EFFECTS OF SPACE CHARGE ON DECOHERENCE IN ION BEAMS

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Abstract

This paper studies dipolar and quadrupolar decoherence of a bunch in presence of space charge. The centroid of a bunch displaced transversely or longitudinally decoheres due to nonlinearities that cause phase space filamentation and mixing. Here we show that space charge can inhibit decoherence and keep the beam centroid oscillations undamped over long times. This feature complicates the detection of echoes for diagnostics purposes. An echo requires in fact a fully decohered beam to show up as a coherent signal at later times than the excitations.
Results are qualitatively compared with experiments in the GSI synchrotron SIS.

INTRODUCTION

The question of longitudinal decoherence of the collective motion of a bunch of charged particles in presence of nonlinearities due to a sinusoidal bucket and space charge is addressed in this paper.
Experimental observations are described in Section II. Section III shows a simplified calculation of decoherence due to nonlinearity and space charge. Semi-analytical expressions for the evolution of the bunch longitudinal centroid and rms-size (first two moments of the detailed longitudinal particle distribution, on which alone we assume the space charge forces to depend) with the non-linearity alone are presented in Section IV. These expressions can then be used as starting point of a recursive numerical procedure, which recalculates the single particle equation of motion with space charge and thus converges to the final profile of longitudinal centroid and rms-size evolution with space charge included after a few iterations.

OBSERVATION OF UNDAMPED CENTROID MOTION IN SIS

Longitudinal bunched beam dipole oscillations are observed during SIS operation, if there is an energy error between the injected beam from the UNILAC and the rf frequency setting in SIS. For low beam intensities the induced bunch center dipole oscillation is damped before the start of the acceleration ramp. For high beam intensities or electron cooled beams the dipole oscillation survive during ramping. For example, the waterfall plot in Fig. 1 shows the persistent dipole oscillation for 0.5×10^{10} C^{6+} ions per bunch with an estimated equivalent coasting beam momentum spread $\Delta p/p|_{FWHM} \sim 5 \times 10^{-4}$. The estimated frequency error is $\Delta f/f \sim 1 \times 10^{-3}$ (corresponding to $\approx 2\sigma_{z0}$ kick in the longitudinal phase space)

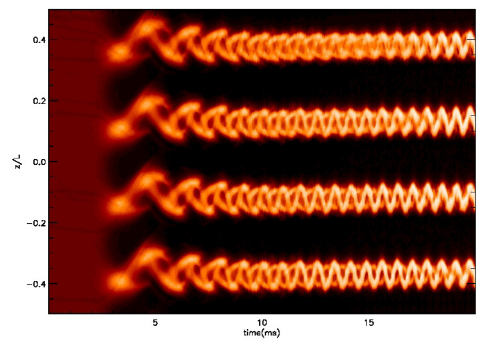

Figure 1: Observed persistent dipole oscillations of four bunches in SIS as function of time.

DECOHERENCE WITH SPACE CHARGE

A bunch with an initial longitudinally offset from the bucket center will perform synchrotron oscillations around it. If all particles have the same synchrotron tune, the centroid motion is expected to be harmonic. However, if the beam contains a spread of tunes, the motion will decohere since the individual synchrotron phases of the particles disperse. As the longitudinal phase space of the beam spreads to an annulus, the observed centroid of the beam will show a decaying oscillation and its rms-size will grow. Space charge can inhibit the centroid decoherence and thus keep the oscillations undamped by local compensation of the synchrotron detuning with amplitude.
The synchrotron frequency spread as a function of the single particle oscillation amplitude $\hat{\phi}$ in a RF bucket is (see e.g. Ref. [1])

$$\omega_s(\hat{\phi}) = \frac{\pi\omega_s(0)}{2K(\sin^2\frac{\hat{\phi}}{2})} \approx \omega_s(0)\left(1 - \frac{\hat{\phi}^2}{16}\right), \qquad (1)$$

Assuming a parabolic bunch form, space charge induces a shift $\Delta\omega$ of the incoherent frequency distribution. This shift can be related to the space charge parameter [2] $\Sigma = 2K_L z_m/\epsilon_L^2$ (longitudinal emittance ϵ_L^2 and perveance K_L, bunch half-length z_m)

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{4}\Sigma \qquad (2)$$

Damping of dipole modes is only possible if the coherent dipole frequency $\Omega = \omega_s(\bar{\phi})$ (amplitude of the dipole mode $\bar{\phi}$) that is not affected by space charge, lies within the incoherent frequency distribution. Persistent dipole oscillations

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after kicking a beam to a finite phase $\bar{\phi}$, are possible if space charge shifts the frequency distribution towards sufficiently low frequencies. Below transition energy $\Omega > \omega_0 - \Delta\omega$ yields the criterion

$$\frac{\Delta\omega}{\omega_0} > \frac{\bar{\phi}^2}{16} \quad \Rightarrow \quad \Sigma > \frac{\bar{\phi}^2}{4} \quad . \tag{3}$$

This criterion is consistent with the SIS observation ($\Sigma \approx$0.4). A similar condition was independently obtained in Ref. [3]. It is interesting to note, that above transition energy or for negative Σ (inductive impedance) one obtains

$$|\Sigma| > \frac{1}{4}\left(\hat{\phi}_m^2 - \bar{\phi}^2\right) \tag{4}$$

with the parabolic bunch half-length $\hat{\phi}_m^2$.

ITERATIVE PROCEDURE

We can express the centroid and rms-size evolutions through the integrals:

$$\begin{aligned} \bar{z}(t) &= \int_{\Re^2} z(\hat{z},\hat{\delta},\bar{z},\sigma_z)\rho(\hat{z},\hat{\delta})\mathrm{d}\hat{z}\mathrm{d}\hat{\delta} \\ \sigma_z(t) &= \int_{\Re^2} z^2(\hat{z},\hat{\delta},\bar{z},\sigma_z)\rho(\hat{z},\hat{\delta})\mathrm{d}\hat{z}\mathrm{d}\hat{\delta} - \bar{z}^2(t) \end{aligned} \tag{5}$$

where $\rho(\hat{z},\hat{\delta})$ is the initial particle distribution in the longitudinal phase space, $\rho(\hat{z},\hat{\delta}) = \frac{1}{2\pi\sigma_{z0}\sigma_{\delta 0}}\exp\left[-\frac{1}{2}\left(\frac{(\hat{z}-z_0)^2}{\sigma_{z0}^2}+\frac{\hat{\delta}^2}{\sigma_{\delta 0}^2}\right)\right]$, and $z(\hat{z},\hat{\delta},\bar{z},\sigma_z)$ is the solution of the single particle equations of motion in presence of space charge

$$\begin{cases} \dot{z} = -\eta c\delta \\ \dot{\delta} = \mathrm{sgn}(\eta)\dfrac{\mathrm{e}V_m}{p_0 2\pi R_0}\sin\left(\dfrac{\omega_{rf}z}{\mathrm{c}}\right) + \mathcal{F}_{sc}(z-\bar{z},\sigma_z) \end{cases} \tag{6}$$

The problem consists of a very complicated integro-differential set of equations having $\bar{z}(t)$ and $\sigma_z(t)$ as unknowns with initial conditions $\bar{z}(t=0) = z_0$ and $\sigma_z(t=0) = \sigma_{z0}$. We solve it by iterations following the procedure that we describe here below. As first step, we neglect space charge. Using now Eqs. (5) with the solution of the equation of motion (6) in nonlinear regime and without driving term leads us to the expressions for the bunch centroid and rms-size evolutions in absence of space charge,

$$\bar{z}(t) = -\frac{1}{2\pi\sigma_{z0}\sigma_{\delta 0}}\mathrm{Re}\left[\frac{\mathrm{i}\cdot \mathrm{e}^{\mathrm{i}\omega_s t}}{\sqrt{-\frac{1}{\pi}\left(\frac{1}{2\sigma_{\delta 0}^2}+\mathrm{i}\cdot\frac{p_0|\eta|\mathrm{c}k}{2}t\right)}}\cdot \int_{\Re} \mathrm{e}^{-\mathrm{i}\cdot\frac{\mathrm{e}V_m \mathrm{c}k}{2\pi R_0\omega_{rf}}C(\hat{z})t - \frac{(\hat{z}-z_0)^2}{2\sigma_{z0}^2}}\mathrm{d}\hat{z}\right] \tag{7}$$

$$\begin{aligned} \sigma_z^2(t) + \bar{z}^2(t) = \sigma_{z0}^2 + \frac{z_0^2}{2} - \frac{1}{4\pi\sigma_{z0}\sigma_{\delta 0}} \cdot \mathrm{Re}\Bigg[&\int_{\Re}\hat{z}^2\mathrm{e}^{-\frac{(\hat{z}-z_0)^2}{\sigma_{z0}^2}-2\mathrm{i}\cdot\frac{k\mathrm{e}V_m\mathrm{c}}{2\pi R_0\omega_{rf}}C(\hat{z})t}\mathrm{d}\hat{z}\cdot \frac{\mathrm{i}\cdot\mathrm{e}^{2\mathrm{i}\omega_s t}}{\sqrt{-\frac{1}{\pi}\left(\frac{1}{2\sigma_{\delta 0}^2}+\mathrm{i}\cdot p_0|\eta|\mathrm{c}kt\right)}} + \frac{\eta^2\mathrm{c}^2}{\omega_s^2}\cdot \\ &\cdot\int_{\Re}\mathrm{e}^{-\frac{(\hat{z}-z_0)^2}{\sigma_{z0}^2}-2\mathrm{i}\cdot\frac{k\mathrm{e}V_m\mathrm{c}}{2\pi R_0\omega_{rf}}C(\hat{z})t}\mathrm{d}\hat{z}\cdot\frac{\mathrm{e}^{2\mathrm{i}\omega_s t}}{\frac{1}{\sigma_{\delta 0}^2}+4\mathrm{i}\cdot p_0|\eta|\mathrm{c}kt}\Bigg] \end{aligned} \tag{8}$$

where $C(\hat{z}) = 1 - \cos\left(\frac{\omega_{rf}z}{\mathrm{c}}\right)$.

Table 1: SIS parameters used in this study.

variable	symbol	value
Circumference	$2\pi R_0$	216 m
Revolution frequency	ω_0	8.7×10^6 rad/s
Relativistic gamma	γ	3.129
Chamber size	a, b	$10 \times \sigma_{x,y}$
Bunch population	N_b	10^{10} C^{6+}
Rms bunch length	σ_{z0}	2 m
Rms energy spread	$\sigma_{\delta 0}$	5.9×10^{-4}
Slip factor	η	-0.0665
Synchrotron tune	Q_{s0}	6.8×10^{-4}
Initial kick amplitude	z_0	4 m
Maximum voltage	V_m	32 kV
Harmonic number	h	4

Figures 2 and 3 (blue lines) show longitudinal centroid and rms-size evolutions as resulting from the expressions (7) and (8) for SIS parameters in Table I. Due to the sinusoidal bucket the centroid oscillation, which would have survived forever undamped in the case of purely linear restoring force, significantly decoheres after a few synchrotron periods. At the same time the bunch longitudinal rms-size grows and tends to level off at the asymptotic value

$$\sigma_z(t\to\infty) = \sqrt{\sigma_{z0}^2 + \frac{z_0^2}{2}} \quad ,$$

as results from Eq. (8) when taking its limit as $t \to \infty$. Our iterative procedure to evaluate the effect of space charge simply consists in using these evolutions back in the single particle equation of motion, find new solutions for a set of initial conditions and thus recalculate the integrals (5). After only two iterations the method converges to the evolutions depicted in Figs. 2 and 3 (red lines). If we evaluate the Σ factor for the parameters in Table I, we easily find out that it amounts to 0.2 compared to a kick phase of about $\pi/7$. Therefore we expect a similar evolution to that of the experiment described in Sec. I. As predicted by the above criterion (3), the damping of the centroid oscillation is no longer to be observed. The bunch longitudinal rms-size still increases even if its growth seems to stop at a lower level than without space charge.

The effect of space charge on longitudinal quadrupole oscillations can also be studied with our method. To excite

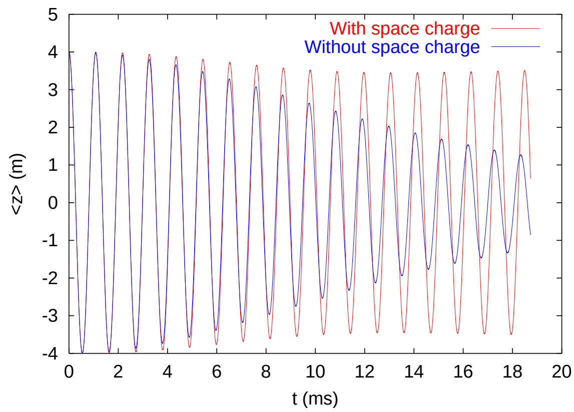

Figure 2: Decoherence of the centroid motion for a longitudinally displaced bunch due to the sinusoidal bucket nonlinearity with (red) and without (blue) space charge effects.

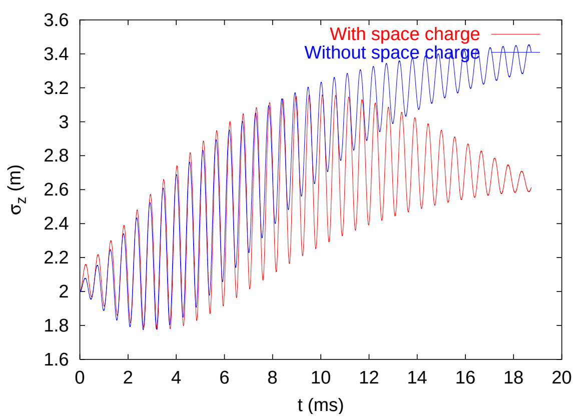

Figure 3: Bunch rms-size evolution for a longitudinally displaced bunch due to the sinusoidal bucket nonlinearity with (red) and without (blue) space charge effects.

a pure quadrupole oscillation, we only need to unmatch the bunch and set the initial longitudinal offset of the bunch, z_0, to 0.

In this case, no centroid oscillation will be observed. The σ_z will still evolve according to Eq. (8), where this time it will be $\bar{z}(t) = 0$ and, because of the unmatched situation, σ_{z0}^2 on the first line must be replaced by $\sigma_{z0}^2/2 + \eta^2 c^2 \sigma_{\delta 0}^2/(2\omega_{s0}^2)$. The evolution of the bunch rms-size is depicted in Fig. 4 (blue line). We have chosen to simulate an SIS bunch with a momentum spread which is scaled by a factor 0.8 with respect to the value reported in the Table I (matched value). A smaller momentum spread than the matched value would cause the bunch to shrink initially and then oscillate around the new $\sigma_z^{(match)} = \sigma_{\delta 0}|\eta| c/\omega_s = 0.8\sigma_{z0}$ if no longitudinal detuning were included in the analysis. It is clear that, owing to the bucket nonlinearity, decoherence appears in the quadrupole oscillation, too. The asymptotic value of the bunch rms-size, which is eventually reached after the oscillation at twice the synchrotron frequency has fully died out, will be:

$$\sigma_z(t \to \infty) = \sqrt{\frac{\sigma_{z0}^2}{2} + \frac{\eta^2 c^2 \sigma_{\delta 0}^2}{2\omega_{s0}^2}}$$

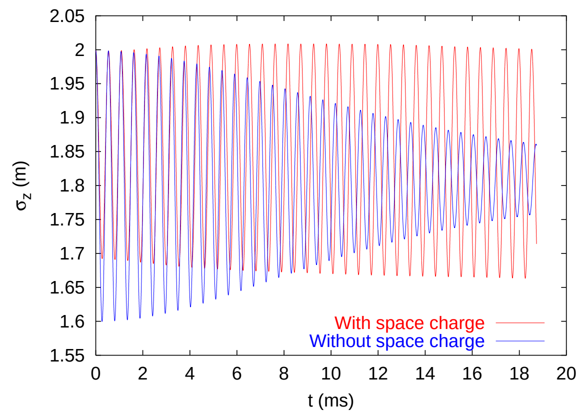

Figure 4: Bunch rms-size evolution for a longitudinally unmatched bunch due to the sinusoidal bucket nonlinearity with (red) and without (blue) space charge effects.

Just like in the case of the pure dipole oscillation, space charge causes the quadrupole oscillation to be undamped. Figure 4 (red line) shows the persistent oscillation as evaluated with the iteration method.

To cross-check the validity of the obtained analytical expressions as well as of the iterative procedure which we have used above, we have also carried out macroparticle simulations using the HEADTAIL code [4] and Vlasov simulations. The results in terms of bunch centroid and rms-size evolution agree very well with the iterative procedure.

CONCLUSIONS

Below transition even relatively low space charge supresses the decoherence of longitudinal dipole oscillations. Therefore, in contrast to the recently described collective coasting beam echoes [5], collective bunched beam echoes might be difficult to observe.

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