\delta(\rho-\rho_l(\xi))}{\rho_l(\xi)}\cdot\exp[-ij\tau_l(\xi)]\right\},$$

with conditions

$$b_{\beta 0} = (b_0 + b_z(r), 0, 0),$$

$$b_0 = \frac{eB_0 R_T}{m_e c^2}, \quad (7)$$

$$\frac{\partial\psi}{\partial\rho}(0) = \frac{\partial a_z}{\partial\rho}(0) = a_\vartheta(0) = a_\rho(0) = 0,$$

$$\psi(1) = a_z(1) = a_\vartheta(1) = \frac{\partial}{\partial\rho}(\rho a_\rho)\bigg|_{\rho=1} = 0.$$

When B_0 is the internal magnetic field, ρ=r/R, ξ=z/R, R – radius drift tube.

BCI INSTABILITY

The most important of the instabilities in the two-beam electron—ion system is the Budker- Chirikov instability (BCI) [4,5]. It is connected with the resonance of the slow-cyclotron wave of electron beam and fast betatron wave of the ion beam. Unlike [1], the spread of longitudinal velocities of an electron beam are took place.

The Budker-Chirikov instability take place in that case. The instability in the time periodic regime is displayed in the growth of the radial modulation amplitude of both beams along the longitudinal coordinate. Also the BCI is developed in exponential growth of the potential amplitude.

The dependence of the BCI increment from the increase the inhomogeneity magnetic field has appeared multiple-valued. With some parameters of the two-beams electron - ion system the BCI increment decreases with the increase the inhomogeneity magnetic field. In that case also take place the increasing of length of propagation of a electron beam. But with other parameters of the two-beams electron - ion system take place the opposite result.

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