STUDY OF A LOW IMPEDANCE BEAM POSITION MONITOR FOR SHORT BUNCHES

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Abstract

A low coupling impedance beam position monitor based on microwave spectroscopy is proposed. A coaxial cavity is coupled to a circular beam pipe through four small longitudinal slots. Displaced beams excite TE cavity modes that can be detected by two antennas. Applying the modified Bethe's theory, the TE₁₁₁ mode power has been derived as a function of the beam displacement. An aluminium prototype has been constructed with the support of numerical (HFSS) simulations. Bench measurements using a coaxial wire excitation show good agreement with both theoretical and numerical expectations.

1 INTRODUCTION

Different methods to measure beam position inside the pipe have been proposed either in the time domain than in the frequency domain. In this paper we describe a beam position monitor (b.p.m.) that measures the beam position detecting the resonant field excited by the bunch train in a coaxial cavity. The coaxial cavity is coupled to the beam pipe through four identical slots (see Fig. 1) having the longer side (*l*) in the beam direction that assure a small coupling impedance of the whole device [1]. The length of the cavity is chosen in order to excite a transverse TE mode. The beam position inside the pipe can be calculated by probing the field in the cavity with a small antenna.

We will describe and compare three different methods to evaluate the signal in the cavity as a function of the beam displacement. The first one is an analytic approach based on the modified Bethe's theory [2]. The second one is based on HFSS [3] simulations and the third one is based on the wire measurements method [4] made on an aluminium prototype.

2 ANALYTICAL APPROACH RESULTS

A sketch of the beam position monitor is shows in Fig.1 with the dimensions of the prototype shown on Table 1.

Considering a bunch train traveling off-axis at a distance r₀ with respect to the center of the beam pipe, the total density current in z=0 can be expressed by the summation:

$$J(t) = \left[\sum_{k=-\infty}^{+\infty} I(t - kT_0)\right] \frac{\delta(r - r_0)}{\pi r_0} \sum_{m=0}^{\infty} \frac{\cos[m(\varphi - \varphi_1)]}{1 + \delta_{m0}} \tag{1}$$

where I(t) is the current distribution of the single bunch, T_0 is the bunches spacing and c is the velocity of light. The spectrum of the beam current is:

$$\sum_{k=-\infty}^{+\infty} I(t - kT_0) = I_0 + \sum_{p=1}^{+\infty} \widetilde{I}_p \cos(p\omega_0 t)$$
 (2)

where $\omega_0 = 2\pi/T_0$, I_0 is the average beam current and \tilde{I}_p is the amplitude of the Fourier component at $p\omega_0$ [5]. By assuming a Gaussian bunch distribution we obtain:

$$I(t) = \frac{I_0 T_0 c}{\sqrt{2\pi} \sigma_z} e^{-\frac{1}{2} \left(\frac{z}{\sigma_z}\right)^2} \Rightarrow \widetilde{I}_p = 2I_0 e^{-\frac{1}{2} \left(\frac{\sigma_z}{c}\right)^2 p^2 \omega_0^2}$$
(3)

where σ_z is the rms bunch length.

If a dipole mode of the cavity is tuned in order to resonate at one of the frequencies of the beam power spectrum lines (\widetilde{I}_n) , the average powers extracted by two probes coupled with the two azimuthal polarities (φ_0) and $\varphi_{\pi/2}$ can be expressed as a function of the beam current and transverse displacement in the general form:

$$\begin{cases}
P^{(0)} = \underline{P}(\omega_r, r_0, \varphi_1) \widetilde{I}_n^2 \\
P^{(\pi/2)} = \underline{P}(\omega_r, r_0, \varphi_1 - \pi/2) \widetilde{I}_n^2
\end{cases}$$
(4)

Knowing by theory, simulations or calibrating measurements the coefficient \underline{P} , the previous system of equations (for a given power spectrum lines \widetilde{I}_n) allows to calculate, in principle, the values of r_0 and φ_n .

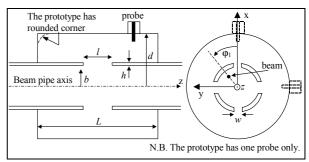


Figure 1: sketch of the beam position monitor.

Table 1: prototype dimensions (mm)

0	d 30 b 10 L 52 w 2	
	d	30
	b	10
	L	52
	w	2
	l	5
	h	1

The TE₁₁₁ mode with the polarity φ_0 , according to the modified Bethe's theory, dissipates an average power given by [2]:

$$P_{Th.}^{(0)} = \frac{1}{2} \frac{R}{Q_0} \cos(\varphi_1)^2 r_0^2 \widetilde{I}_n^2$$
 (5)

where R/Q is given by:

$$\frac{R}{Q} = \frac{4\omega_r \mu A^2 \alpha_E^2}{\left(1 - 2\alpha_E (1 + Q)A^2\right)^2 + \left(1 - 2\alpha_E A^2\right)^2}$$
(6)

where ω_r and Q_0 are the resonant angular frequencies and the quality factor of the TE₁₁₁ mode, α_E is the electrical polarizability of the single slot and the A coefficient is given by the following integral:

$$A = \frac{\omega_r L \widetilde{J}_1(b)}{b\pi c} \left[\int_b^d \frac{L\pi}{2} \left(k_t^2 \left(\widetilde{J}_1^* \right)^2 r + \frac{\left(\widetilde{J}_1^* \right)^2}{r} + \frac{k_t^4 L^2}{\pi^2} \left(\widetilde{J}_1^* \right)^2 r \right) dr \right]^{\frac{1}{2}}$$
(7)

with $\widetilde{J}_1(r) = J_1(k_r r) - Y_1(k_r r) J_1(k_r b) / Y_1(k_r b)$, the prime denote the total derivative with respect to the argument and $k_t = \sqrt{(\omega_r/c)^2 - (\pi/L)^2}$.

Considering the prototype dimensions, one obtains the values of ω_r , Q_0 and R/Q reported in Table 2 (first column).

The value of the normalized power $P_{Th.}^{(0)}/Q_0 \tilde{I}_n^2$ as a function of r_0 for $\varphi_1 = 0$ is plotted in Fig. 2 and compared with the simulation and measurement results (described in the next section). In Fig. 3 the same quantity is plotted as a function of φ_1 for r_0 =0.6 mm.

Table 2: theory, simulations and measurements results

	THEORY	HFSS	MEAS
ω _r [GHz]	3.78	3.74	3.74 (tunable)
Q_0	9600	9100	3500
\mathbf{R}/\mathbf{Q} [m Ω]	36	31	39

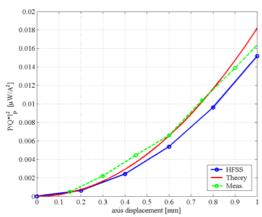


Figure 2: $P^{(0)}/O\tilde{I}_{n}^{2}$ as a function of r_{0} for $\varphi_{0}=0$.

3 SIMULATION RESULTS

To compare the analytical results with the numerical simulations, we have studied the coupling impedance of the cavity for the TE_{111} mode as a function of the transverse displacement r_0 and φ_l . We have, therefore, calculated the longitudinal shunt impedance of the mode (R_s) for different axial positions, using the Eigenmode Solver of HFSS.

The HFSS model is shown in Fig. 4 with the obtained E field lines. It is only 1/8 of the structure with proper

magnetic and electric boundary conditions on the symmetry planes. The average dissipated power in the cavity corresponding to the TE_{111} mode is given by:

$$P_{Sim.}^{(0)} = \frac{1}{2} R_s(\varphi_1, r_0) \widetilde{I}_n^2$$
 (8)

The value of $P_{Sim.}^{(0)}/Q\widetilde{I}_n^2$ is plotted in Figs. 2-3 and compared with the analytical and measurement results. From the previous plot it is possible to verify that it is possible to write the eq. (8) in the form:

$$P_{Sim.}^{(0)} \cong \frac{1}{2} \frac{R}{Q} Q_0 \cos(\varphi_1)^2 r_0^2 \widetilde{I}_n^2$$
 (9)

The values of ω_r , Q_0 and R/Q obtained by the simulations are reported in Table 2 (second column).

If the cavity is coupled through a small probe to an external load, then the average dissipated power in the system is still given by eq. (5) or (9) but with Q_0 replaced by the loaded Q_L . The average power dissipated in the load ($P_{Ext}^{(0)}$) is related to the power dissipated in the cavity by:

$$P_{Eyr}^{(0)} = P_{cov}^{(0)} \beta / (1 + \beta) \tag{10}$$

where β is the coupling coefficients between the probe and the cavity mode TE₁₁₁ and it can be obtained from reflection measurement at the probe port.

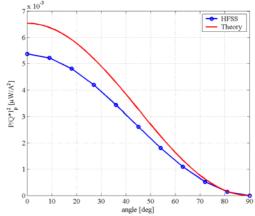


Figure 3: $P^{(0)}/Q\tilde{I}_{p}^{2}$ as a function of φ_{1} for $r_{0}=0.6$ mm.

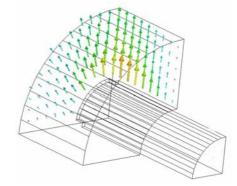


Figure 4: HFSS model with E field lines.

4 PROTOTYPE MEASUREMENTS

Wire measurements have been performed on the aluminum prototype shown on Fig. 5. An antenna probes the signal in the cavity.

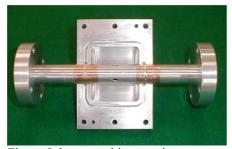
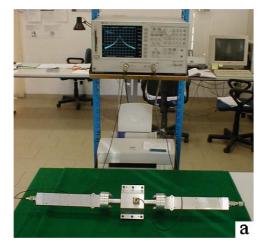


Figure 5: beam position monitor prototype.



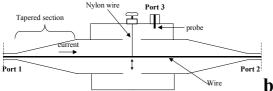


Figure 6: measurement setup

The measurements setup is shown Fig. 6a and schematically represented in Fig 6b. The wire has a radius of 1.5 mm. In order to minimize reflections at the ports 1 and 2 we have inserted two tapered sections that match the 50 Ω impedance of the Network Analyzer with the ~114 Ω impedance of the wire in the beam pipe. To excite the dipolar mode TE₁₁₁ the wire inside the beam pipe has been properly displaced from the axis of the beam pipe. To this purpose, a thin nylon wire has been connected to the wire in order to displace it from the beam pipe axis in a controlled way and to excite the polarity φ_0 . The measured transmission coefficient S₂₁ and S₃₁ are shown in Fig. 7 for a displacement of the wire of ~1.5 mm.

The average dissipated power in the cavity is given by:

$$P_{cav}^{(0)} = P_{Ext.}^{(0)} \frac{(1+\beta)}{\beta} \cong \frac{1}{2} Z_c \frac{|S_{31}(\omega_r)|^2}{|S_{21}(\omega_r)|} \frac{(1+\beta)}{\beta} \widetilde{I}_n^2$$
 (11)

The plot of $P_{cav}^{(0)} / Q\widetilde{I}_p^2$ as a function of r_0 is reported in Fig. 2. It is clear that $P_{cav}^{(0)}$ can be written in the form (5) or (9) and the equivalent R/Q is reported in Table 2. The Q factor of the resonant mode is lower than the theoretical one because of RF losses in the mechanical contacts. Similar considerations than those developed in [6] can be

done from the point of view of the induced errors in the measurement setup.

By the system of equations (4) it is possible to calculate the position of the bunches in the beam pipe when the b.p.m is inserted in the accelerator, by the formulae:

$$\begin{cases}
r_0 = \frac{\sqrt{P_{Ext.}^{(\pi/2)} / \underline{P} + P_{Ext}^{(0)} / \underline{P}}}{\widetilde{I}_n} \\
\varphi_1 = \pm \arctan\left(\sqrt{\frac{P_{Ext.}^{(\pi/2)}}{P_{Ext.}^{(0)}}}\right) \pm \pi
\end{cases}$$
(12)

where the calibration coefficient P is given by:

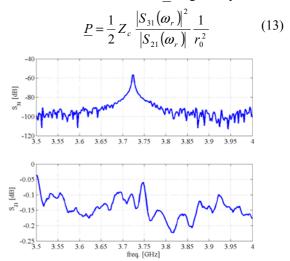


Figure 7: measured transmission coefficients (S₁₂, S₃₁).

5 CONCLUSIONS

A novel bunch position monitor has been described. Measurements on an aluminium prototype have been compared to the theoretical and numerical expectations of the power dissipated by the first dipolar mode in the cavity. The compare shows a good agreement between theory and simulations expecially considering the mechanical differencies between the prototype and the ideal structure. These results confirm the potential application of this device as a beam position monitor. The very low coupling impedance of the device and the possibility of calibration by simple wire measurements make the device hopefully usable in the accelerator machines.

6 REFERENCES

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